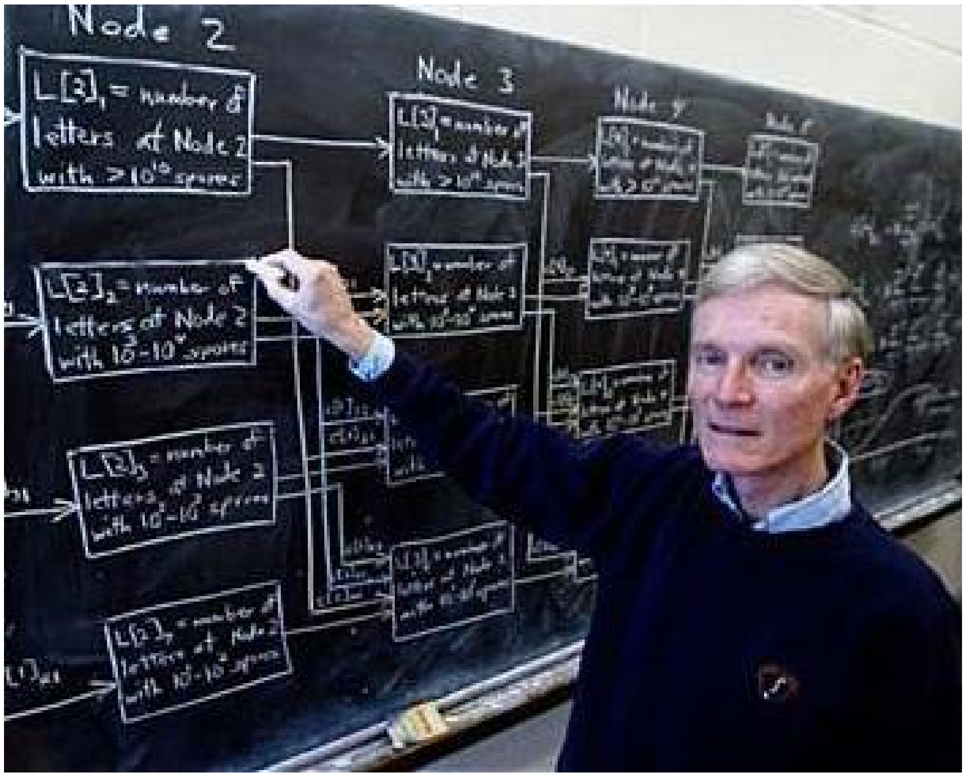
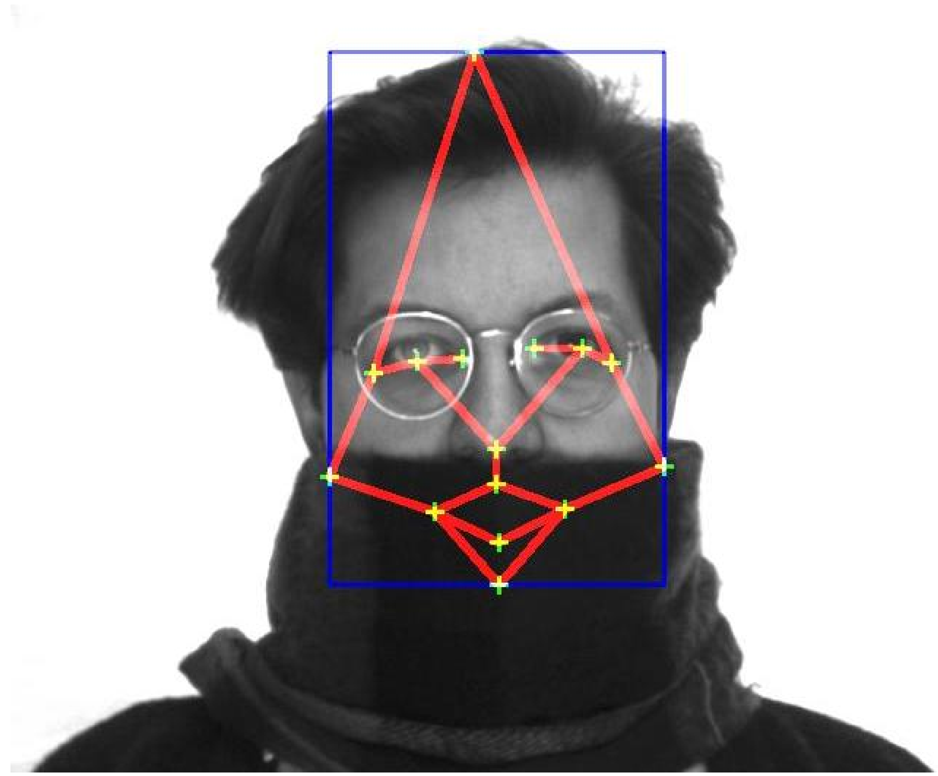
Mathematics in a Dangerous Time





Applied Math Workshop, UIUC, May 17, 2003

Douglas N. Arnold, Institute for Mathematics & its Applications

Institute for Mathematics and its Applications

The breaking of the German enigma code by Polish and British mathematicians (Rejewski, Turing, . . .) and the breaking of the Japanese PURPLE code by the Americans shortened the war by ≈ 2 years.

WW II cryptography

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Far from over: NSA is the largest single employer of mathematics Ph.D.s in the world, hiring 40–60 per year.

DES was broken in 2000, AES is not provably secure.

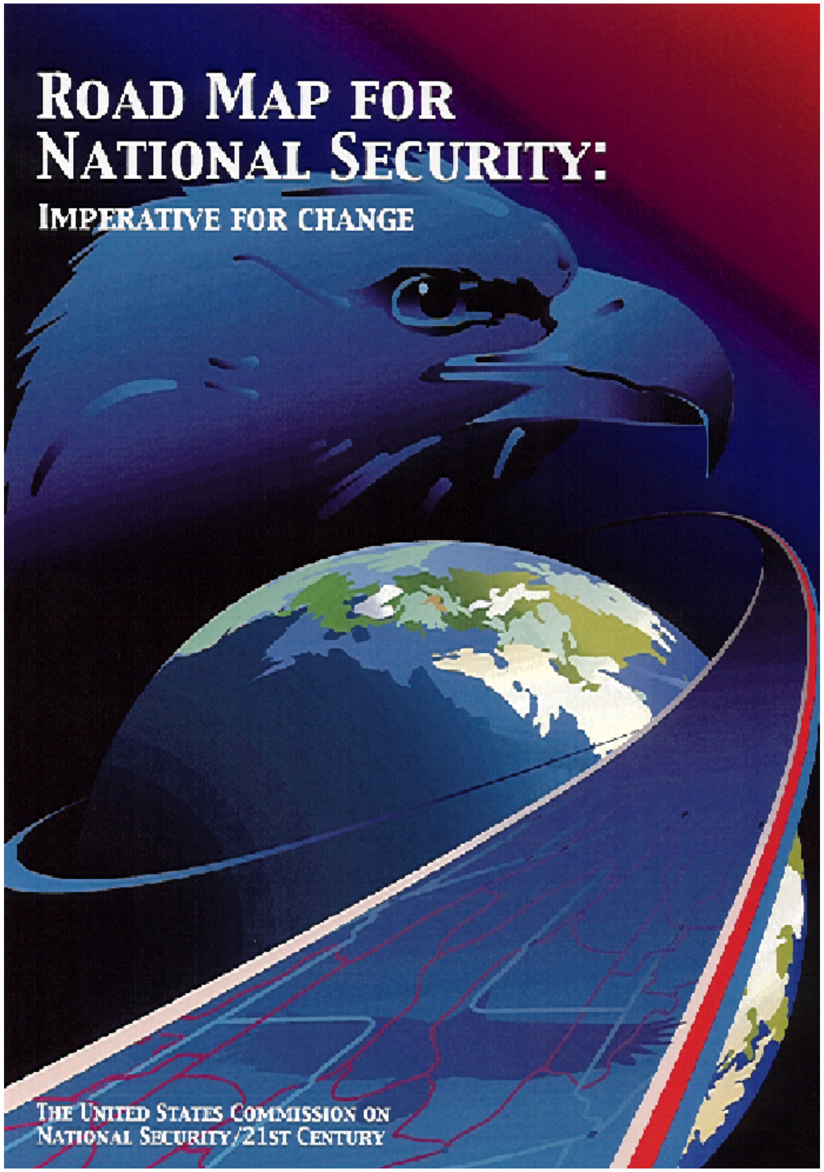
Will quantum cryptography and quantum factorization could change the landscape?

