

# Financial Mathematics

## Pricing/hedging in many subperiods

### Part 1

0058-1. Let  $S$  be the price of a stock at a time  $T$  units in the future. Let  $\mu$  and  $\sigma$  the mean and standard deviation of  $\ln S$ , resp. Divide the time interval into  $N$  subintervals, all of length  $T/N$ . Assume an i.i.d. model with a 40%-60% chance of uptick-downtick on each subinterval, and with uptick and downtick factors of  $e^u$  and  $e^d$ , resp.

a. Write formulas for  $u$  and  $d$ ,  
in terms of  $\sigma$ ,  $\mu$  and  $N$ .

Let  $r$  denote the logarithmic risk-free factor over this time period of  $T$  time units. That is, assume that \$1, invested risk-free, will grow to  $e^r$  dollars after the  $T$  units of time.

b. Write formulas for the risk-neutral uptick and downtick probabilities,  
in terms of  $r$ ,  $\sigma$ ,  $\mu$  and  $N$ .