Financial Mathematics

Clicker review session, Midterm 02

$$L_M(x, y) = (3x - 4y, 2x + y, x)$$

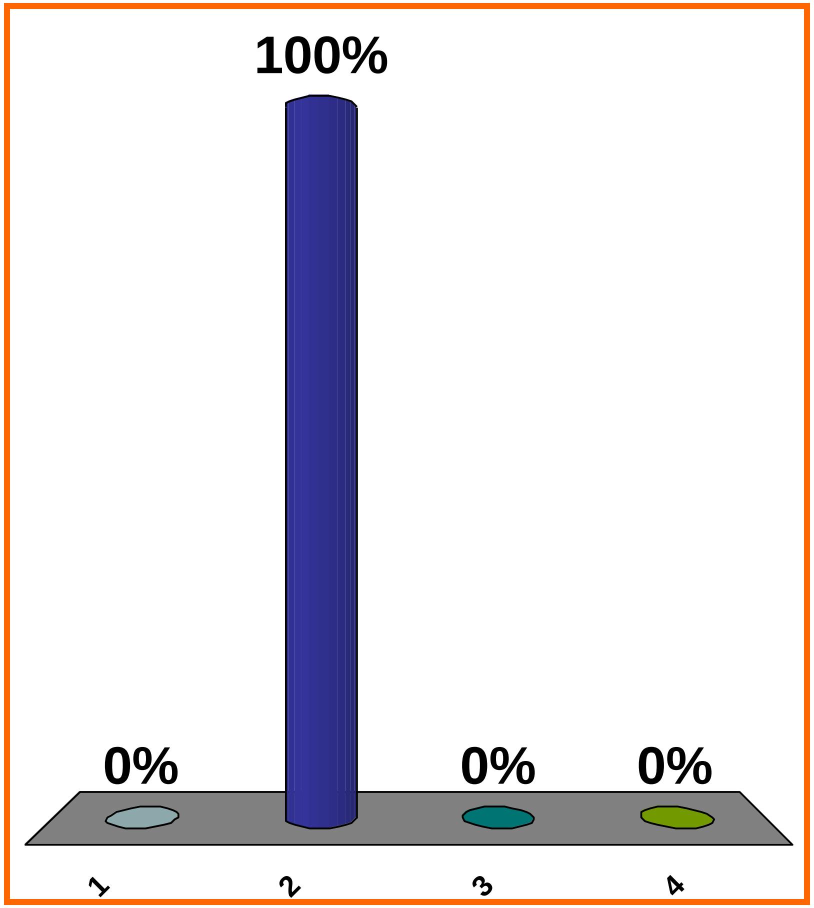
Dimensions of M?

(a) 2×3

(b) $\boxed{}$ 3×2

(c) 3×3

(d) none of the above



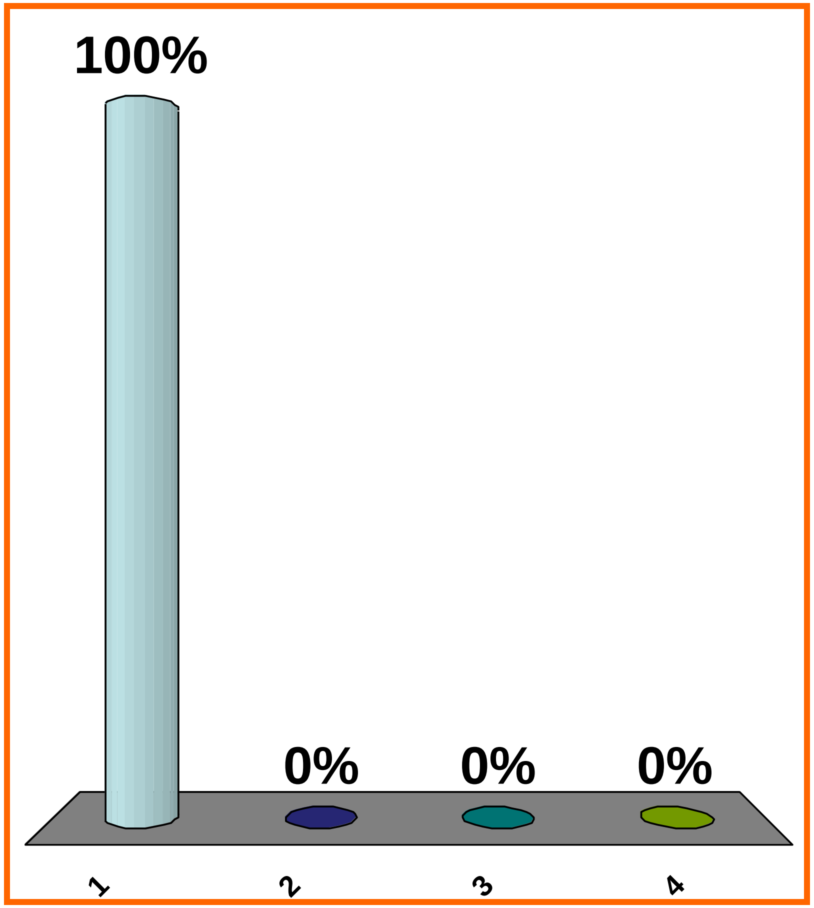
$$L_M(x,y) = (3x-4y, 2x+y, x)$$
$$M = ??$$

(a) $\begin{bmatrix} 3 & -4 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 & 1 \\ -4 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 & 1 \\ -4 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) none of the above



$$L_M(x,y,z) = (3x - 4y + z, 2x + y - z)$$

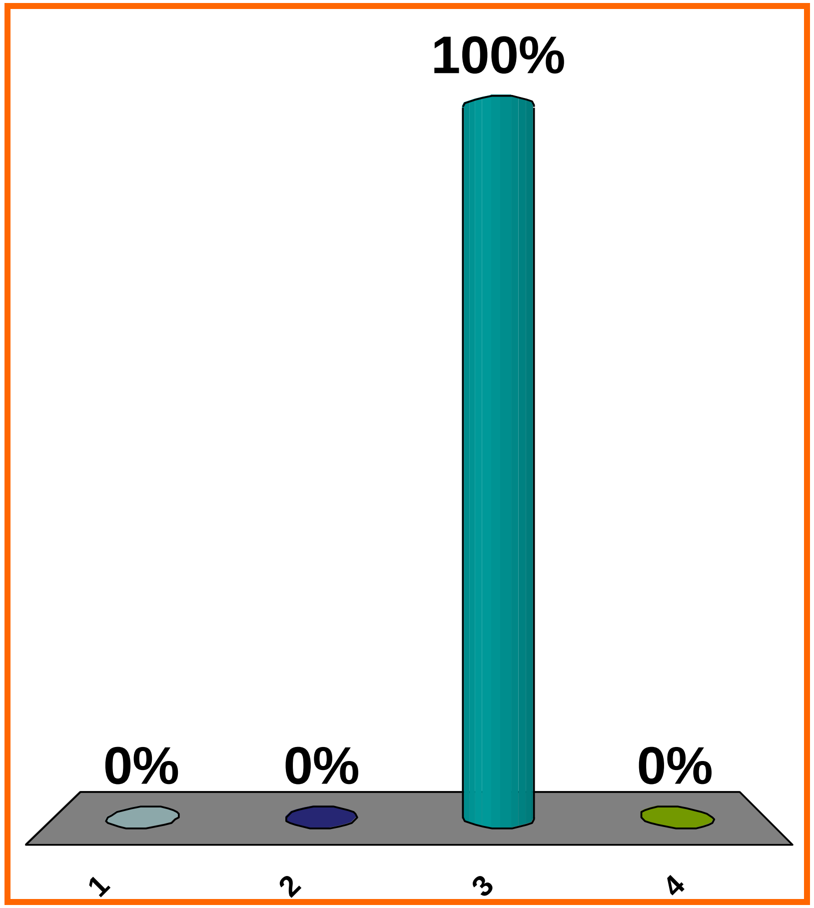
Dimensions of M?