Some other mathematicians in U.S. war effort

Norbert Weiner
George Dantzig
James Givens
Richard Hamming
Alston Householder
John Kemeny
Cornelius Lanczos
Norbert Weiner
Garrett Birkhoff
Witold Hurewicz
Cathleen Morawetz
Olga Taussky-Todd
Nathan Jacobson
J. Ernest Wilkins

Peter Lax
John von Neumann
Isaac Schoenberg
John Synge
Stanislaw Ulam
John Crank
James Wilkinson
James Alexander
Daniel Gorenstein
Solomon Lefschetz
Marshall Stone
Paul Halmos
John Tukey
Nicholas Metropolis

February 2001: Hart–Rudman report



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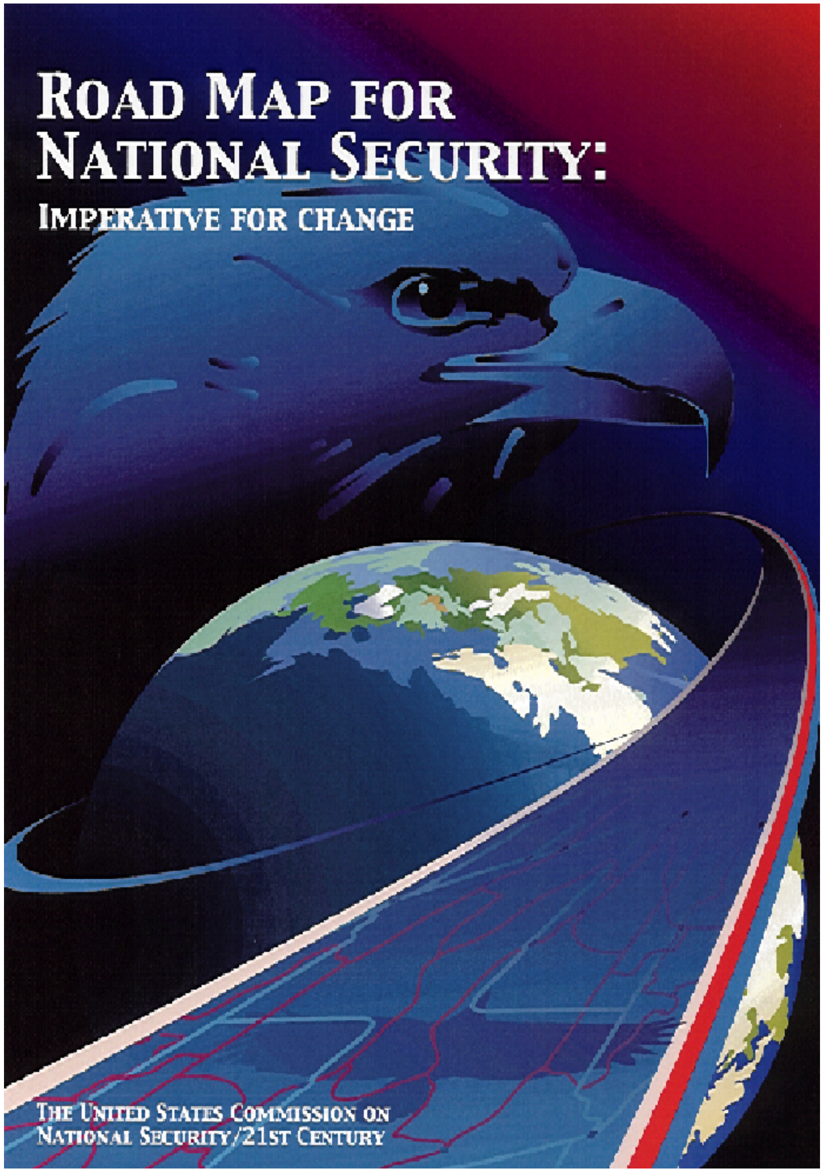

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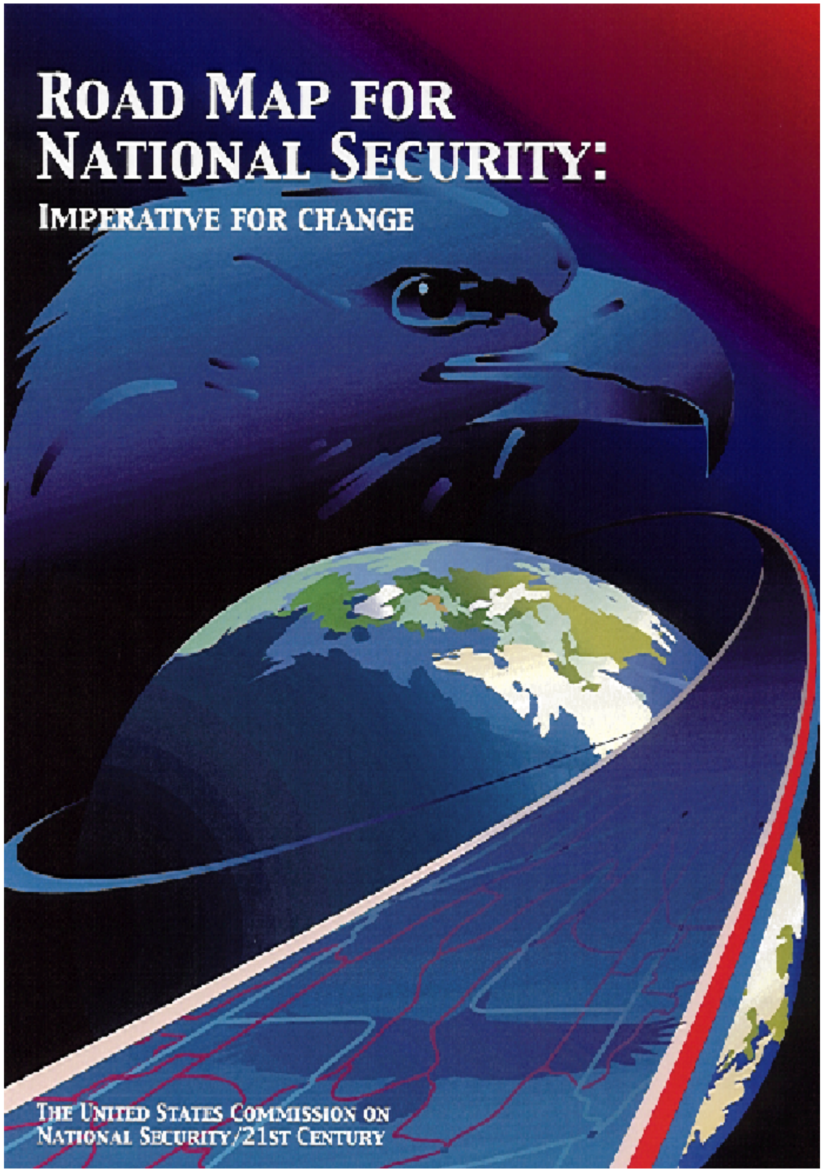

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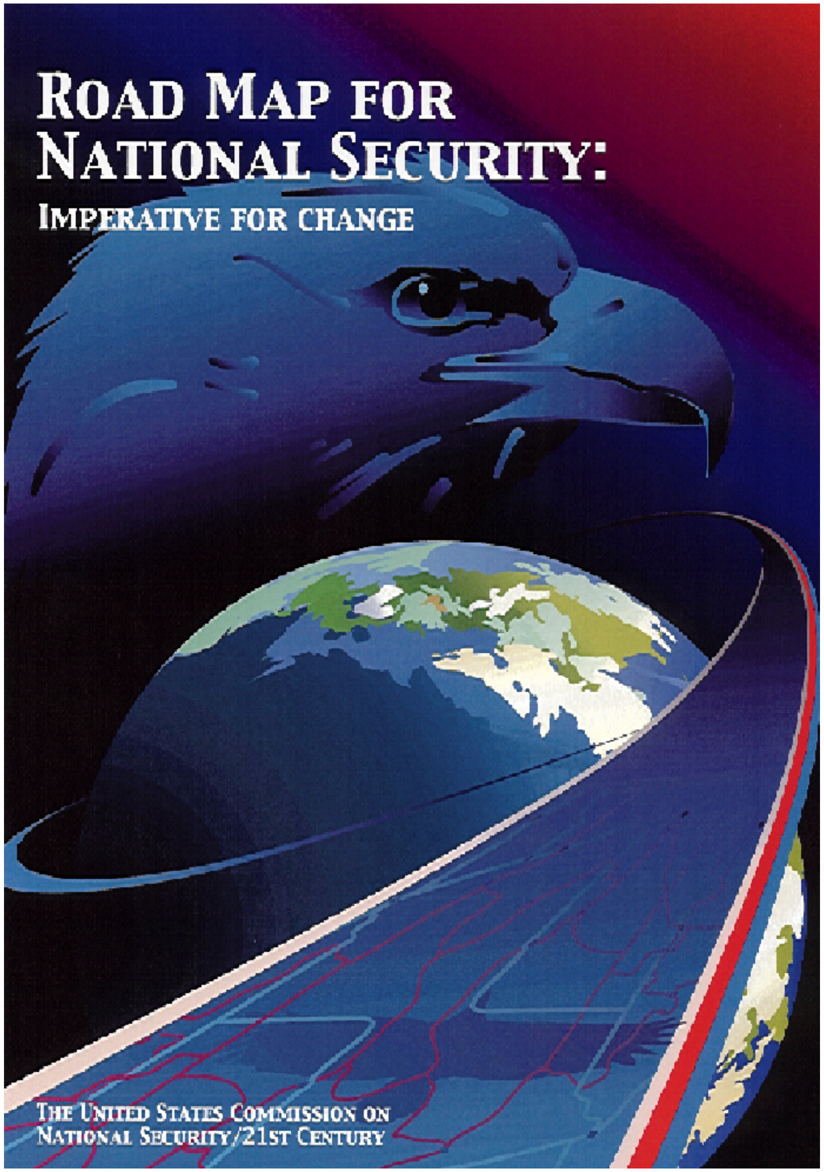

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The President should propose, and the Congress should support, doubling the U.S. government's investment in science and technology research and development by 2010.

