Financial Mathematics

Preliminaries to
the Triangular Central Limit Theorem
and a first proof of Black-Scholes

0063-1. $\mathcal{S} := \{\text{standard PCRVs}\}$

a. Compute $E[8 - 3\mathcal{S}]$.

b. Compute $Var[8 - 3\mathcal{S}]$.

c. Compute $SD[8 - 3\mathcal{S}]$.

0063-2. a. Compute $E[\mathcal{B}_{0.35,5}^{0.65,9}]$.

b. Compute $Var[\mathcal{B}_{0.35,5}^{0.65,9}]$.

c. Compute $SD[\mathcal{B}_{0.35,5}^{0.65,9}]$.

0063-3. a. Compute $E[\sum^{400} \mathcal{B}_{0.35,5}^{0.65,9}]$.

b. Compute $Var[\sum^{400} \mathcal{B}_{0.35,5}^{0.65,9}]$.

c. Compute $SD[\sum^{400} \mathcal{B}_{0.35,5}^{0.65,9}]$.

0063-4. a. Compute $E[\sum^{100} \mathcal{B}^{0.65,9+3}_{0.35,5+3}]$.

b. Compute $Var[\sum^{100} \mathcal{B}^{0.75,9+3}_{0.25,5+3}]$.

c. Compute $SD[\sum^{100} \mathcal{B}^{0.65,9+3}_{0.35,5+3}]$.

0063-5. a. Compute $E[250 + \sum^{400} \mathcal{B}^{0.65,9}_{0.35,5}]$.

b. Compute $Var[250 + \sum^{400} \mathcal{B}^{0.65,9}_{0.35,5}]$.

c. Compute $SD[250 + \sum^{400} \mathcal{B}^{0.65,9}_{0.35,5}]$.

0063-6. a. Compute $E[\sum^{400} \mathcal{B}^{0.65,(7)(9)}_{0.35,(7)(5)}]$.

b. Compute $Var[\sum^{400} \mathcal{B}^{0.65,(7)(9)}_{0.35,(7)(5)}]$.

c. Compute $SD[\sum^{400} \mathcal{B}^{0.65,(7)(9)}_{0.35,(7)(5)}]$.

0063-7. a. Compute $E[7\sum^{400} \mathcal{B}^{0.65,9}_{0.35,5}]$.

b. Compute $Var[7\sum^{400} \mathcal{B}^{0.65,9}_{0.35,5}]$.

c. Compute $SD[7\sum^{400} \mathcal{B}^{0.65,9}_{0.35,5}]$.

0063-8. a. Compute $\mathrm{E}[\exp(\mathcal{B}_{0.35,5}^{0.65,9})]$.

b. Compute $\mathrm{Var}[\exp(\mathcal{B}_{0.35,5}^{0.65,9})]$.

c. Compute $\mathrm{SD}[\exp(\mathcal{B}_{0.35,5}^{0.65,9})]$.

0063-9. a. Compute $\mathrm{E}[\mathcal{B}_{0.35,e^5}^{0.65,e^9}]$.

b. Compute $\mathrm{Var}[\mathcal{B}_{0.35,e^5}^{0.65,e^9}]$.

c. Compute $\mathrm{SD}[\mathcal{B}_{0.35,e^5}^{0.65,e^9}]$.

0063-10. Compute $\mathrm{E}[\prod^{400}(\exp(\mathcal{B}_{0.35,5}^{0.65,9}))]$.

0063-11. Compute $\mathrm{E}[\exp(\sum^{400}(\mathcal{B}_{0.35,5}^{0.65,9}))]$.

0063-12. Say $A \in \mathcal{B}^{0.3,4}_{0.7,1}$ and $B \in \mathcal{B}^{0.1,8}_{0.9,5}$.

Then the possible values of $A + B$ are

$$1 + 5,\ 1 + 8,\ 4 + 5 \text{ and } 4 + 8.$$

This is redundant, because $1 + 8 = 4 + 5$.

Eliminating this redudancy, we have:

$$A + B \in \{6, 9, 12\}.$$

Assume A and B are independent.

Compute the distribution of $A + B$,

i.e., compute each of these:

$$\Pr[A + B = 6],$$
$$\Pr[A + B = 9],$$
$$\text{and } \Pr[A + B = 12].$$

(The moral: When and B are indep., the dist. of $A + B$ is det'd by the dist.s of A and B.)