(a) 2×3

(b) 3×2

(c) 3×3

(d) none of the above



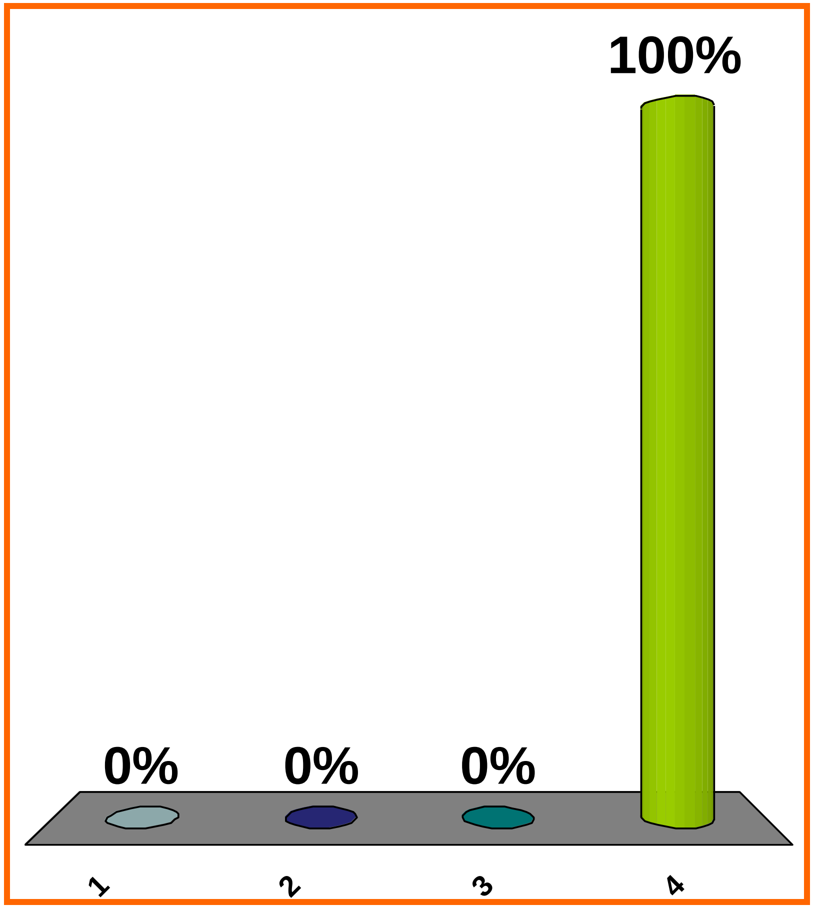
$$L_M(x,y,z) = (3x - 4y + z, 2x + y - z)$$
$$M = ??$$

(a) $\begin{bmatrix} 3 & 2 \\ -4 & 1 \\ 1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & -3 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -4 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

(d) none of the above



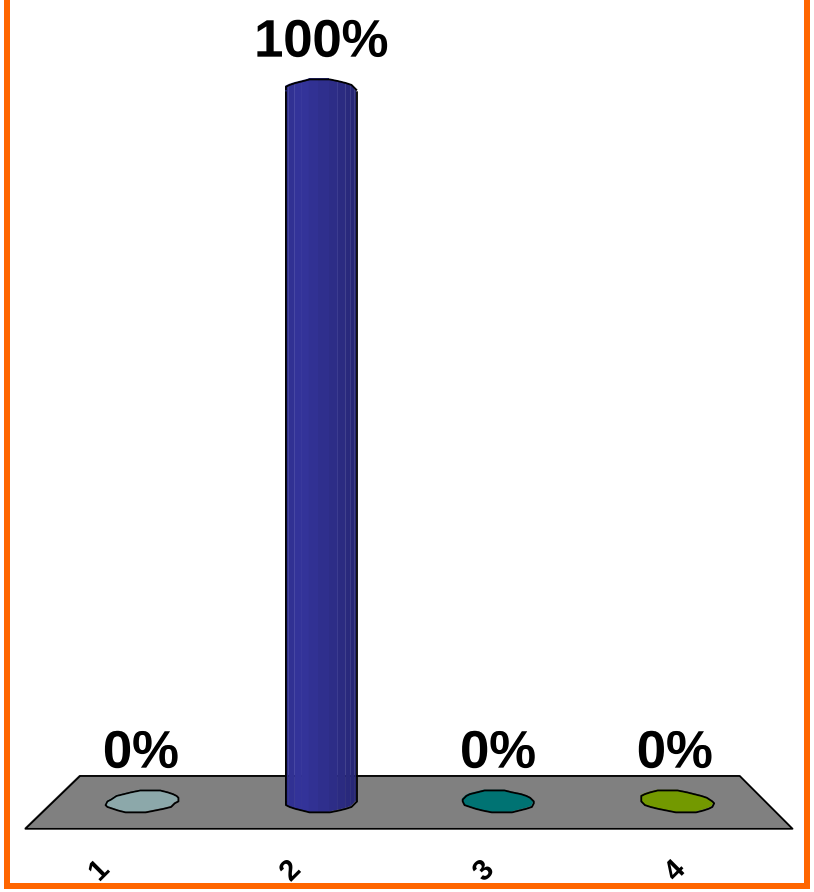
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

(a) $\boxed{\begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}}$

(b) $\begin{bmatrix} 1\cdot 5 & 2\cdot 6 \\ 3\cdot 7 & 4\cdot 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix}$

(d) none of the above



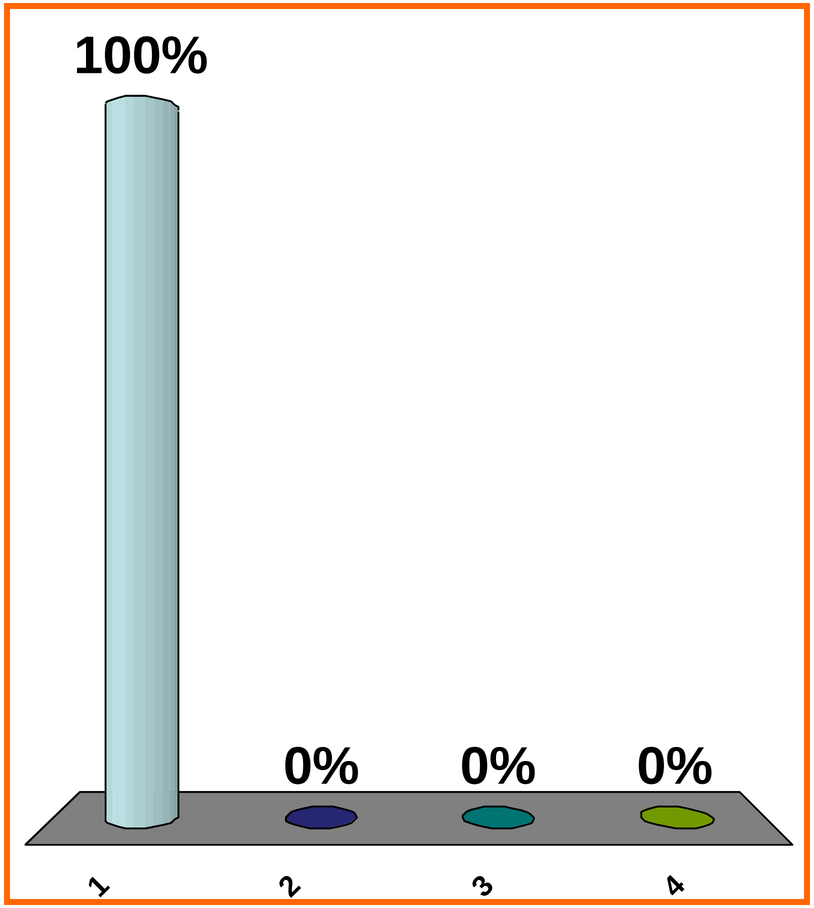
$$\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

