

# Financial Mathematics

## Conditional expectation

0049-1. Define PCRVs  $X$  and  $Y$  by

$$X(\omega) = \begin{cases} 2, & \text{if } 0 \leq \omega \leq 0.3 \\ 3, & \text{if } 0.3 < \omega \leq 0.65 \\ 7, & \text{if } 0.65 < \omega \leq 1 \end{cases}$$
$$Y(\omega) = \begin{cases} 8, & \text{if } 0 \leq \omega \leq 0.45 \\ 9, & \text{if } 0.45 < \omega \leq 1 \end{cases}$$

Compute  $E[X|Y]$ .

0049-2. Define PCRVs  $U$  and  $V$  by

$$U(\omega) = \begin{cases} 2, & \text{if } 0 \leq \omega \leq 0.3 \\ 3, & \text{if } 0.3 < \omega \leq 0.65 \\ 7, & \text{if } 0.65 < \omega \leq 1 \end{cases}$$
$$V(\omega) = \begin{cases} 800, & \text{if } 0 \leq \omega \leq 0.45 \\ 900, & \text{if } 0.45 < \omega \leq 1 \end{cases}$$

Compute  $E[U|V]$ .

0049-3.

- a. Show the graph of some PCRV  $X$  s.t.  
 $\Pr[X = 3] = 0.2$  and  $\Pr[X = 5] = 0.8$ .
- b. Show the graph of some PCRV  $Y$  s.t.  
 $X$  and  $Y$  are independent,  
 $\Pr[Y = -1] = 0.5$  and  $\Pr[Y = -2] = 0.5$ .
- c. Show the graph of  $E[X|Y]$ .
- d. Show the graph of  $E[Y|X]$ .
- e. Find the partition  $\mathcal{P}$  of  $X$ .
- f. Find the partition  $\mathcal{Q}$  of  $Y$ .
- g. Is  $X$   $\mathcal{P}$ -measurable?
- h. Is  $Y$   $\mathcal{P}$ -measurable?
- i. Is  $E[Y|X]$   $\mathcal{P}$ -measurable?
- j. Find  $\text{Cov}[X, Y]$ .

0049-4. Find two PCRVs  $X$  and  $Y$   
s.t.  $E[X|Y]$  is deterministic,  
but  $X$  and  $Y$  are not independent.

NOTE:

$X, Y$  independent  $\Rightarrow E[X|Y]$  is deterministic

0049-5. Find two PCRVs  $X$  and  $Y$   
s.t.  $E[XY] = (E[X])(E[Y])$ ,  
but  $E[X|Y]$  is not deterministic.

NOTE:

$E[X|Y]$  is deterministic  $\Rightarrow E[XY] = (E[X])(E[Y])$   
 $X, Y$  uncorrelated  $\Leftrightarrow E[XY] = (E[X])(E[Y])$