Given

$$\begin{tikzcd} \operatorname{Spec} R_1 \arrow[rr] \arrow[dr] & & \operatorname{Spec} R_2 \arrow[dl] \\ & \mathcal{M}_{Ell} & \end{tikzcd}$$

the obstruction theory will produce

$$\mathcal{O}^{top}(\operatorname{Spec} R_1) \to \mathcal{O}^{top}(\operatorname{Spec} R_2),$$

but this map will not necessarily be unique. However, if we choose $\mathcal{O}^{top}(\mathcal{M}_{Ell})$, and require that the diagr.

$$\begin{tikzcd} \mathcal{O}^{top}(\operatorname{Spec} R_1) \arrow[rr] \arrow[dr] & & \mathcal{O}^{top}(\operatorname{Spec} R_2) \arrow[dl] \\ & \mathcal{O}^{top}(\mathcal{M}_{Ell}) & \end{tikzcd}$$

commutes, then, it turns out, the top map will be unique. This uses a rel. Frob. trick.

ex $p=3$.

$$y^2 = x^3 + b_2 x^2 + b_4 x$$

scaling: $y \mapsto \lambda^{-3} y$, $x \mapsto \lambda^{-2} x$,

$x \mapsto x+r$; $r^3 + b_2 r^2 + b_4 r = 0$

$$\begin{tikzcd}
\operatorname{Spec} A[\lambda^{\pm 1}][r]/(r^3+b_2r^2+b_4r) \arrow[r] \arrow[d] & \operatorname{Spec} \overbrace{\mathbb{Z}_{(3)}[b_2, b_4]}^{A} \arrow[d, "\text{flat}"] \\
\operatorname{Spec} A \arrow[r] & \mathcal{M}_{Ell}
\end{tikzcd}$$

cohomologically like $H^*(\mathbb{Z}/3\mathbb{Z}, \text{something})$.

Ordinary locus:

$$y^2 - (x^3 + b_2 x^2 + b_4 x + b_6) = F(x,y)$$

$$F(x,y)^{p-1} = \cdots + v_1 (xy)^{p-1} + \cdots$$

Hasse invariant $= b_2$

ordinary $\iff$ b_2 unit.

$$y^2 = x^3 + b_2x^2 + b_4x + b_6$$

unique r s.t. $x \mapsto x+r$ makes $b_4 = 0$.

$$\begin{tikzcd} \operatorname{Spec} A[\lambda]/(\lambda^2-1) \arrow[r] \arrow[d, "\text{étale}"'] & \operatorname{Spec} \overbrace{\mathbb{Z}_3[b_6]}^{A} \arrow[d] \\ \operatorname{Spec} A \arrow[r] & (\mathcal{M}_{Ell})_{ord}^{\wedge} \end{tikzcd}$$

No higher coh., so can feed the alg. to our obstruction machine.

<u>ex</u> $p=2$, ordinary locus ~~can simplify~~

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$$

a_1: Hasse inv.

(by $x \mapsto x+r$) to get

$$y^2 + a_1xy = x^3 + a_2x^2 + a_4x + a_6$$

Can use $y \mapsto y + sx + t$ to get

$$y^2 + a_1xy = x^3 + a_4x \, ;$$

can scale to get $a_1 = 1$. The only

remaining symmetries of the resulting eq.

$$y^2 + xy = x^3 + a_4 x$$

are $y \mapsto y + 3x$, $s^2 + \cdots = 0$. So we have a 2-fold étale cover

$$\begin{array}{c} \operatorname{Spec} \mathbb{Z}_2[a_4] \\ \downarrow \\ (\mathcal{M}_{Ell})_{ord}^{\wedge} \end{array}$$

This will have higher coh., though. //

Given

$$\begin{array}{c} \operatorname{Spec} R \\ \downarrow \text{étale} \\ (\mathcal{M}_{Ell})_{ord} \end{array}$$

we have from the algebraic theory

$$K_* \mathcal{O}^{top}(R),$$

a θ-algebra in K_*K-comodules.

At $p=3$, take $R = \mathbb{Z}_3[b_3]$, construct $\mathcal{O}^{top}(R)$ with $\mathbb{Z}/2\mathbb{Z}$-action and define

$$\mathcal{O}^{top}(\mathcal{M}_{ord}) := \mathcal{O}^{top}(R)^{h\mathbb{Z}/2\mathbb{Z}}$$

~~For p = 2 we first~~ some facts:

$$K_*K = \mathrm{Map}_{cts}(\mathbb{Z}_p^*, K_*)$$

$$K_*KO = \mathrm{Map}_{cts}(\mathbb{Z}_p^*/\{\pm 1\}, K_*)$$

$$KO_*KO = \mathrm{Map}_{cts}(\mathbb{Z}_p^*/\{\pm 1\}, KO_*).$$

$$\mathbb{Z}_2^* = \{\pm 1\} \times (1+4\mathbb{Z}_2)^* \cong \{\pm 1\} \times 4\mathbb{Z}_2.$$

At $p=2$, we use our obstruction theory but with KO instead of K; get θ-alg. in KO_*KO-comodules ($\Leftrightarrow \mathbb{Z}_2^*/\{\pm 1\}$-action).