(a) $\begin{bmatrix} 10 & 0 \\ 0 & 40 \end{bmatrix}$

(b) $\boxed{\begin{bmatrix} 10 & 12 \\ 35 & 40 \end{bmatrix}}$

(c) $\begin{bmatrix} 7 & 6 \\ 7 & 13 \end{bmatrix}$

(d) none of the above



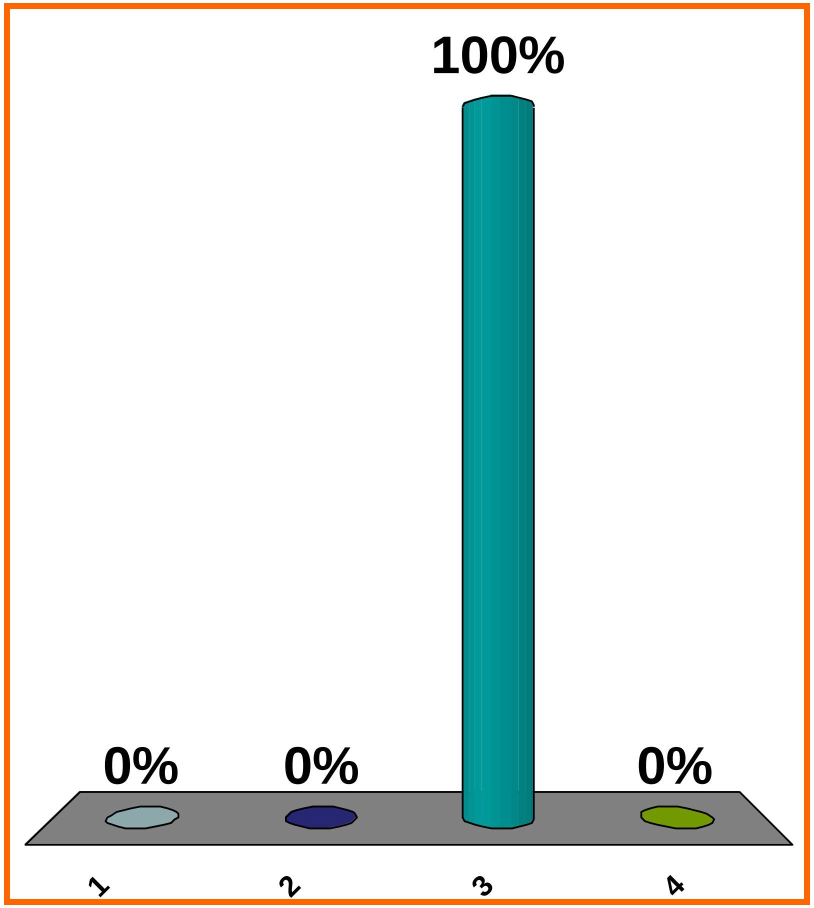
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

(b) $\boxed{\begin{bmatrix} 1 & 1+2 \\ 3 & 3+4 \end{bmatrix}}$

(c) $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

(d) none of the above



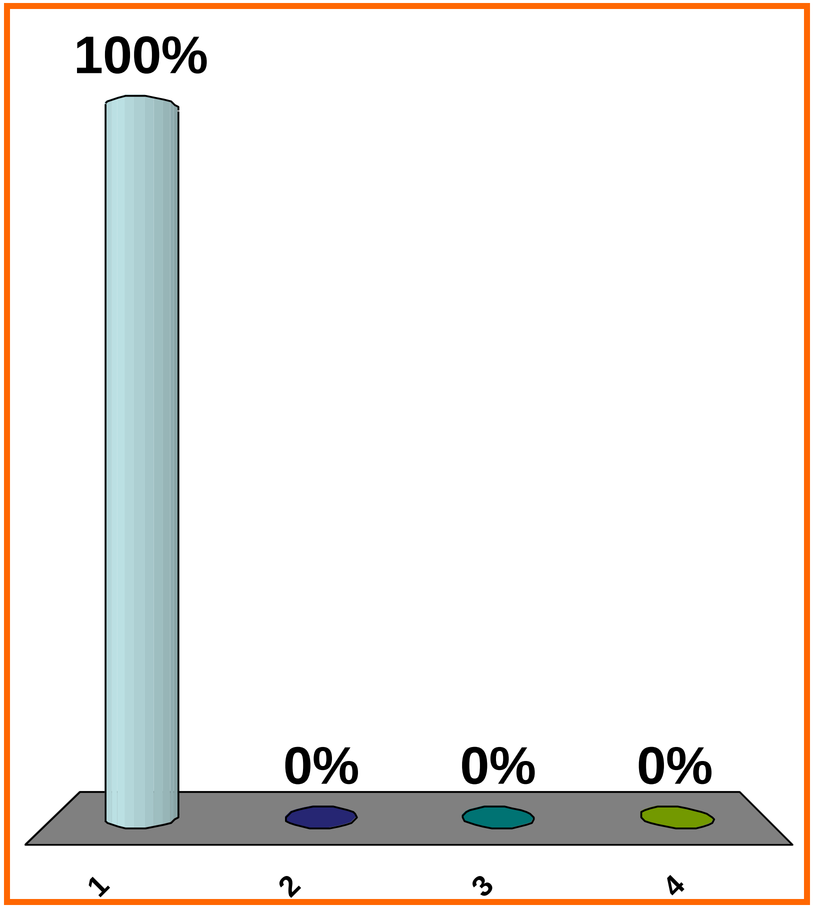
$$\dim \operatorname{im} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) 2

(d) none of the above



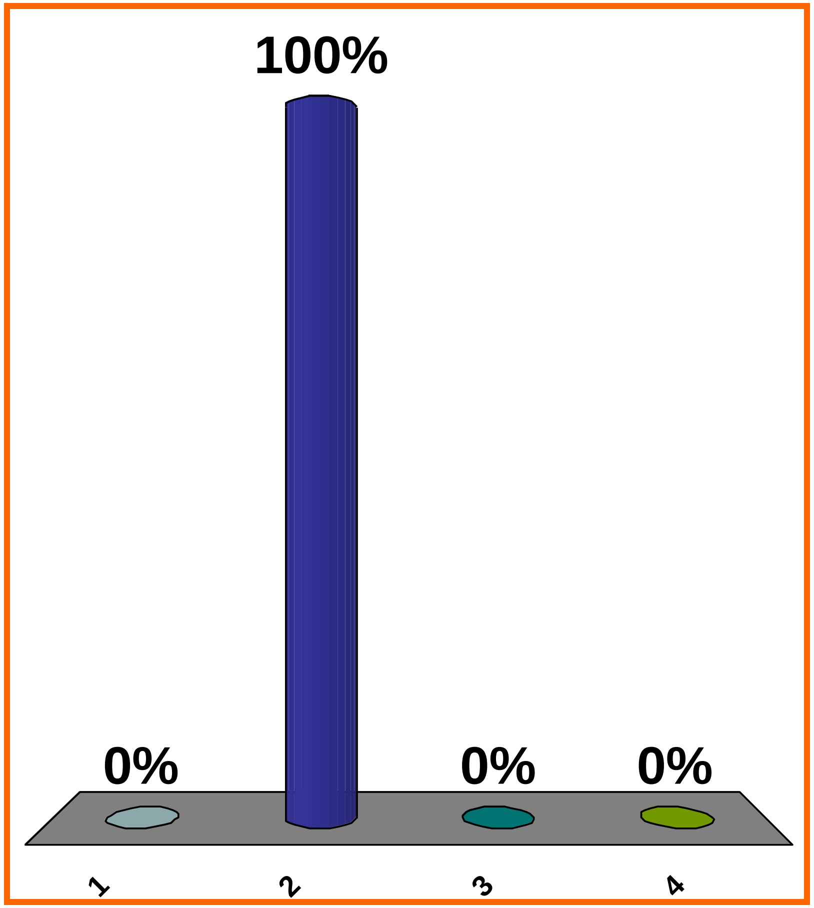
$$\dim \operatorname{im} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) 2

