

Corrections to

Kogan, I.A., and Olver, P.J., The invariant variational bicomplex, *Contemp. Math.* **285** (2001), 131–144.

In (30), the second formula is missing a summation over i :

$$d_{\mathcal{V}} I^\alpha = \sum_{\beta=1}^q \mathcal{A}_\beta^\alpha(\vartheta^\beta), \quad d_{\mathcal{V}} \varpi^j = \sum_{i=1}^p \sum_{\beta=1}^q \mathcal{B}_{i,\beta}^j(\vartheta^\beta) \wedge \varpi^i, \quad (30)$$

A sign error in the third displayed equation on page 142 propagated. The corrected version of the affected text follows:

Further,

$$\begin{aligned} d_{\mathcal{V}} \varpi^1 &= -\kappa^1 \vartheta \wedge \varpi^1 + \frac{1}{\kappa^1 - \kappa^2} (\mathcal{D}_1 \mathcal{D}_2 - Z_2 \mathcal{D}_1) \vartheta \wedge \varpi^2, \\ d_{\mathcal{V}} \varpi^2 &= \frac{1}{\kappa^2 - \kappa^1} (\mathcal{D}_2 \mathcal{D}_1 - Z_1 \mathcal{D}_2) \vartheta \wedge \varpi^1 - \kappa^2 \vartheta \wedge \varpi^2, \end{aligned}$$

which yields the Hamiltonian operator complex

$$\begin{aligned} \mathcal{B}_1^1 &= -\kappa^1, & \mathcal{B}_2^1 &= \frac{1}{\kappa^1 - \kappa^2} (\mathcal{D}_1 \mathcal{D}_2 - Z_2 \mathcal{D}_1) = \frac{1}{\kappa^1 - \kappa^2} (\mathcal{D}_2 \mathcal{D}_1 - Z_1 \mathcal{D}_2) = -\mathcal{B}_1^2. \\ \mathcal{B}_2^2 &= -\kappa^2, \end{aligned}$$

Therefore, according to our fundamental formula (9.24), the Euler-Lagrange equation for a Euclidean-invariant variational problem is

$$\begin{aligned} 0 = \mathbf{E}(L) &= [(\mathcal{D}_1 + Z_1)^2 - (\mathcal{D}_2 + Z_2) \cdot Z_2 + (\kappa^1)^2] \mathcal{E}_1(\tilde{L}) \\ &+ [(\mathcal{D}_2 + Z_2)^2 - (\mathcal{D}_1 + Z_1) \cdot Z_1 + (\kappa^2)^2] \mathcal{E}_2(\tilde{L}) + \kappa^1 \mathcal{H}_1^1(\tilde{L}) + \kappa^2 \mathcal{H}_2^2(\tilde{L}) \\ &+ [(\mathcal{D}_2 + Z_2)(\mathcal{D}_1 + Z_1) + (\mathcal{D}_1 + Z_1) \cdot Z_2] \cdot \left(\frac{\mathcal{H}_2^1(\tilde{L}) - \mathcal{H}_1^2(\tilde{L})}{\kappa^1 - \kappa^2} \right). \end{aligned}$$

As before, $\mathcal{E}_\alpha(\tilde{L})$ are the invariant Eulerians with respect to the principal curvatures κ^α , while $\mathcal{H}_j^i(\tilde{L})$ are the invariant Hamiltonians. In particular, if $\tilde{L}(\kappa^1, \kappa^2)$ does not depend on any differentiated invariants, the Euler-Lagrange equation reduces to

$$[(\mathcal{D}_1^\dagger)^2 + \mathcal{D}_2^\dagger \cdot Z_2 + (\kappa^1)^2] \frac{\partial \tilde{L}}{\partial \kappa^1} + [(\mathcal{D}_2^\dagger)^2 + \mathcal{D}_1^\dagger \cdot Z_1 + (\kappa^2)^2] \frac{\partial \tilde{L}}{\partial \kappa^2} - (\kappa^1 + \kappa^2) \tilde{L} = 0.$$

For example, the problem of minimizing surface area has invariant Lagrangian $\tilde{L} = 1$, and so has the well-known Euler-Lagrange equation $\mathbf{E}(L) = -(\kappa^1 + \kappa^2) = -2H = 0$, and hence minimal surfaces have vanishing mean curvature. The mean curvature Lagrangian $\tilde{L} = H = \frac{1}{2}(\kappa^1 + \kappa^2)$ has Euler-Lagrange equation

$$\frac{1}{2} [(\kappa^1)^2 + (\kappa^2)^2 - (\kappa^1 + \kappa^2)^2] = -\kappa^1 \kappa^2 = -K = 0.$$