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September 2001

May 2002: NSF/MPS responds to Hart–Rudman

White paper from MPS Advisory Committee
(W. Pulleyblank, chair)

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- *The MPS Directorate should take actions that will support the formation and maintenance of an active, national community involved in carrying out research in areas relevant to homeland security.*

November 2002: 3-day NSF MPS/IC workshop: Approaches to Combat Terrorism: Opportunities for Basic Research

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- supplements to existing grants
- proposals for Small Grants for Exploratory Research
- proposals for workshops

deadline: July 17

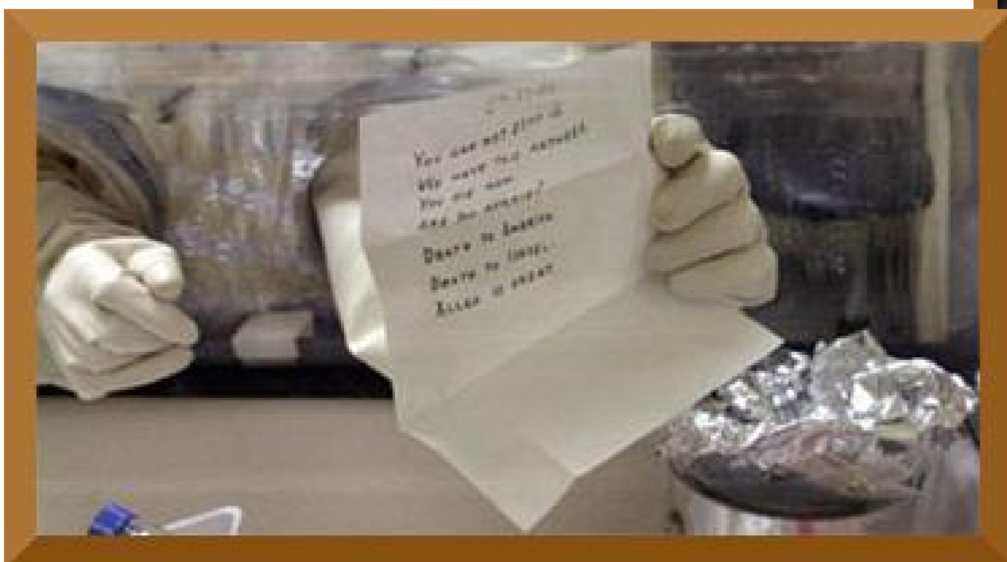
Some relevant meetings

- March 2002: DIMACS workshop: Mathematical Sciences Methods for the Study of Deliberate Releases of Biological Agents and their Consequences
- April 2002: BMSA meeting: The Mathematical Sciences' Role in Homeland Security
- September 2002: IMA workshop: Operational Modeling and Biodefense: Problems, Techniques, and Opportunities

An example: bioterrorism preparedness

How do we prepare our response to the deliberate release of pathogens?

Our plans effect the probability of an event (deterrence) as well as the consequences.



Aspects of bioterror preparedness

- Intelligence, prediction, and detection

- Planning and policy

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  - Evacuation protocols.
  - Infrastructure modifications. airport, road closures; postal system, communication networks. . .

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- Intelligence, prediction, and detection
  - data mining, information integration
  - sensor design
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    - harmonic analysis
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    - all of the above
    - control theory
    - optimization, OR
    - game theory

www.pnas.org/cgi/doi/10.1073/pnas.162282799 PNAS | August 6, 2002 | vol. 99 | no. 16 | 10935–10940

Emergency response to a smallpox attack: The case for mass vaccination

Edward H. Kaplan*†, David L. Craft‡, and Lawrence M. Wein§

*Yale University School of Management, and Department of Epidemiology and Public Health, Yale School of Medicine, New Haven, CT 06520-8200; and ‡Operations Research Center and §Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA 02139-3407

Edited by Burton H. Singer, Princeton University, Princeton, NJ, and approved June 11, 2002 (received for review May 10, 2002)

In the event of a smallpox bioterrorist attack in a large U.S. city, the interim response policy is to isolate symptomatic cases, trace and vaccinate their contacts, quarantine febrile contacts, but vaccinate more broadly if the outbreak cannot be contained by these measures. We embed this traced vaccination policy in a smallpox disease transmission model to estimate the number of cases and deaths that would result from an attack in a large urban area. Comparing the results to mass vaccination from the moment an

breaks. Surely strategies that contain smallpox epidemics under the free mixing assumption will also work under mixing patterns less favorable to the spread of disease. In this sense, our approach is to plan for the worst but hope for the best.

Absent intervention, each infectious individual generates an average of $R_0 = \beta S^0(0)/r_3$ infections early in the epidemic (8), where $S^0(0)$ is the number of susceptibles immediately after an attack at time $t = 0$, and β is the effective disease transmission

Queueing States. Until $t = 0$, there is no tracing and hence no queuing. Once $t > 0$, note that because only n tracers/vaccinators are available, the total flow out of the queuing states can never exceed $n\mu$ per day. If the system becomes congested (more than n persons are in the queue), then those in queue in disease stage j receive service at rate $n\mu Q_j/Q$, that is, the service provided is proportional to the relative numbers in queue. This explains the $\min(1, n/Q)$ in Eqs. **5–7**. Disease transmission and progression continue unabated among those in the queue.

$$\frac{dQ_0}{dt} = [c - pR_0(t)]\,\frac{S^0}{N}\,r_3 I_3 - \beta I_3 Q_0 - \mu Q_0 \min(1, n/Q) \qquad [5]$$

$$\frac{dQ_1}{dt} = \beta I_3 Q_0 + \left\{[c - pR_0(t)]\,\frac{I_1^0}{N} + p\lambda_1(t)\right\} r_3 I_3 - \mu Q_1 \min(1, n/Q) - r_1 Q_1 \qquad [6]$$

immune. However, the fraction δ of those who develop smallpox die of the disease, whereas the fraction f of all those vaccinated die of vaccine-related complications.

$$\frac{dZ}{dt} = (1 - f)(v_0 Q_0 + v_1 Q_1)\mu \min(1, n/Q) + (1 - \delta) r_4 (I_4^0 + I_4^1) \qquad [14]$$

$$\frac{dD}{dt} = f\mu Q \min(1, n/Q) + \delta r_4 (I_4^0 + I_4^1). \qquad [15]$$

The Functions $R_0(t)$ and $\lambda_j(t)$. When a newly symptomatic smallpox case is discovered at time t, she will have infected on average $R_0(t)$ persons over her duration of infectiousness, where

$$R_0(t) = \int_0^{t+\tau} e^{-r_3 x}\beta[S^0(t - x) + Q_0(t - x) + S^1(t - x)]\,dx$$

www.nytimes.com
The New York Times ON THE WEB

July 7, 2002

U.S. to Vaccinate 500,000 Workers Against Smallpox

By WILLIAM J. BROAD

The federal government will soon vaccinate roughly a half-million health care and emergency workers against smallpox as a precaution against a bioterrorist attack, federal officials said. The government is also laying the groundwork to carry out mass vaccinations of the public - a policy abandoned 30 years ago - if there is a large outbreak.

Until last month, officials had said they would soon vaccinate a few thousand health workers and would respond to any smallpox attack with limited vaccinations of the public. Since 1983, only 11,000 Americans who work with the virus and its related diseases have received a vaccination, according to the Centers for Disease Control and Prevention.

The plan to increase the number of "first responders" who receive the vaccination to roughly 500,000 from 15,000 and to prepare for a mass undertaking of vaccinations in effect acknowledges that the government's existing program is insufficient to fight a large outbreak.

Mathematical epidemiology

Mathematical epidemiology is a well-established field which has had a real effect in medicine and public health. The pre-emptive culling strategy the British used against foot-and-mouth disease was one example.

There are lots of books, conferences, even courses at the undergraduate level.

It divides the population into groups based on disease status, age, and other considerations, and models the transitions amongs these groups by differential equations, integrodifferential equations, etc. These are studied by analysis (dynamical systems, bifurcation theory) and simulations.

SIR model of mathematical epidemiology

Although Daniel Bernoulli published a mathematical study of smallpox spread in 1760, the precursor of modern mathematical epidemiology is the SIR model of Kermack and McKendrick from the 1920's:

$$\boxed{\text{S}} \longrightarrow \boxed{\text{I}} \longrightarrow \boxed{\text{R}}$$

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \gamma I, \quad \frac{dR}{dt} = \gamma I,$$

where $S + I + R = 1$ give the division of the population into susceptible, infective, and recovered segments, $\beta > 0$ the *infection rate*, $\gamma > 0$ the *removal rate*.