(d) none of the above   3



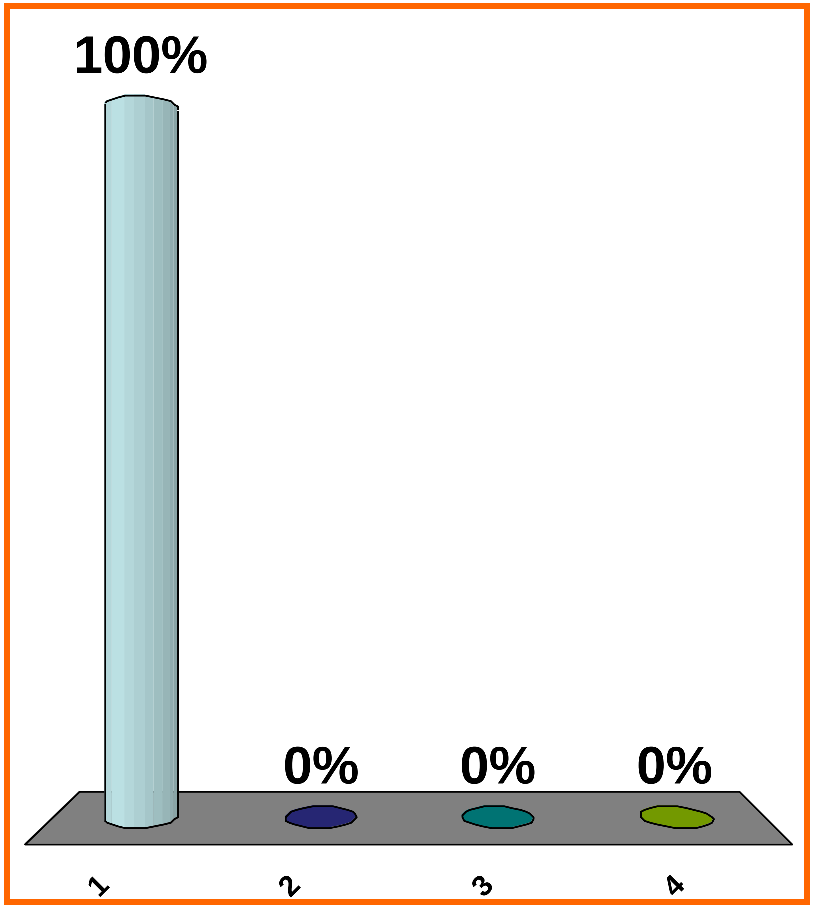
$$\dim \operatorname{im} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 1

(b) 2

$\boxed{\text{(c) } 3}$

(d) none of the above



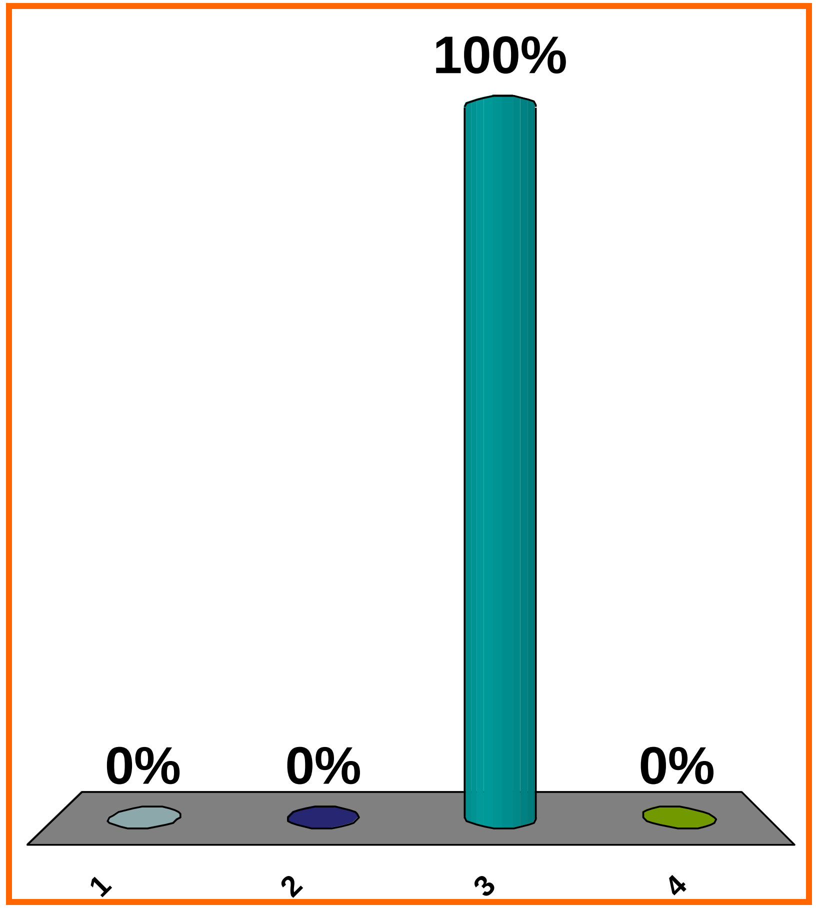
$$\dim \operatorname{im} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) $\boxed{2}$

(d) none of the above



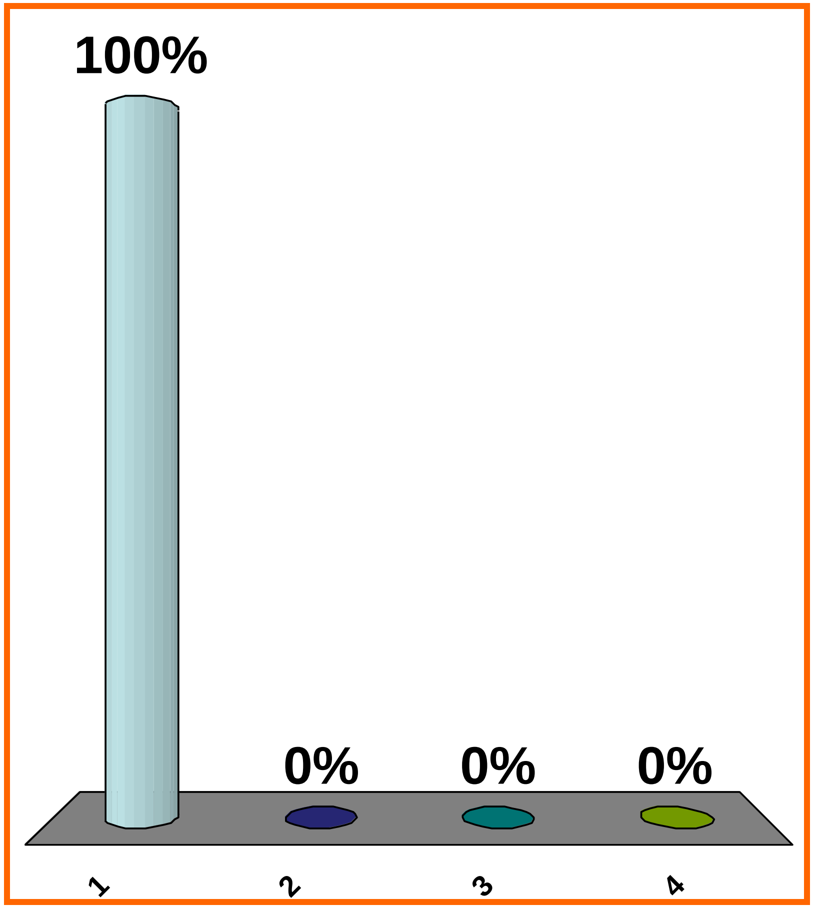
$$\dim \operatorname{im} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) $\boxed{2}$

