Financial Mathematics

Linear approximation

0037-1. Let $f(x,y) = e^{2x-y}(\sin(x+y))$.

a. Compute $\frac{\partial}{\partial x}[f(x,y)]$.

b. Compute $\left[\frac{\partial}{\partial x}[f(x,y)]\right]_{\substack{x:\to\pi/4\\ y:\to\pi/4}}$.

c. Compute $\frac{\partial}{\partial y}[f(x,y)]$.

d. Compute $\left[\frac{\partial}{\partial y}[f(x,y)]\right]_{\substack{x:\to\pi/4\\ y:\to\pi/4}}$.

0037-1. Let $f(x,y) = e^{2x-y}(\sin(x+y))$.

e. Compute the gradient of
$f(x,y)$ w.r.t. (x,y).

f. Evaluate the gradient of
$f(x,y)$ w.r.t. (x,y)
at the point $(x,y) = (\pi/4, \pi/4)$.

g. Find the 1-jet of
$f(x,y)$ w.r.t. (x,y)
at the point $(x,y) = (\pi/4, \pi/4)$.

h. Find the 1st order Macl. approx.
of $f(x + \frac{\pi}{4}, y + \frac{\pi}{4})$.

0037-1. Let $f(x,y) = e^{2x-y}(\sin(x+y))$.

i. Compute $\frac{\partial^2}{\partial x^2}[f(x,y)]$.

j. Compute $\left[\frac{\partial^2}{\partial x^2}[f(x,y)]\right]_{\substack{x:\to\pi/4 \\ y:\to\pi/4}}$.

k. Compute $\frac{\partial^2}{\partial x\,\partial y}[f(x,y)]$.

l. Compute $\left[\frac{\partial^2}{\partial x\,\partial y}[f(x,y)]\right]_{\substack{x:\to\pi/4 \\ y:\to\pi/4}}$.

m. Compute $\frac{\partial^2}{\partial y^2}[f(x,y)]$.

n. Compute $\left[\frac{\partial^2}{\partial y^2}[f(x,y)]\right]_{\substack{x:\to\pi/4 \\ y:\to\pi/4}}$.

0037-1. Let $f(x,y) = e^{2x-y}(\sin(x+y))$.

o. Compute the 2×2 Hessian of $f(x,y)$ w.r.t. (x,y).

p. Evaluate the 2×2 Hessian of $f(x,y)$ w.r.t. (x,y) at the point $(x,y) = (\pi/4, \pi/4)$.

q. Find the 2-jet of $f(x,y)$ w.r.t. (x,y) at the point $(x,y) = (\pi/4, \pi/4)$. (Use lexicographic ordering.)

r. Find the 2nd order Macl. approx. of $f(x + \frac{\pi}{4}, y + \frac{\pi}{4})$.

0037-2. How many terms appear in the 6th order Macl. approx. of a fn of 9 variables?

0037-3. Find the second-order Maclaurin approximation, $q(x, y)$, (w.r.t. (x, y)) to the expression

$$g(x, y) := [\tan(e^{2x+y})] + [2xy^2] - [4\cos x] + 8x.$$

Unassigned exercise:

With $q(x, y)$ and $g(x, y)$ as above see if you can show that

$$\lim_{(x,y)\to(0,0)} \frac{g(x, y) - q(x, y)}{x^2 + y^2} = 0,$$

i.e., that $g(x, y) = (q(x, y)) + (o(x^2 + y^2))$ near $(x, y) = (0, 0)$.

0037-4. Let $f(x,y) := \sqrt{7^2 - x^2 - 2y^2}$.

(Note: The graph of $z = f(x,y)$ is the upper half of the ellipsoid $x^2 + 2y^2 + z^2 = 7^2$. The lower half would be the graph of $z = -f(x,y)$. The north pole appears on the graph of $z = f(x,y)$ over $(x,y) = (0,0)$, at $(0,0,7)$.)

a. Compute $(\nabla f)(0,0) \in \mathbb{R}^2$.

b. Compute $f'(0,0) \in \mathbb{R}^{1\times 2}$.

c. Compute $(Hf)(0,0) \in \mathbb{R}^{2\times 2}$.

d. Compute $f''(0,0) \in \mathbb{R}^{2\times 2}$.

e. Compute $Q_{f''(0,0)}(x,y)$.

f. Show $Q_{f''(0,0)}(x,y)$ is neg. def.

0037-5. Let $Q(x,y) = 3x^2 - 4xy + 3y^2$.

Let $f(x,y) = \sqrt{1 - (Q(x,y))}$.

Let $M := (Hf)(0,0) \in \mathbb{R}^{2\times 2}$.

a. Show that $(0,0)$ is a critical point for f, *i.e.*, show that $(\nabla f)(0,0) = (0,0)$.

b. Compute M.

c. Find the char. poly. of M.

d. Find the eigenvalues of M.

e. For each eigenvalue of M, find a basis of the corresponding eigenspace.

f. Find a 2×2 rotation matrix R s.t. $R^{-1}MR$ is diagonal.