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Theorem. *Let* $S(0), I(0) > 0$, $R(0) = 1 - S(0) - I(0) \geq 0$ *be given. For the solution of the SIR model with* $S(0) > \gamma/\beta$, $I(t)$ *increases initially until it reaches its maximum value and then decreases to zero at* $t \to \infty$*. Otherwise* $I(t)$ *decreases monotonically to zero as* $t \to 0$*.*

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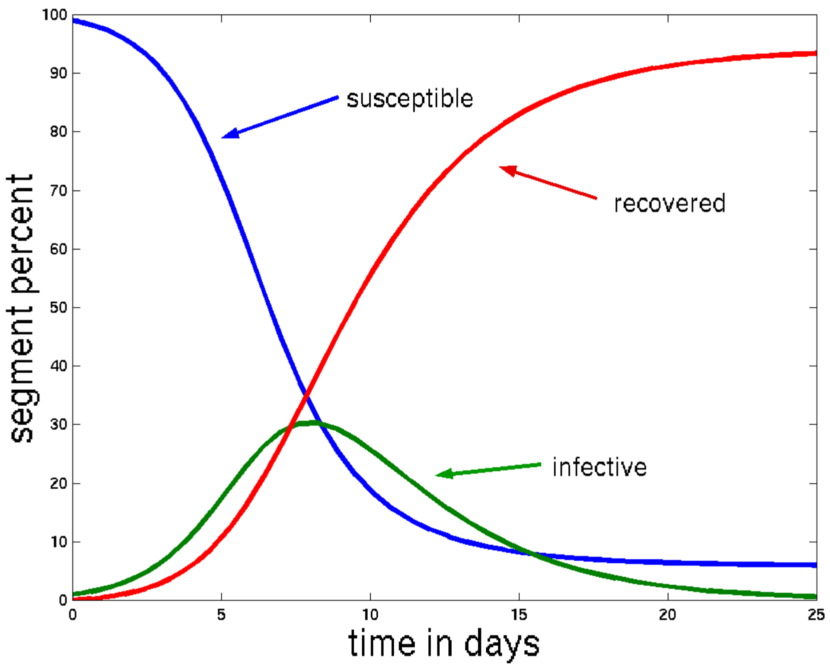

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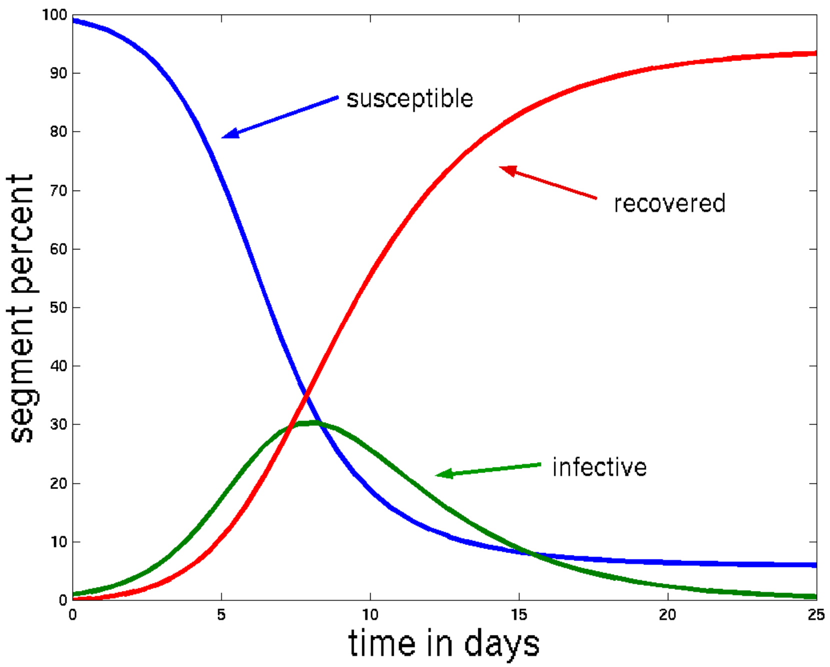

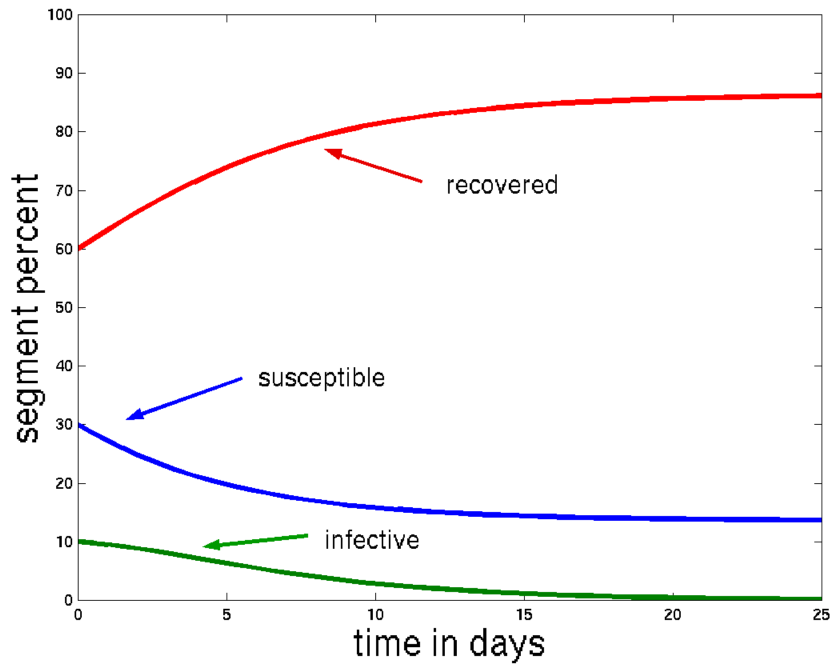

herd immunity

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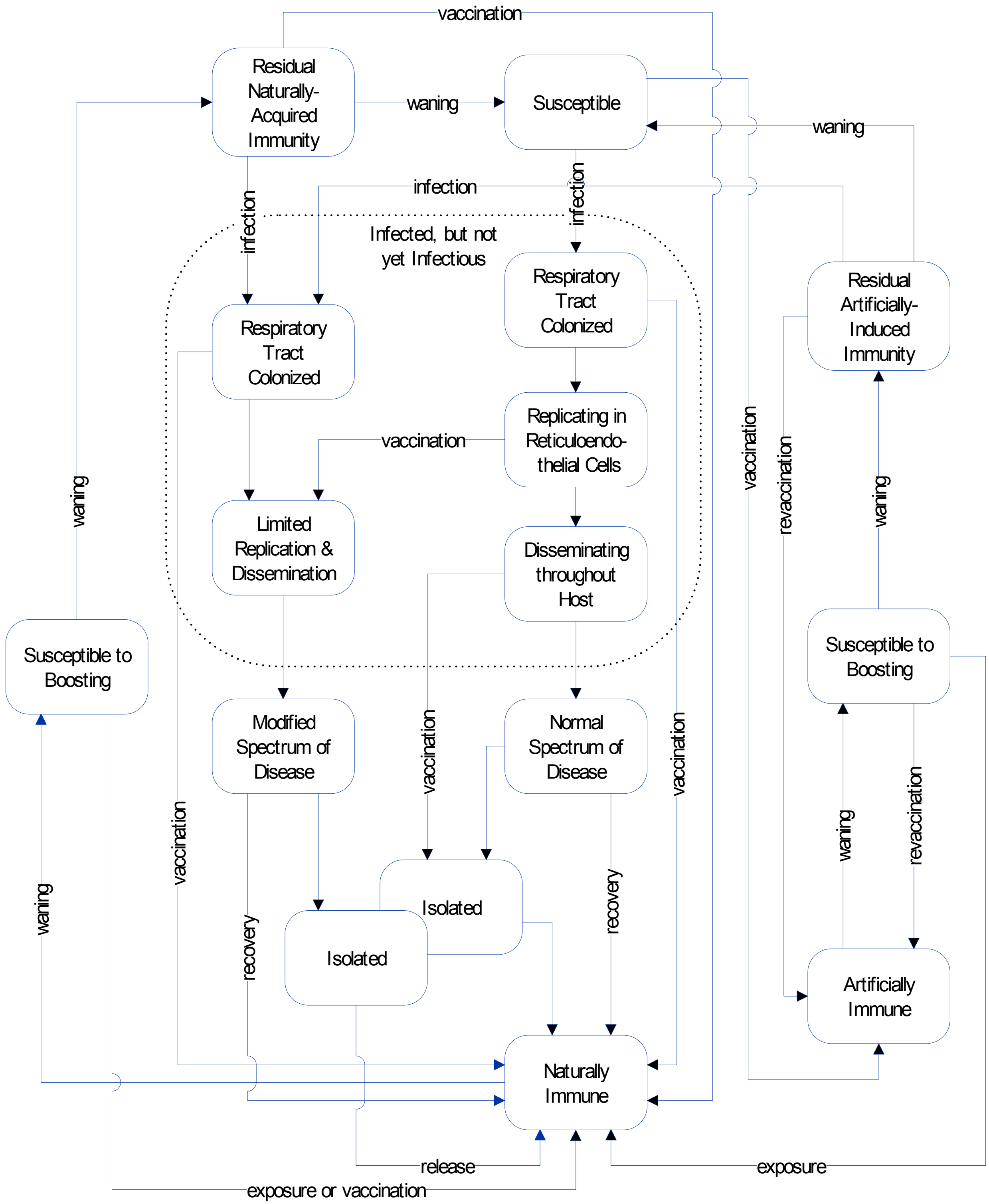

vaccination
Residual Naturally-Acquired Immunity
waning
Susceptible
waning
infection
infection
infection
Infected, but not yet Infectious
Respiratory Tract Colonized
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Residual Artificially-Induced Immunity
Replicating in Reticuloendo-thelial Cells
vaccination
Limited Replication & Dissemination
Disseminating throughout Host
vaccination
revaccination
waning
waning
Susceptible to Boosting
Susceptible to Boosting
Modified Spectrum of Disease
Normal Spectrum of Disease
vaccination
vaccination
vaccination
Isolated
Isolated
recovery
recovery
waning
revaccination
Artificially Immune
Naturally Immune
release
exposure
exposure or vaccination

Plague modeling at Dynamic Technology, Inc.