(d) none of the above



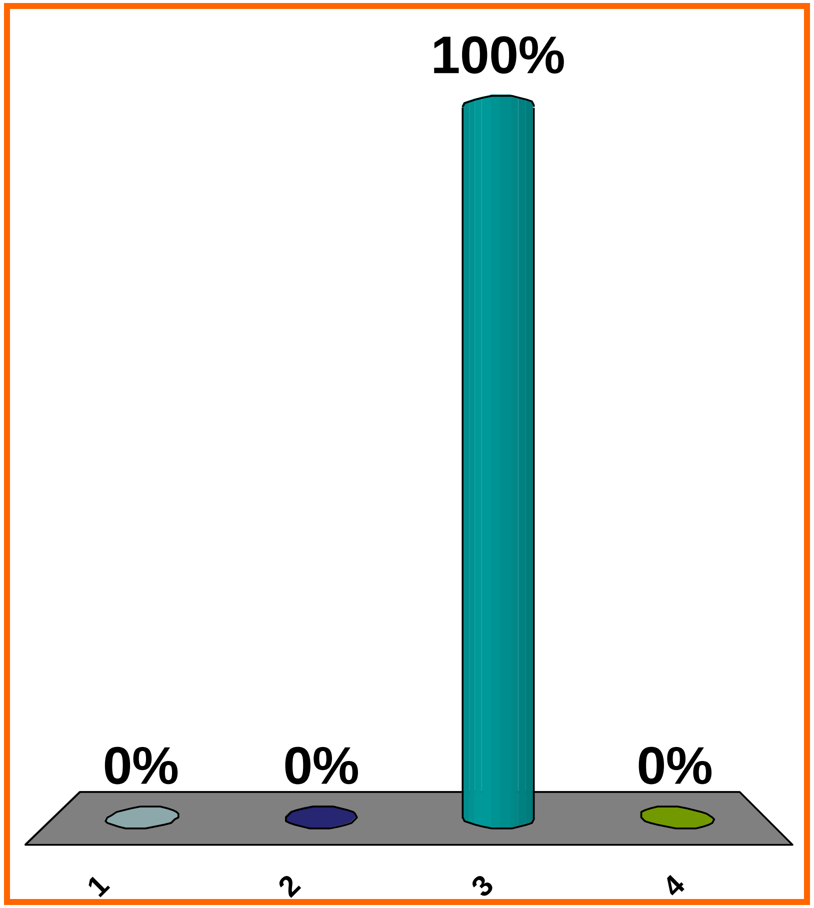
$$\dim\ker\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}=$$

(a) 0

(b) 1

(c) $\boxed{2}$

(d) none of the above



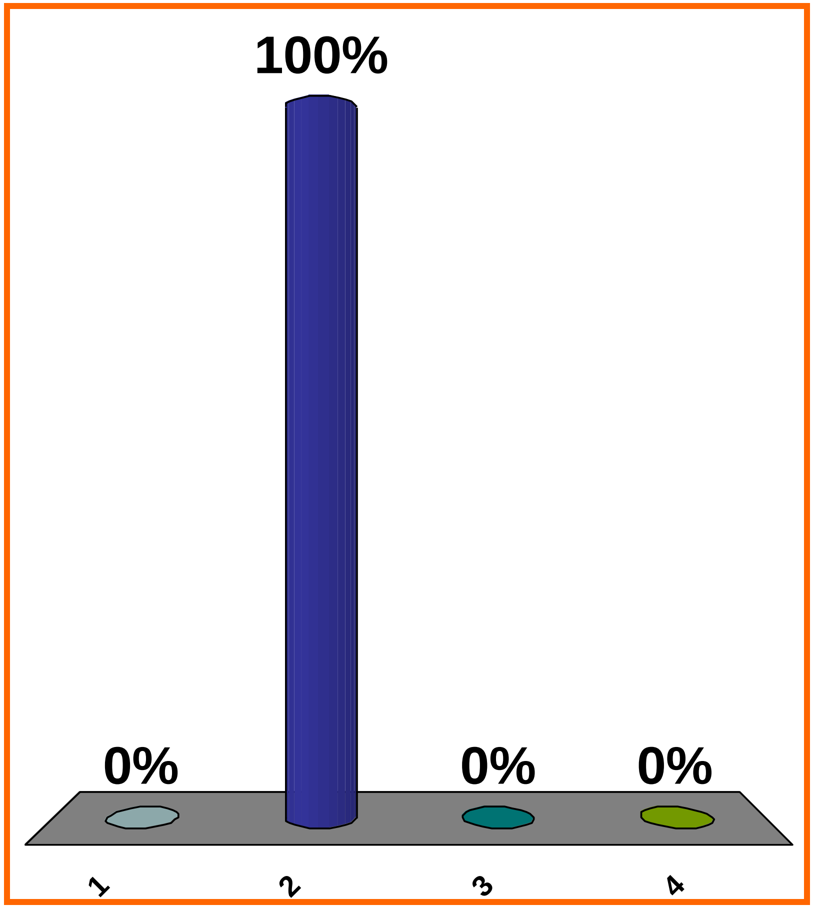
$$\dim \ker \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) 2

(d) none of the above



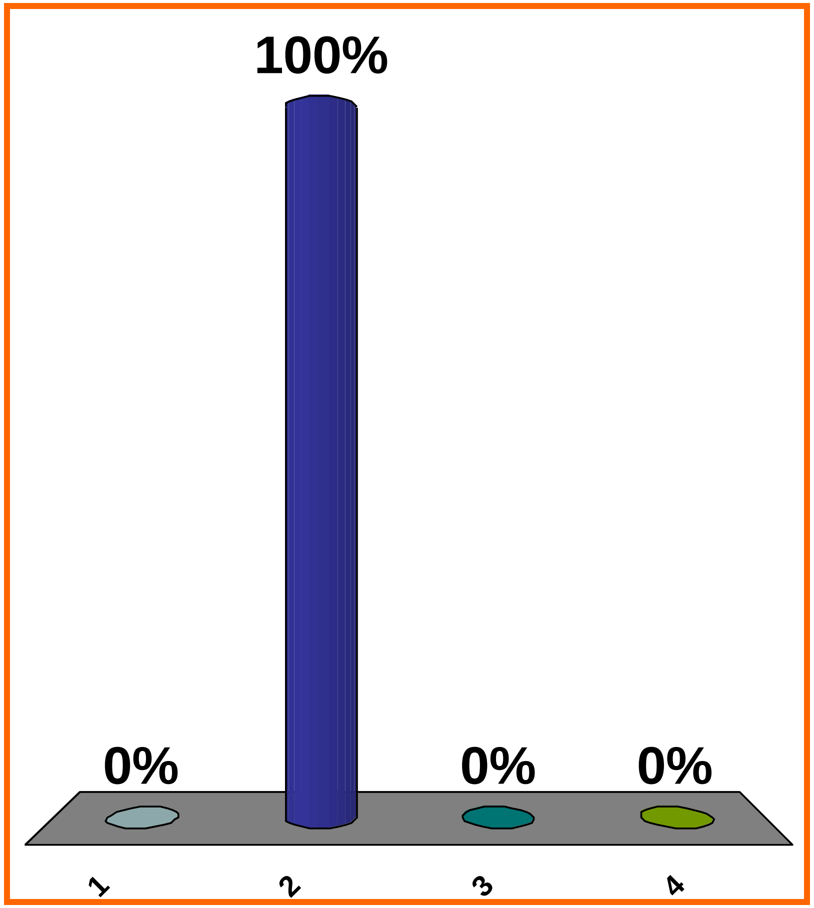
$$\dim \ker \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 0

