Homework #6. MF 5012. Due on May 15, 2008.

Consider the problem of pricing an American put option for $r=0.1$, $\sigma=0.2$, $K=1$ and $T=1$.

(4pts) ① Take $S_M = 10$. Apply a finite difference method to the un-transformed model and use the projected SOR method to solve at each time step. Fix the ratio $\Delta t/\Delta S$ and find experimentally how the CPU time of your implementation depends on the number of intervals I. Note that the CPU time might strongly depend on the stopping criterion you are using for your projected SOR method. Argue that your CPU time is reasonable.

(4pts) ② Since we do not know the exact solution, how would you assess the quality of your approximation? If you assume that the error for a given ΔS (remember we are maintaining $\Delta t/\Delta S$ fixed!), $e_{\Delta S}$, is of the form

$$e_{\Delta S} = C\,\Delta S^{\alpha}$$

How would you estimate α without knowing the exact solution?

(Hint: If $e_{\Delta S} = \| f - f_{\Delta S} \|$, then

$$\| f_{\Delta S/2} - f_{\Delta S} \| \le \| f - f_{\Delta S/2} \| + \| f - f_{\Delta S} \|$$
$$= C\left(1 + \tfrac{1}{2^{\alpha}}\right)\Delta S^{\alpha} \quad)$$

(4pts) ③ Use the results of the previous exercise to assess at what order of convergence the method in ① is converging.

(4pts) ④ Compare the price of the American option you obtained with that of the corresponding European option. Are your results reasonable. Why?