Financial Mathematics

Cauchy-Schwarz

0030-1. Let $B : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be the polarization of the quadratic form $Q : \mathbb{R}^2 \to \mathbb{R}$ defined by $Q(x, y) = 2x^2 + 6y^2$.

a. Compute $B((1,0),(1,0))$.

b. Compute $B((1,0),(0,1))$.

c. Compute $B((0,1),(1,0))$.

d. Compute $B((0,1),(0,1))$.

e. Let S be the set of all pairs

$$(v, w) \in \mathbb{R}^2 \times \mathbb{R}^2$$

s.t. $Q(v) \leq 2$ and $Q(w) \leq 8$.

Find $\max\{B(v,w) \mid v, w \in S\}$.

That is, maximize $B(v,w)$, subject to the constraint that $(v,w) \in S$.

0030-2. Let $R : \mathbb{R}^3 \to \mathbb{R}$ be the quadratic form defined by $R(x, y, z) = 3x^2 + 2y^2 + z^2$

Let $B : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ be the polarization of R.

Let $D := \{(x, y, z) \in \mathbb{R}^3 \,|\, R(x, y, z) \leq 20\}$.

Let $v_0 := (1, 2, -3)$.

Find $w_0 \in D$ such that

$$B(v_0, w_0) = \max\{B(v_0, w) \,|\, w \in D\}.$$