(b) $\boxed{1}$

(c) 2

(d) none of the above



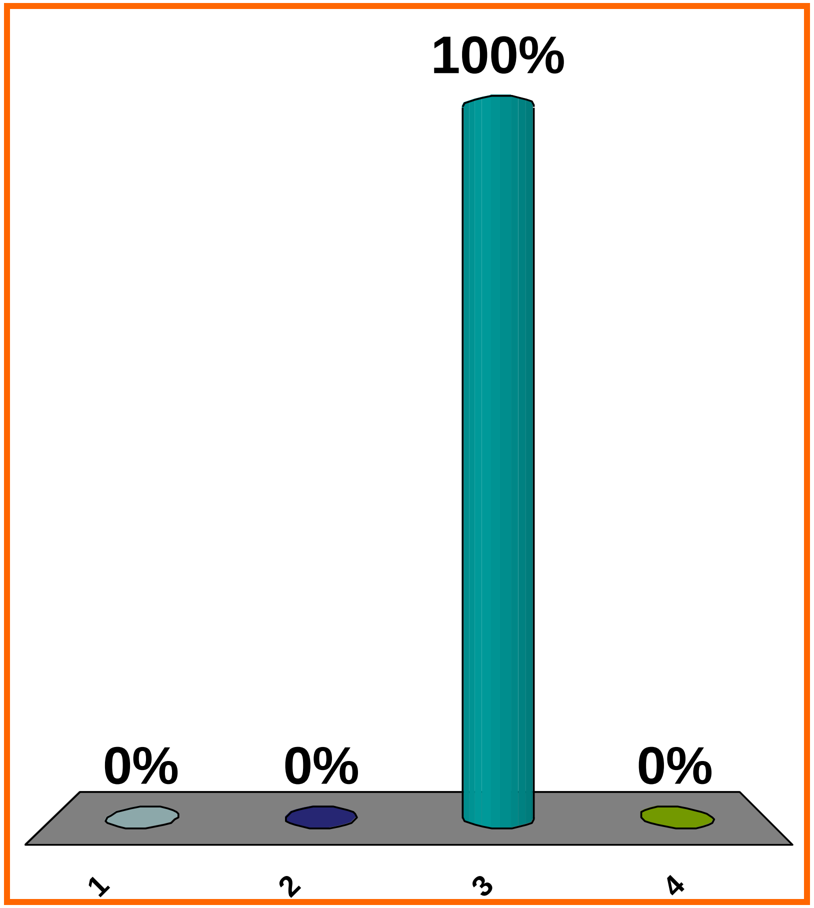
$$\dim \ker \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) $\boxed{2}$

(d) none of the above



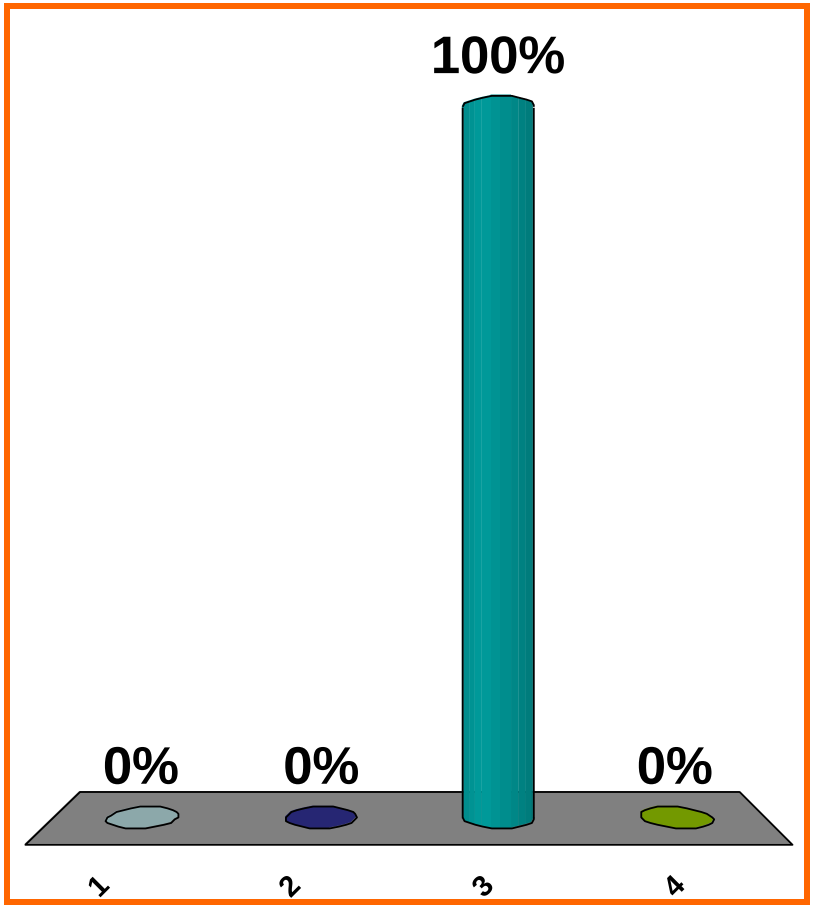
$$\dim \ker \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

