Financial Mathematics
Pricing/hedging in many subperiods
Part 1
0058-1. Let $S$ be the price of a stock at a time $T$ units in the future. Let $\mu$ and $\sigma$ the mean and standard deviation of $\ln S$, resp. Divide the time interval into $N$ subintervals, all of length $T/N$. Assume an i.i.d. model with a 40%-60% chance of uptick-downtick on each subinterval, and with uptick and downtick factors of $e^u$ and $e^d$, resp.

a. Write formulas for $u$ and $d$, in terms of $\sigma, \mu$ and $N$.

Let $r$ denote the logarithmic risk-free factor over this time period of $T$ time units. That is, assume that $1$, invested risk-free, will grow to $e^r$ dollars after the $T$ units of time.

b. Write formulas for the risk-neutral uptick and downtick probabilities, in terms of $r, \sigma, \mu$ and $N$. 