# Financial Mathematics

## Row and column operations and linear algebra

0026-1.

Determine if $(-1, 1, -1, 2, 8)$ is in the span of

$$(0, 1, -1, 5, -3),$$
$$(1, 8, -7, 38, -9),$$
$$(5, -2, 2, 1, 5),$$
$$(-3, 6, -5, 19, 16).$$

Hint: The last four vectors are the rows of

$$M := \begin{bmatrix} 0 & 1 & -1 & 5 & -3 \\ 1 & 8 & -7 & 38 & -9 \\ 5 & -2 & 2 & 1 & 5 \\ -3 & 6 & -5 & 19 & 16 \end{bmatrix}.$$

WARNING: ROW canonical form,
NOT column or fully canonical form.

0026-2.

Let $S \subseteq \mathbb{R}^4$ be the span of

$$(8, 2, 9, 2, 1)$$
$$(5, -7, -6, 8, -2)$$
$$(6, -4, -1, 6, -1)$$
$$(7, -1, 4, 4, 0)$$
$$(0, 8, 1, -1, 2)$$
$$(6, 4, 0, 5, 1)$$

Extract a basis of $S$ from these six vectors.

Your answer should be a subset of the set of these six vectors.

0026-3.

Are the vectors

$$(2,4,6,8),$$
$$(1,2,5,-1),$$
$$(3,6,4,-6),$$
$$(3,6,-7,-9)$$

linearly independent?

If not, express one as a linear combination of the others.

0026-4.

Find the image of

$$M := \begin{bmatrix} 0 & 1 & -1 & 5 & -3 \\ 1 & 8 & -7 & 38 & -9 \\ 5 & -2 & 2 & 1 & 5 \\ -3 & 6 & -5 & 19 & 16 \end{bmatrix}.$$

WARNING: COLUMN canonical form,
NOT row or fully canonical form.

0026-5.

Find the dimension of the kernel of

$$M := \begin{bmatrix} 0 & 1 & -1 & 5 & -3 \\ 1 & 8 & -7 & 38 & -9 \\ 5 & -2 & 2 & 1 & 5 \\ -3 & 6 & -5 & 19 & 16 \end{bmatrix}.$$

# 0026-6.

a. For each of the following two matrices, compute the dimension of its kernel and the dimension of its image.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 3 & 4 & -7 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 4 & -3 \end{bmatrix}.$$

b. For each of the following two matrices, compute the dimension of its kernel and the dimension of its image.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 3 & 4 & 7 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 1 & 1 & 10 \end{bmatrix}.$$

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Hint:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 3 & 4 & -7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 4 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 1 & 1 & 10 \end{bmatrix}$$

0026-7. a. Determine which of these two matrices is invertible.

$$\begin{bmatrix} 6 & 3 & 4 \\ 2 & 2 & 3 \\ 8 & -1 & -3 \end{bmatrix}, \quad \begin{bmatrix} 6 & 3 & 4 \\ 2 & 2 & 3 \\ 7 & -1 & -3 \end{bmatrix}.$$

b. Invert it.

0026-8. Solve:

$$\begin{aligned} 2x + 2y + 5z &= p \\ 5x + 3y + 7z &= q \\ 8x + 3y + 6z &= r, \end{aligned}$$

where $p$, $q$ and $r$ are arbitrary.