(a) 0

(b) 1

(c) $\boxed{2}$

(d) none of the above



$$A = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$$

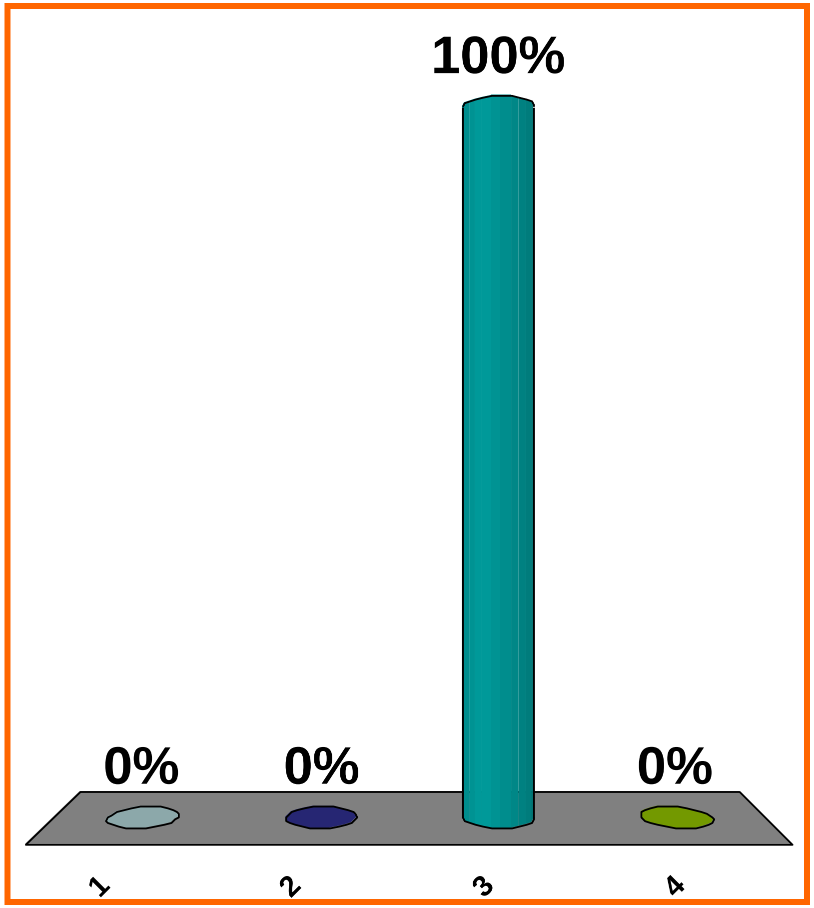
$$A \otimes B =$$

(a) $\begin{bmatrix} \begin{bmatrix} -2 & 0 \\ 4 & 0 \end{bmatrix} & \begin{bmatrix} -4 & 0 \\ 8 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(b) $\begin{bmatrix} -2 & -4 \\ 4 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} \begin{bmatrix} -2 & -4 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 4 & 8 \\ -2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(d) none of the above



$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$$

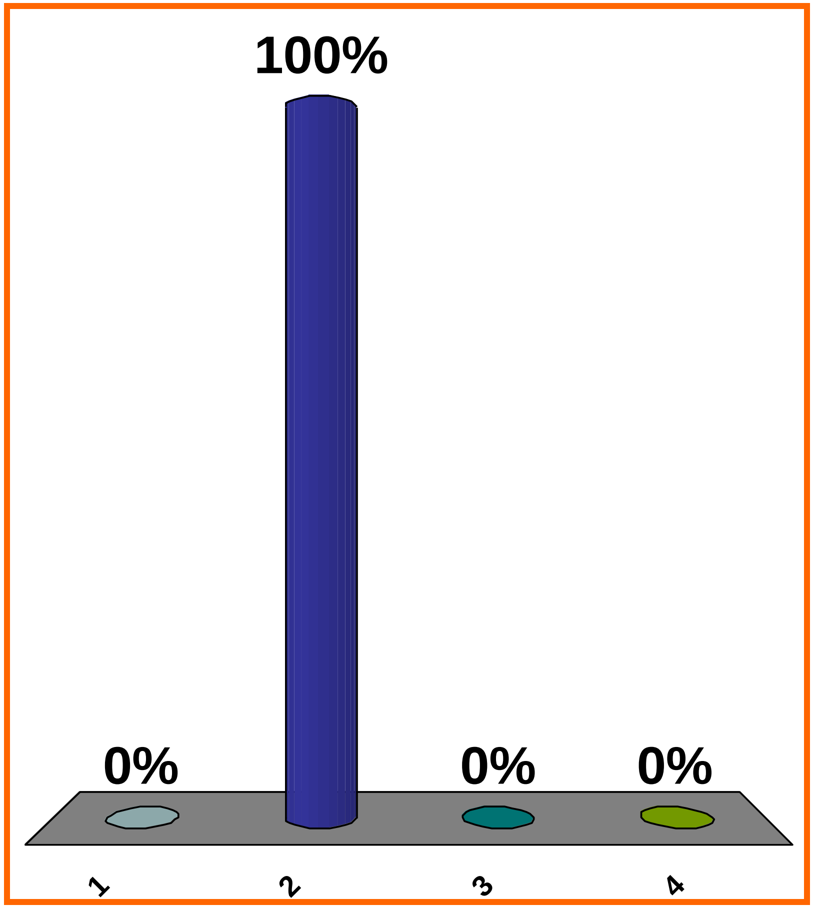
$$A \otimes B =$$

(a) $\begin{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \end{bmatrix}$

(b) $\begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \\ \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(c) $\begin{bmatrix} \begin{bmatrix} -2 & -4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(d) none of the above



$$A = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}$$

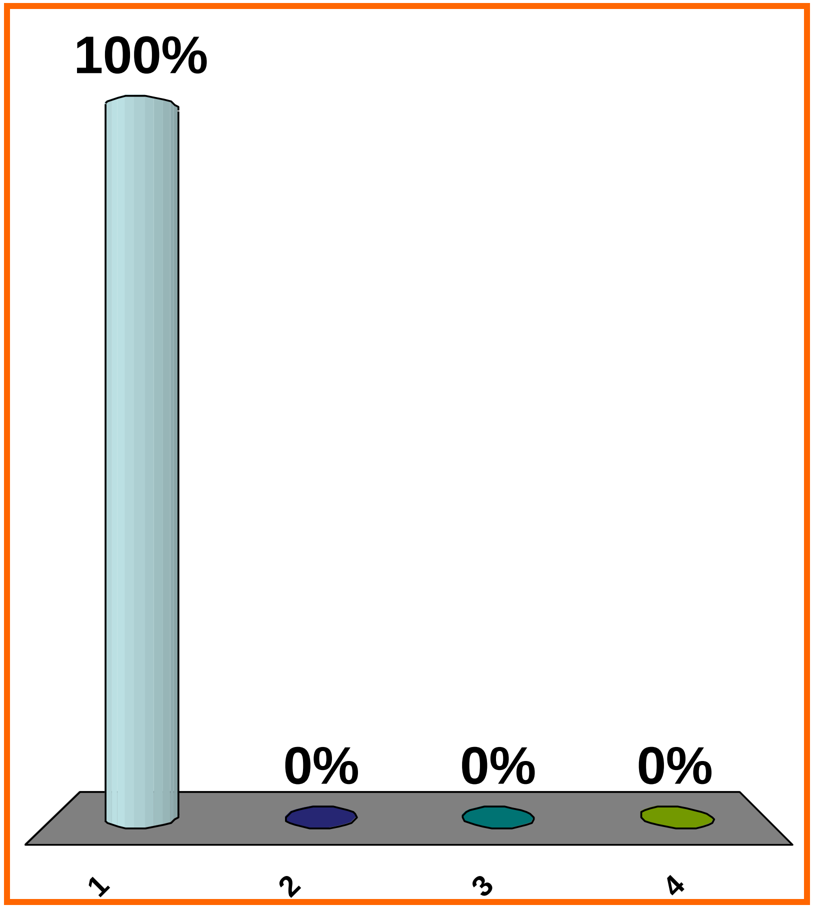
$$A \otimes B =$$

(a) $\begin{bmatrix} \begin{bmatrix} -2 & 0 \\ 4 & 0 \end{bmatrix} & \begin{bmatrix} -4 & 0 \\ 8 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(b) $\begin{bmatrix} \begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix} & \begin{bmatrix} -4 & 0 \\ 8 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(c) $\begin{bmatrix} \begin{bmatrix} -2 & -4 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 4 & 8 \\ -2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(d) none of the above