Multi-patch generalization of Keeling and Gilligan, 2000

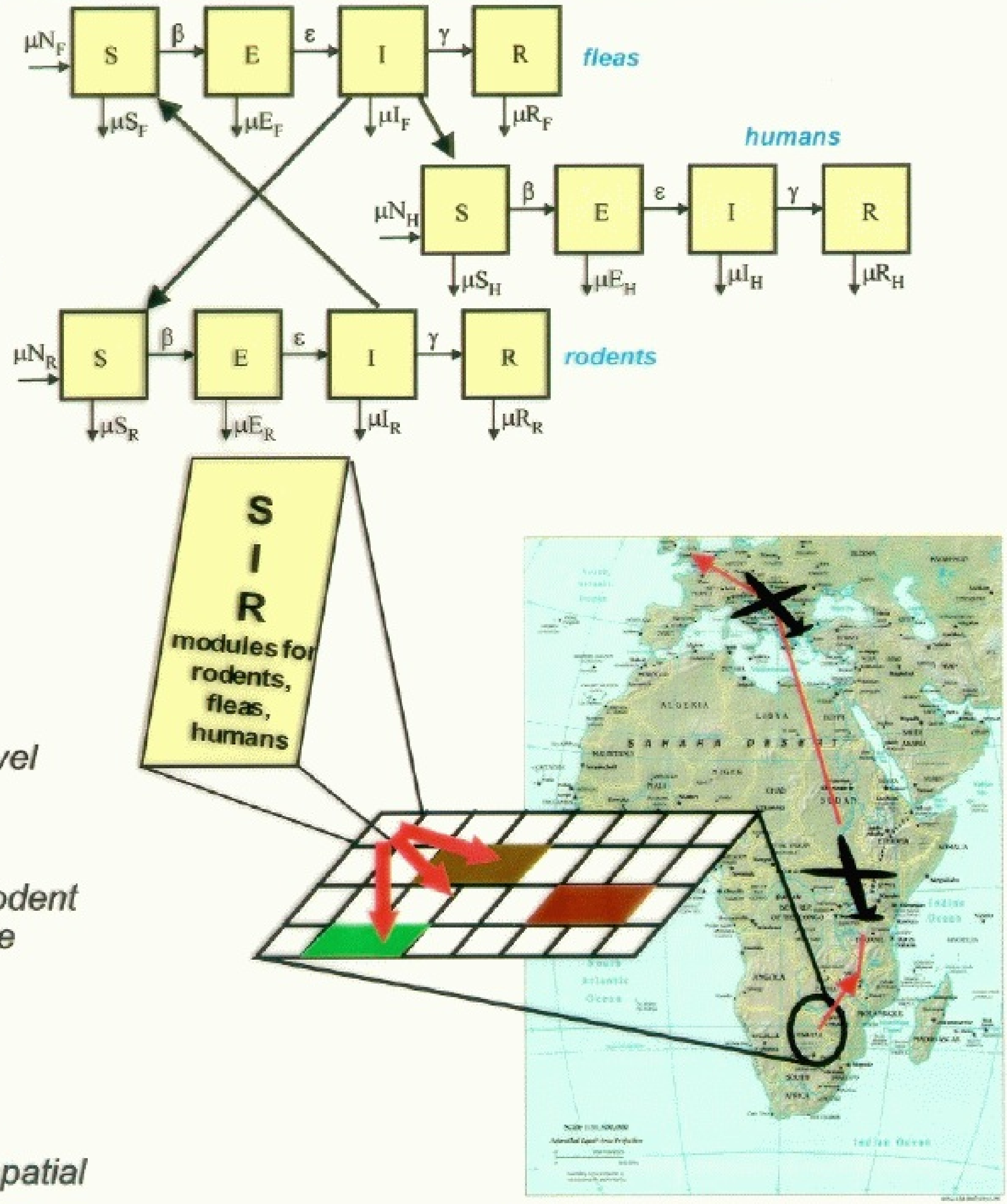
- *Treating patch-patch heterogeneity by Lloyd and May's (1996) approach*
- *Incorporating spatial spread (city-city to transnational) by modeling transportation networks, rates via Rvachev et al. (1977) approach*
- *Including human pneumonic transmission term*

Includes essential dimensions of plague epidemiology

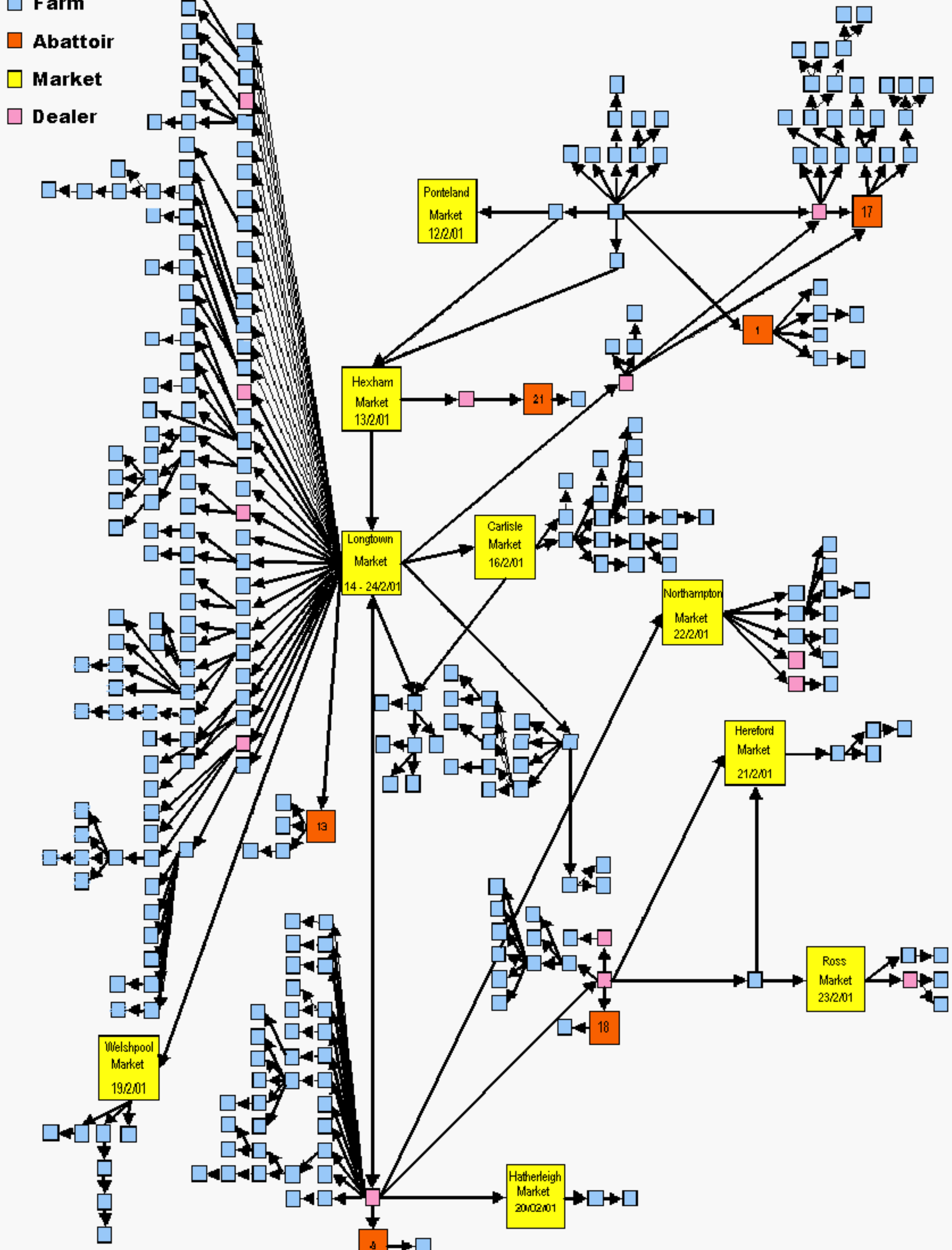
- *Human, rodent and flea interactions*
- *Patch-patch ecological variation*
- *Regional, national and international travel and migrations*
- *Climatology and meteorology*
- *Effects of vaccination, rodent control, rodent genetic resistance to Y. pestis, pesticide application, and others*

Evaluation to include

- *Single-patch incidence, prevalence, R_0*
- *Patch-patch disease propagation and spatial spread*



Links between FMD cases



FMD cases till 23/03/01 16:00 MAFF
NB links are subject to current information

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Agent-based modeling and simulation is a more recent approach which looks very promising. The mathematics of agent-based modeling (in all fields) is in its infancy with big potential.

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Data mining, information integration

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- Emerging area: privacy-preserving data mining.

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Data and the mathematical sciences

Data mining is the semi-automatic discovery of patterns, associations, changes, anomalies, rules, and statistically significant structures and events in data.

– Data Mining Research: Opportunities and Challenges

Mathematics is, almost by definition, concerned with finding structure and patterns in numerically represented sets.

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So data mining *is* math.

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statistical methods (clustering, Bayesian networks, classification, decision trees), discrete mathematics (graph theory, combinatorial optimization, boolean functions), algebraic methods (latent semantic analysis, principal component analysis, neural nets), pattern recognition (. . .), geometrical methods (low dimensional structures in high dimensional spaces), harmonic analysis, multiscale geometric analysis, machine learning, numerical analysis, high performance computing . . .