Calculate spectral sequence

$$H^*(\mathbb{Z}/2\mathbb{Z}, K_*) \Rightarrow KO_*$$

$$K_* = \mathbb{Z}_p[u^{\pm 1}] \qquad \deg u = 2$$

action: $u \longmapsto -u$

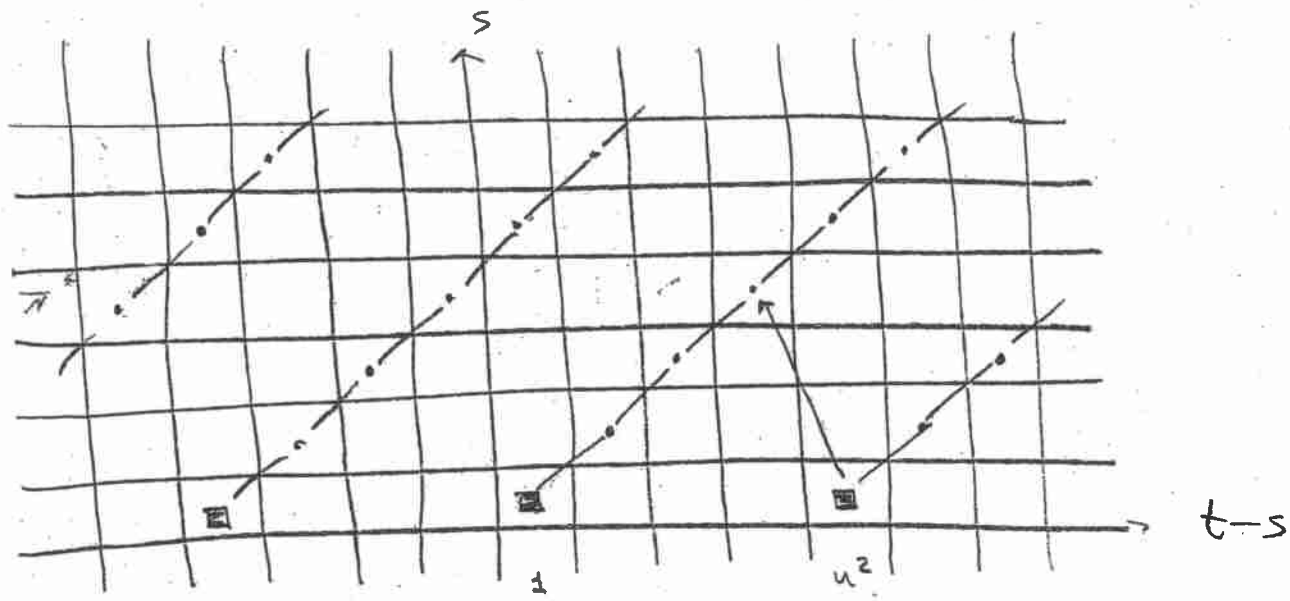
$$H^*(\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}) = \mathbb{Z}[\alpha]/(2\alpha) \qquad \deg \alpha = 2.$$

$$H^*(\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}(1)) = \mathbb{Z}/2\mathbb{Z}\text{ - vector space w. basis}$$

$$\eta, \alpha\eta, \alpha^2\eta, \ldots \qquad \eta \in H^1.$$

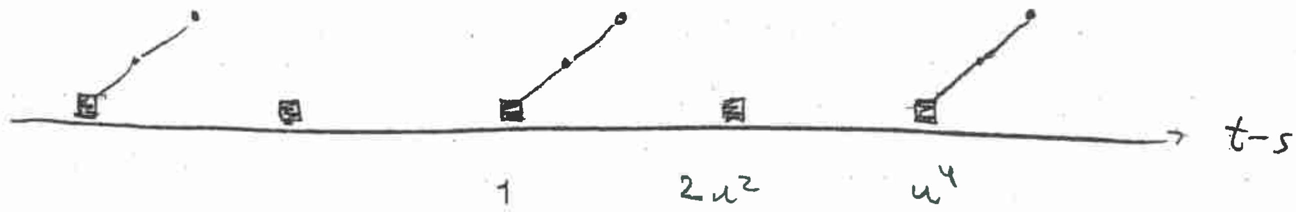
So in Adams grading, the spectral sequence takes the form

$$H^s(\mathbb{Z}/2\mathbb{Z}, K_t) \Longrightarrow KO_{t-s}$$



$\boxed{} = \mathbb{Z}$, $\bullet = \mathbb{Z}/2$, $x \to \eta x$

Know that η^3 must be annihilated; can only happen by $d_3(u^2) = \eta^3$. So the E_4-term takes the form



$\mathbb{Z}\quad \mathbb{Z}/2\quad \mathbb{Z}/2\quad 0\quad \mathbb{Z}\quad 0\quad 0\quad 0\quad \mathbb{Z}\quad \cdots$

Pretend that the sheaf $\mathcal{O}^{top}$ of E_∞-ring spectra on $\mathcal{M}_{Ell}$ in the étale topology has been constructed. Consider $\mathcal{O}^{top}(\mathcal{M}_{Ell})$ or rather

$$tmf \text{ `` = '' } \mathcal{O}^{top}(\mathcal{M}_{Weier}).$$

$p = 5$:

$$y^2 = x^3 + a_4 x + a_6$$

$$c_4 = \text{const. } a_4 \qquad (\text{const.} = \text{known const.})$$

$$c_6 = \text{const. } a_6$$

$$v_1 = c_4$$

Only automorphism is scaling $a_i \mapsto \lambda^i a_i$.

Super-singular :

$$y^2 = x^3 + c_6$$

Must also impose the condition that the curve is smooth, i.e. c_6 is a unit :

$$\Delta = \frac{c_4^3 - c_6^2}{1728}$$

Covering

$$(\mathcal{M}_{Ell})^\wedge_{ss} \leftarrow \operatorname{Spec} \overbrace{\mathbb{Z}_5[c_4, c_6^{\pm 1}]^\wedge_{(c_4)}}^{A} \leftleftarrows \operatorname{Spec} A[\lambda^{\pm 1}]$$

$$c_4 \longmapsto \lambda^4 c_4$$