Financial Mathematics

The heat equation

0054-1. Let $f_t(x) = f(x,t) = \dfrac{e^{-x^2/(4t)}}{2\sqrt{\pi t}}$ be the fundamental solution to the heat equation.

Then $f_3(x) = \dfrac{e^{-x^2/12}}{2\sqrt{3\pi}}$ and $f_5(x) = \dfrac{e^{-x^2/20}}{2\sqrt{5\pi}}$

and $f_8(x) = \dfrac{e^{-x^2/32}}{2\sqrt{8\pi}}$.

Remember that $p * q$ denotes the convolution of p and q.

a. Compute $(f_3 * f_5)(9)$.

b. Show, $\forall x \in \mathbb{R}$, that $(f_3 * f_5)(x) = f_8(x)$.

c. Show, $\forall t, u \geq 0$, that $f_t * f_u = f_{t+u}$.