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Workshop on Data Mining for Counter Terrorism and Security

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- Scalable methods to warehouse disparate data sources
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- Data mining in the field of aviation security, port security, bio-security
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Almost all speakers came from CS departments.

Some new mathematics aimed at data mining

Peter Jones:
The Traveling Salesman Meets Large Data Sets

David Donoho:
High-Dimensional Data Analysis: The Curses and Blessings of Dimensionality

Ronald Coifman:
Harmonic Analysis on Data Sets

Tomasso Poggio & Steve Smale:
The Mathematics of Learning: Dealing with Data

Gunnar Carlsson, Persi Diaconis, Joshua Tenenbaum:
FRG on Topological Methods in Data Analysis

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Intelligence is acquired: You learn that both neighbors have at least one boy because you ask the left neighbor and you see one of the kids run out of the right neighbor's house.

What are the probabilities of one boy and one girl now?

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Left neighbor: $2/3$ Right neighbor: $1/2$

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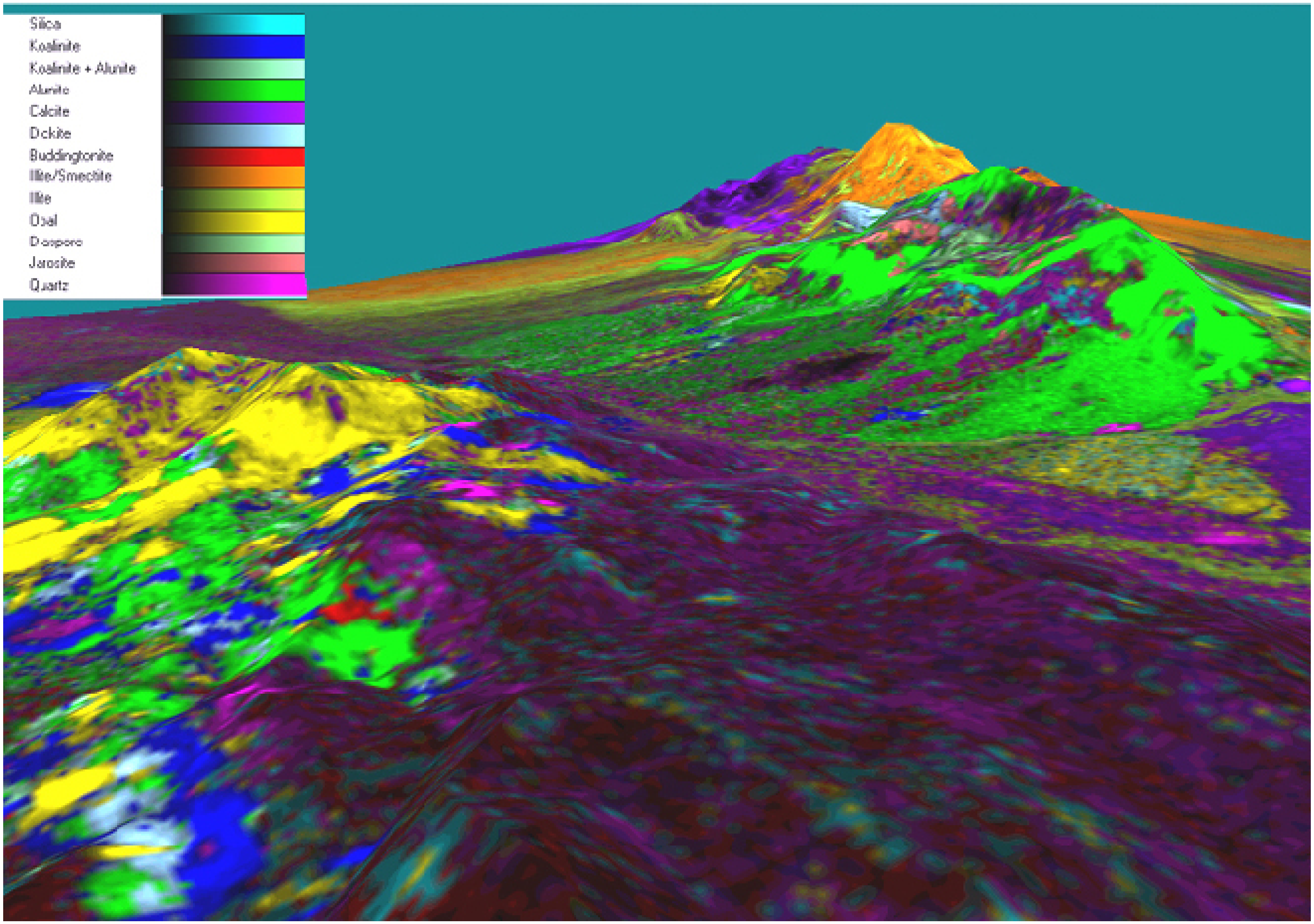
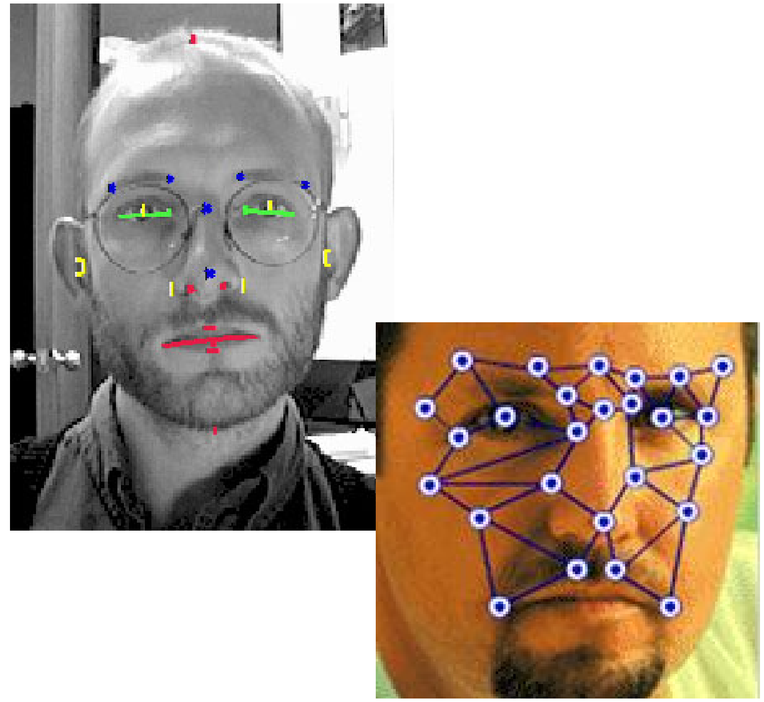
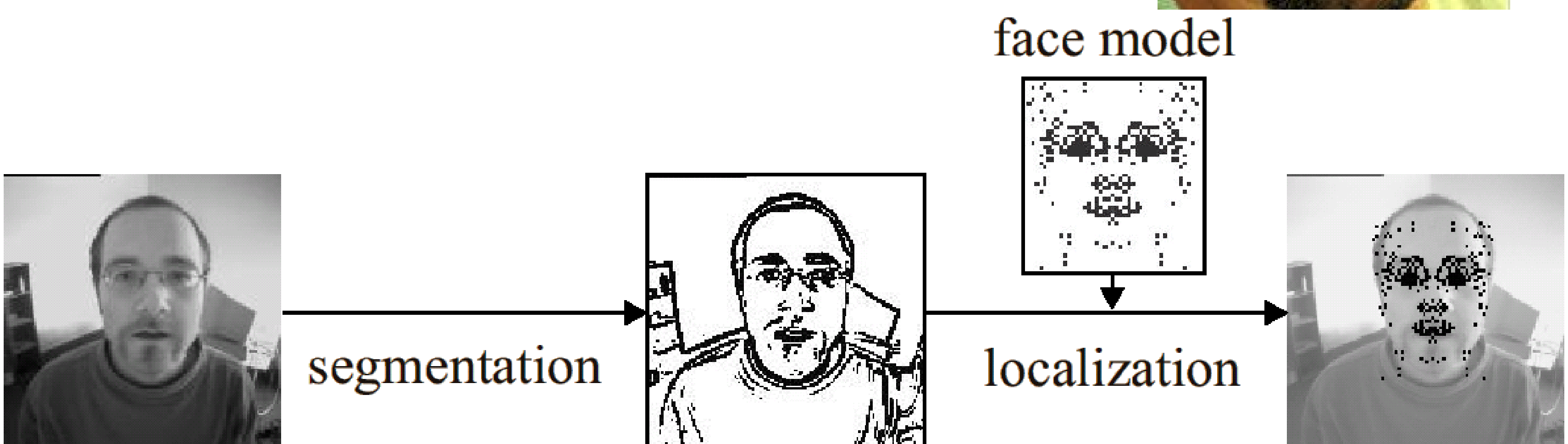
Imaging is an area where the unifying power of mathematics is evident. Imaging techniques developed independently by different disciplines turn out to be related or even identical, e.g., SAR and tomography.

