Financial Mathematics
First proof of Black-Scholes

0065-1. a. Using the Black-Scholes Option Pricing Formula, price a 0.5-year call option on a stock with current share price $2.50 with strike price $2.45 with annual volatility 0.42 and with annual risk-free rate ln(1.02) (meaning that $1 "in the bank" grows to $1.02 after one year).

b. Let $S_0 := 2.5$, $K := 2.45$, $\sigma_* := 0.42$ and $r_* := \ln(1.02)$. Let $\sigma := \sigma_*/\sqrt{2}$, $r := r_*/2$ and $\nu := r - [\sigma^2/2]$. Compute

$$\frac{e^{-r}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (S_0 e^{\sigma x + \nu} - K)_+ e^{-x^2/2}\, dx.$$