Financial Mathematics
Cholesky decomposition

0046-1. Let $X := \begin{bmatrix} 5 & -2 & 3 \\ -2 & 1 & 0 \\ 3 & 0 & 10 \end{bmatrix}$.

a. Find a 3×3 matrix A s.t. $AA^t = X$.

b. Does there exist a *symmetric* 3×3 matrix S s.t. $S^2 = X$?

(You must explain why or why not, but you needn't write out such an S.)

0046-2. Let $X := \begin{bmatrix} 5 & -2 & 3 \\ -2 & 1 & 0 \\ 3 & 0 & 11 \end{bmatrix}$.

Does there exist a
3 × 3 matrix S s.t. $S^2 = X$?
(You must explain why or why not,
but you needn't write out such an S.)

Let $X := \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 6 \end{bmatrix}$, $C := \begin{bmatrix} 3 & -4 & -7 \\ -1 & 2 & 4 \\ 6 & -11 & -21 \end{bmatrix}$.

a. Is X positive definite?

b. Is X positive semi-definite?

c. Does CXC^{-1} have
   only positive eigenvalues?

d. Does CXC^{-1} have
   only nonnegative eigenvalues?

e. Does there exist a symmetric matrix S
   s.t. $S^2 = CXC^{-1}$?
   (You must explain why or why not,
   but you needn't write out such an S.)

0046-4. Let $X := \begin{bmatrix} 1 & 0 & -2 \\ 0 & 9 & 3 \\ -2 & 3 & 9 \end{bmatrix}$

a. Find a 3×3 matrix A s.t. $AA^t = X$ and s.t. A is lower triangular.

b. Find a 3×3 matrix B s.t. $BB^t = X$ and s.t. B is upper triangular.

c. Find a 3×3 matrix C s.t. $C^tC = X$ and s.t. C is upper triangular.

d. Find a 3×3 matrix D s.t. $D^tD = X$ and s.t. D is lower triangular.

e. Does there exist a *symmetric* 3×3 matrix S s.t. $S^2 = X$? (You must explain why or why not, but you needn't write out such an S.)