Hyperspectral imaging



- Deformable templates
- Basis expansions, eigenfaces
- Statistical learning
- Multiscale representation
- Graph-based methods





Face detection and recognition

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- Basis expansions, eigenfaces
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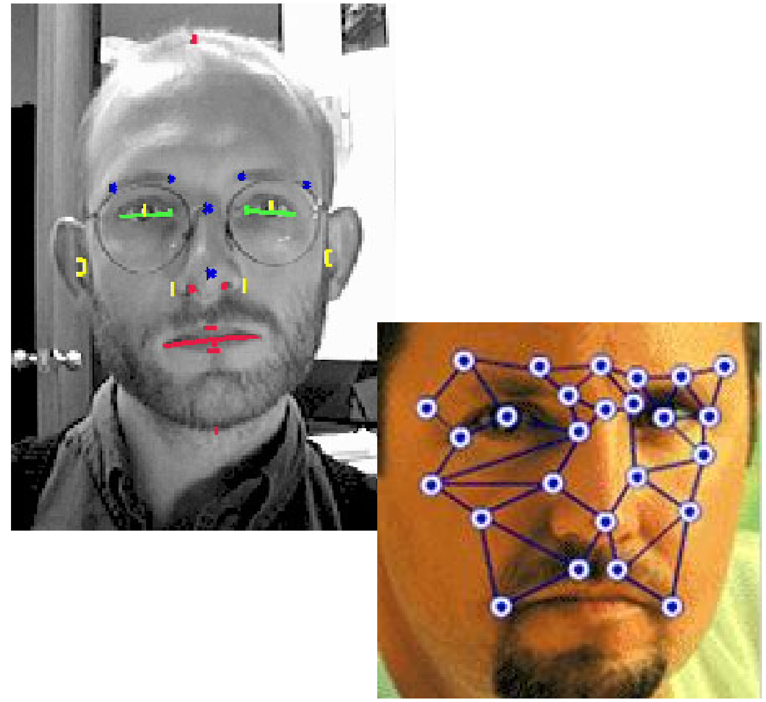
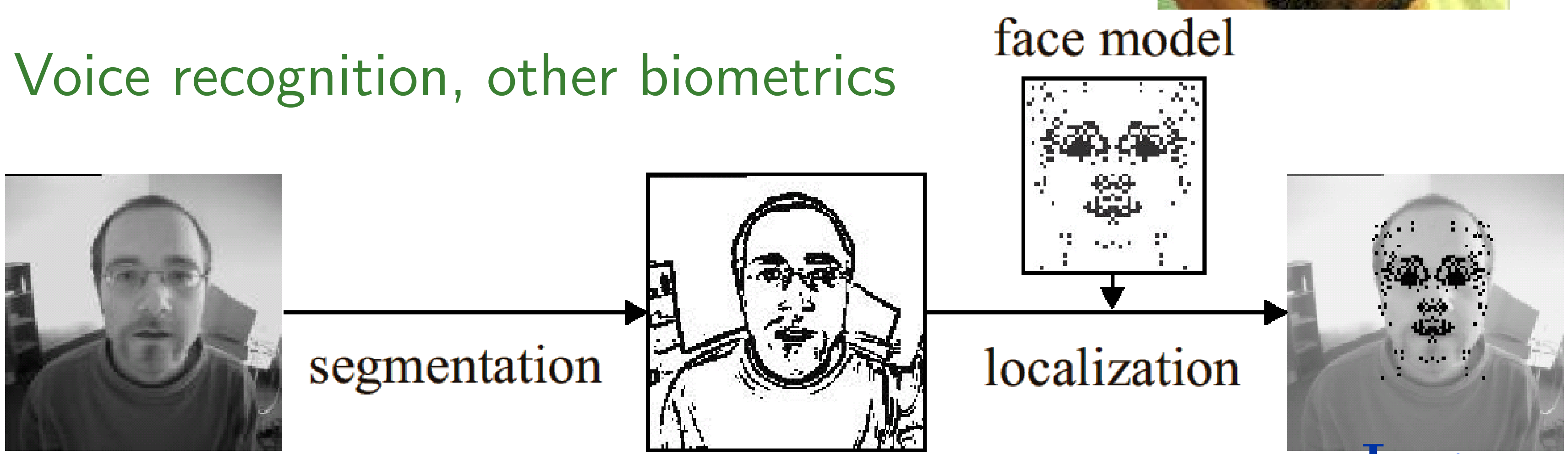
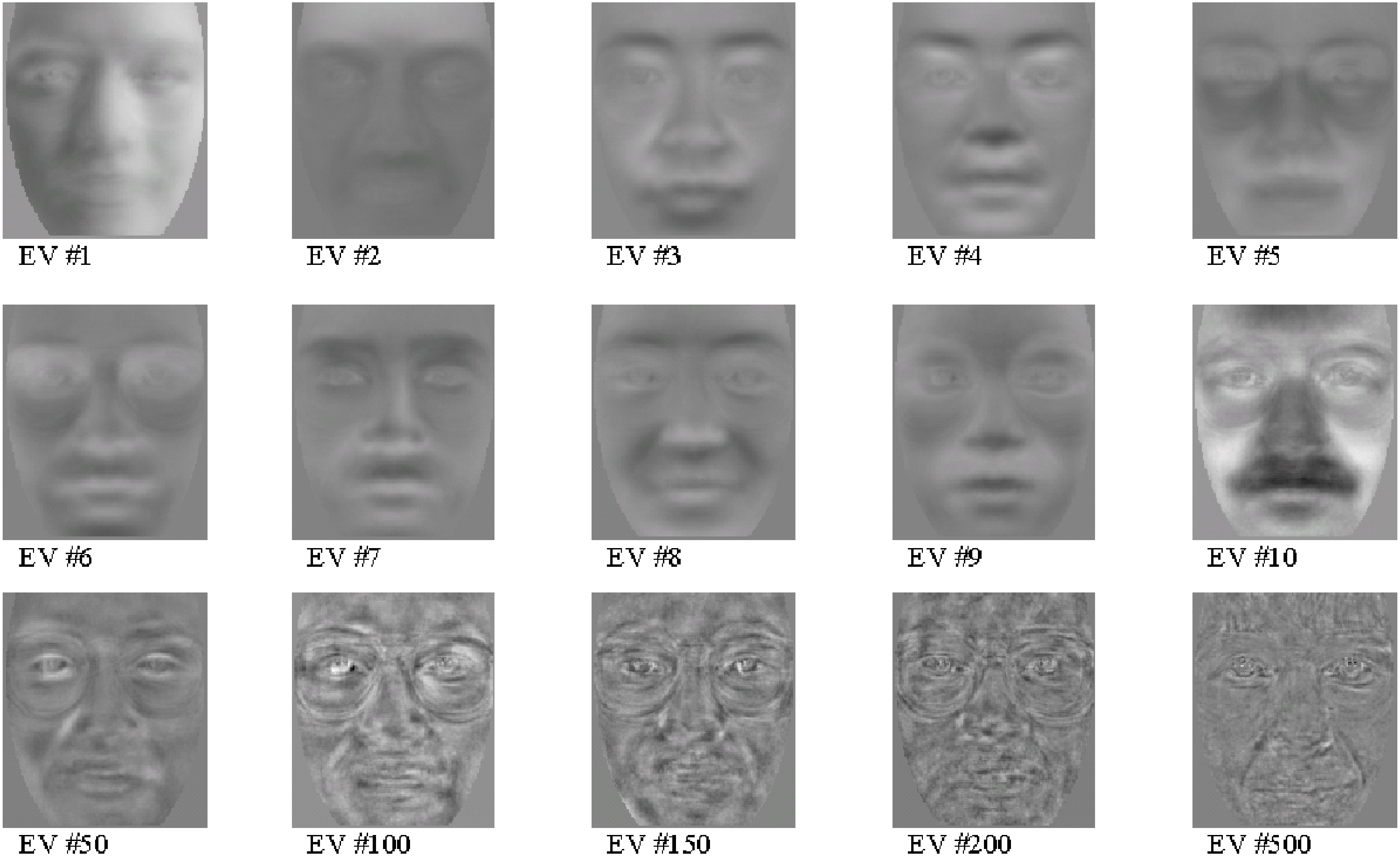
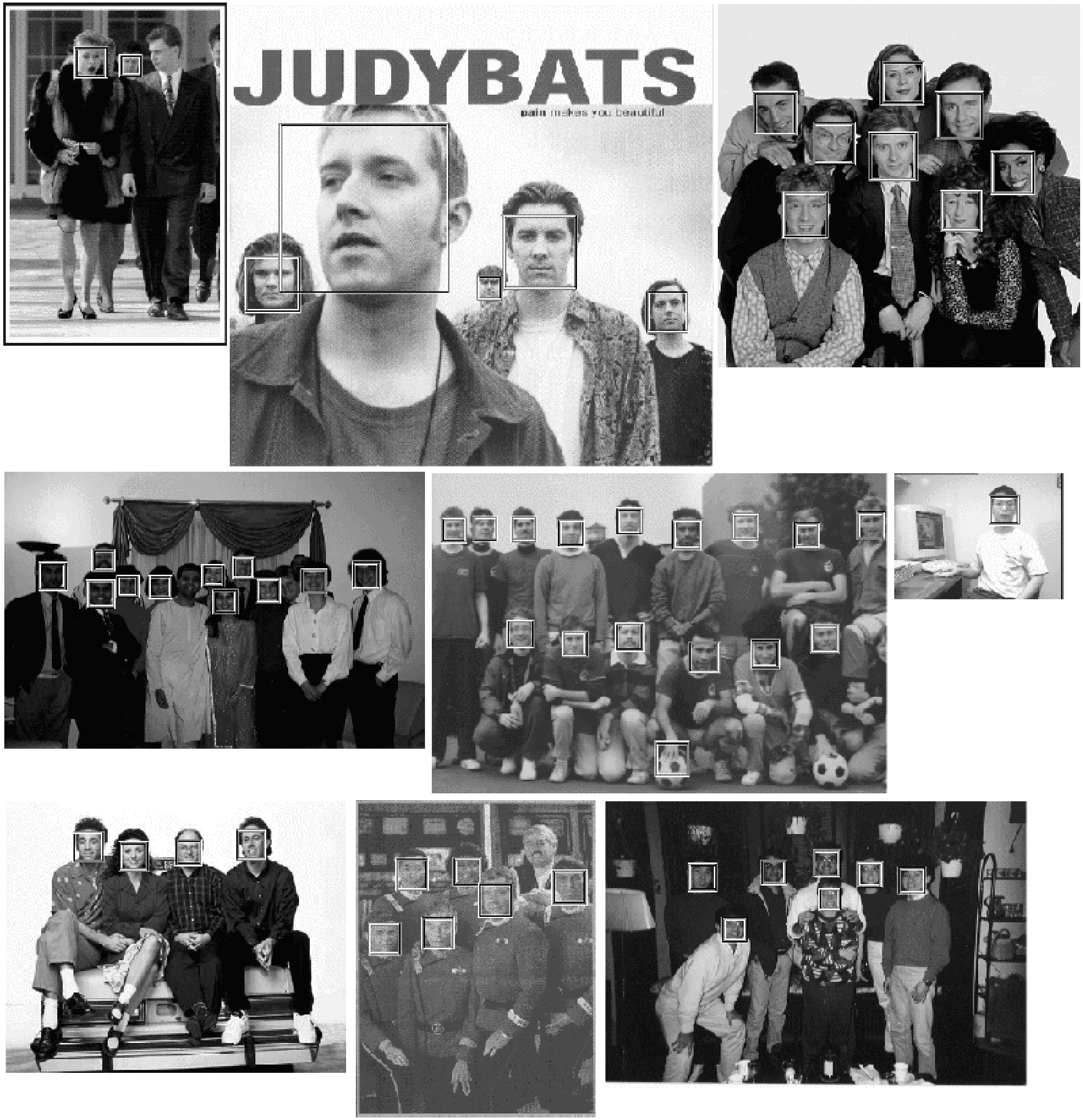


Voice recognition, other biometrics

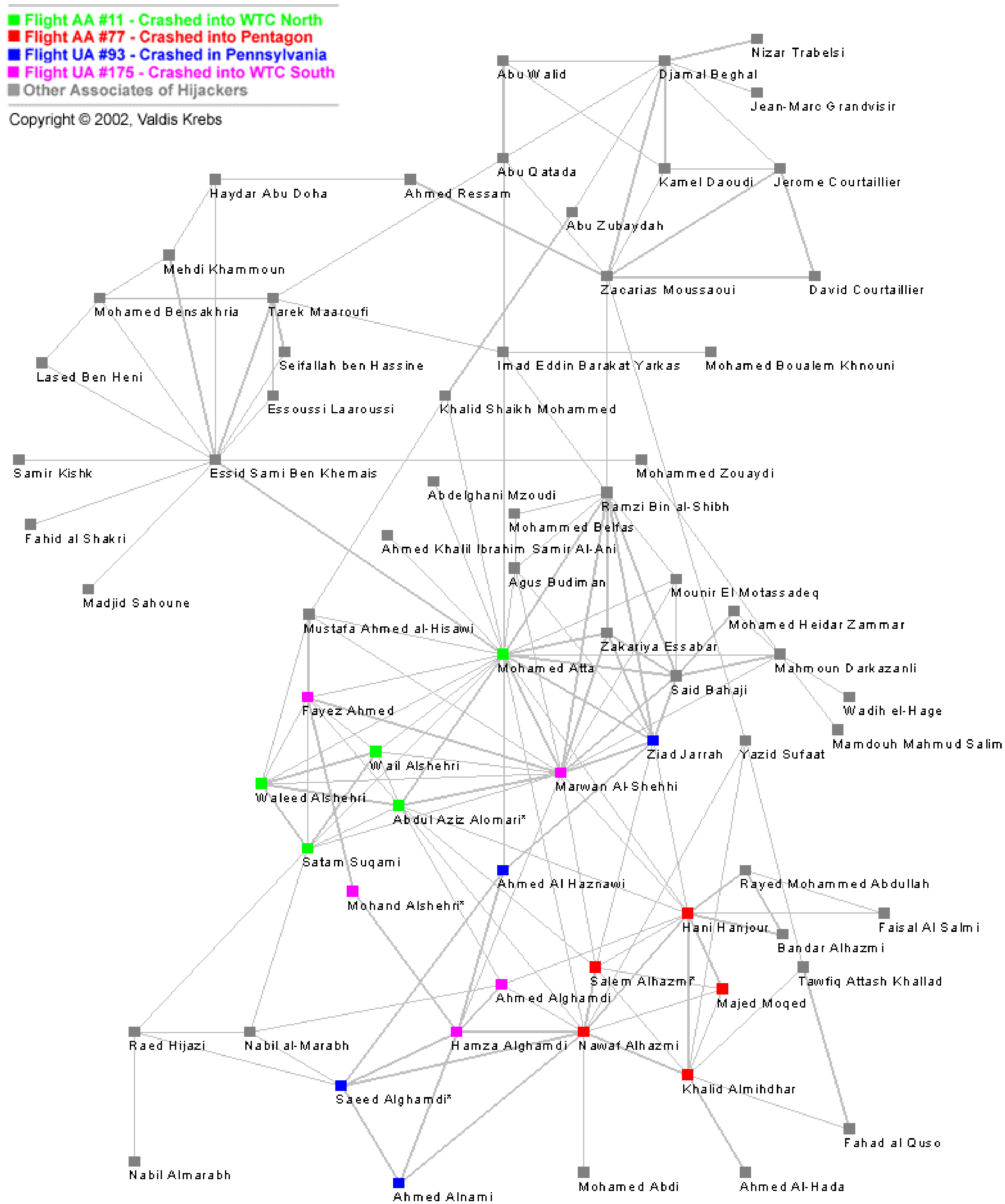



EV #1
EV #2
EV #3
EV #4
EV #5
EV #6
EV #7
EV #8
EV #9
EV #10
EV #50
EV #100
EV #150
EV #200
EV #500


JUDYBATS
pain makes you beautiful

Social network analysis

- A quantitative approach to the analysis of interpersonal relationships in a graph theoretic framework.
- Most commonly studied in sociology departments. Several journals, degree programs, International Network for Social Network Analysis, conferences, etc.
- Classical applications are to corporate structures, primitive societies, disease networks, etc.
- Recent applications are to terrorist networks.



Flight AA #11 - Crashed into WTC North
Flight AA #77 - Crashed into Pentagon
Flight UA #93 - Crashed in Pennsylvania
Flight UA #175 - Crashed into WTC South
Other Associates of Hijackers
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Abu Walid
Djamal Beghal
Jean-Marc Grandvisir
Abu Qatada
Kamel Daoudi
Jerome Courtaillier
Haydar Abu Doha
Ahmed Ressam
Abu Zubaydah
Mehdi Khammoun
Zacarias Moussaoui
David Courtaillier
Mohamed Bensakhria
Tarek Maaroufi
Seifallah ben Hassine
Imad Eddin Barakat Yarkas
Mohamed Boualem Khnouni
Lased Ben Heni
Essoussi Laaroussi
Khalid Shaikh Mohammed
Samir Kishk
Essid Sami Ben Khemais
Mohammed Zouaydi
Abdelghani Mzoudi
Ramzi Bin al-Shibh
Fahid al Shakri
Mohammed Belfas
Ahmed Khalil Ibrahim Samir Al-Ani
Agus Budiman
Madjid Sahoune
Mounir El Motassadeq
Mohamed Heidar Zammar
Mustafa Ahmed al-Hisawi
Zakariya Essabar
Mohamed Atta
Mahmoun Darkazanli
Said Bahaji
Fayez Ahmed
Wadih el-Hage
Mamdouh Mahmud Salim
Ziad Jarrah
Yazid Sufaat
Wail Alshehri
Marwan Al-Shehhi
Waleed Alshehri
Abdul Aziz Alomari*
Satam Suqami
Ahmed Al Haznawi
Rayed Mohammed Abdullah
Mohand Alshehri*
Hani Hanjour
Faisal Al Salmi
Bandar Alhazmi
Salem Alhazmi*
Tawfiq Attash Khallad
Ahmed Alghamdi
Majed Moqed
Raed Hijazi
Nabil al-Marabh
Hamza Alghamdi
Nawaf Alhazmi
Saeed Alghamdi*
Khalid Almihdhar
Fahad al Quso
Nabil Almarabh
Ahmed Alnami
Mohamed Abdi
Ahmed Al-Hada

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Game theory

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- The combination of game theory with the other mathematical techniques discussed seems to have great potential.
- Will quantum mechanics change everything too? Quantum game theory is hot (but the pay-off, if any, is probably decades away).

Homeland Security Advisory System

SEVERE
Severe Risk of Terrorist Attacks

HIGH
High Risk of Terrorist Attacks

ELEVATED
Significant Risk of Terrorist Attacks

GUARDED
General Risk of Terrorist Attacks

LOW
Low Risk of Terrorist Attacks

www.homelandsecurity.gov

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  - familiarity with part of the vast corpus of mathematical theories, techniques, algorithms, problem-solving tools

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- Interdisciplinary work is undervalued in the university structure.

Let's meet the challenge!