$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$$

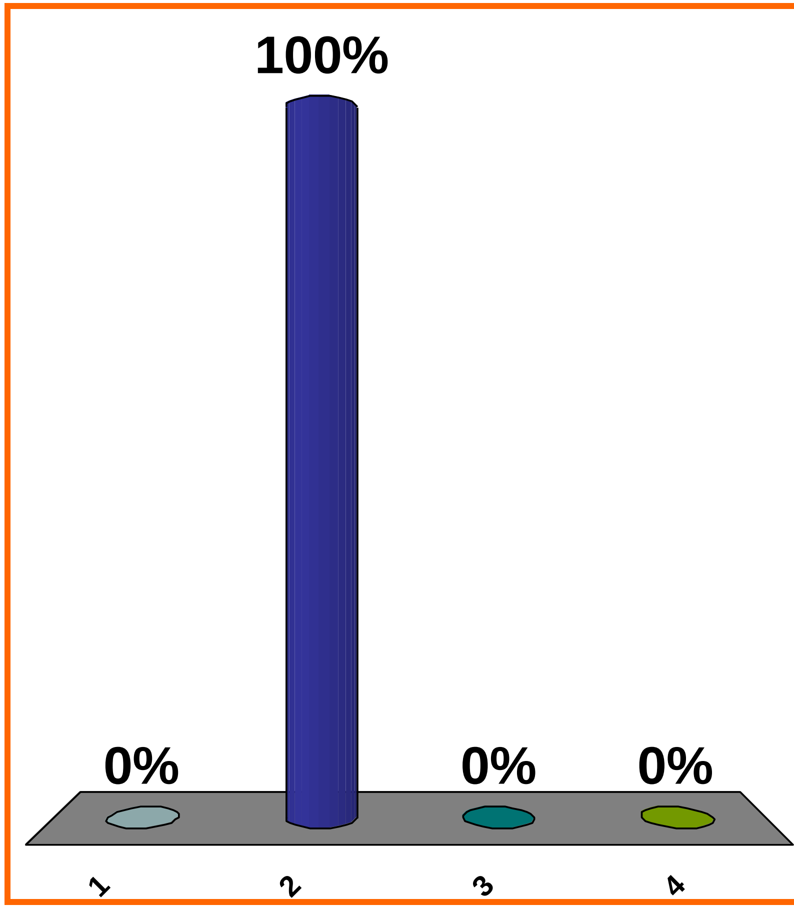
$$A \otimes B =$$

(a) $\begin{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \end{bmatrix}$

(b) $\begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \\ \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(c) $\begin{bmatrix} \begin{bmatrix} -2 & -4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(d) none of the above



$$A = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$$

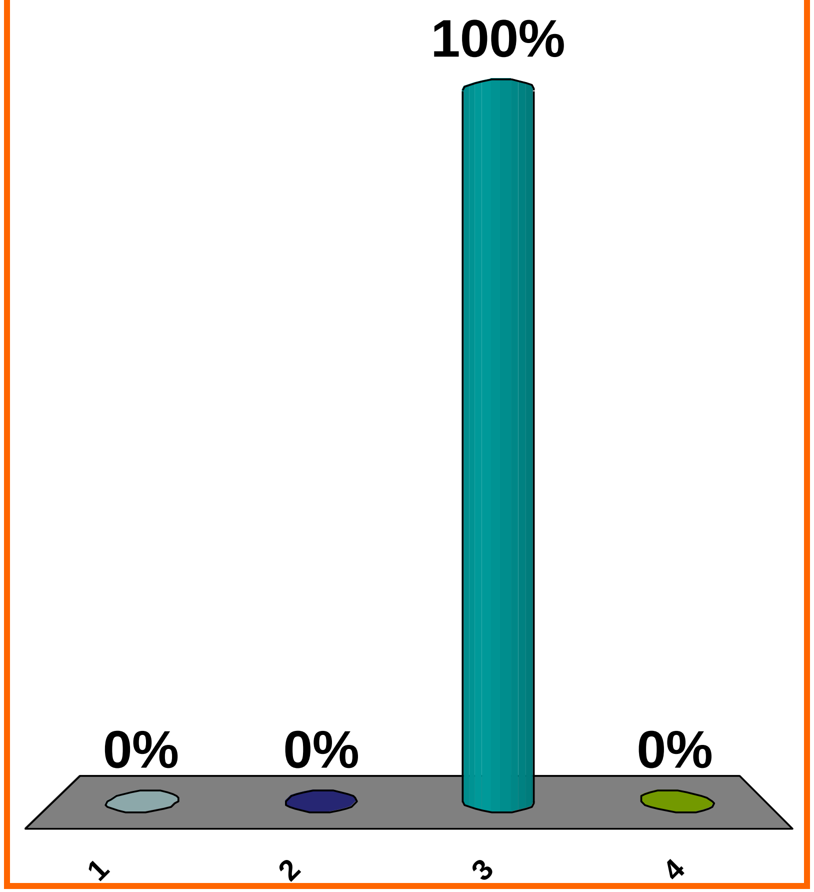
$$\mathcal{M}(A \otimes B) =$$

(a) $\begin{bmatrix} \begin{bmatrix} -2 & 0 \\ 4 & 0 \end{bmatrix} & \begin{bmatrix} -4 & 0 \\ 8 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$

(d) none of the above



$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$$

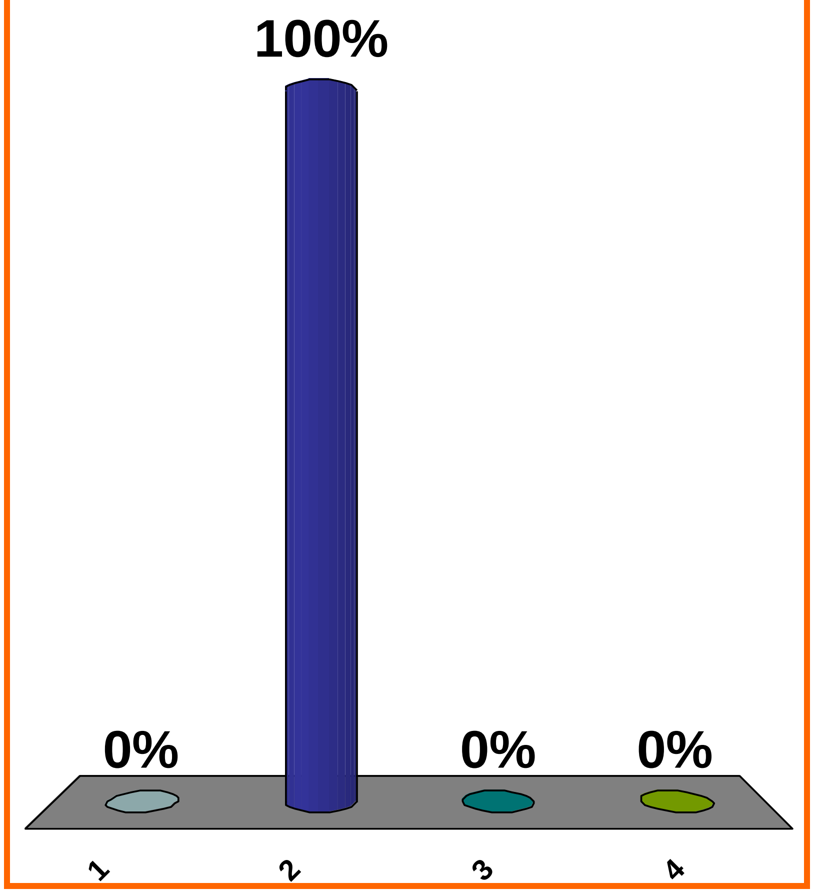
$$\mathcal{M}(A \otimes B) =$$

(a) $\begin{bmatrix} 4 & 2 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$

(d) none of the above



$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$$

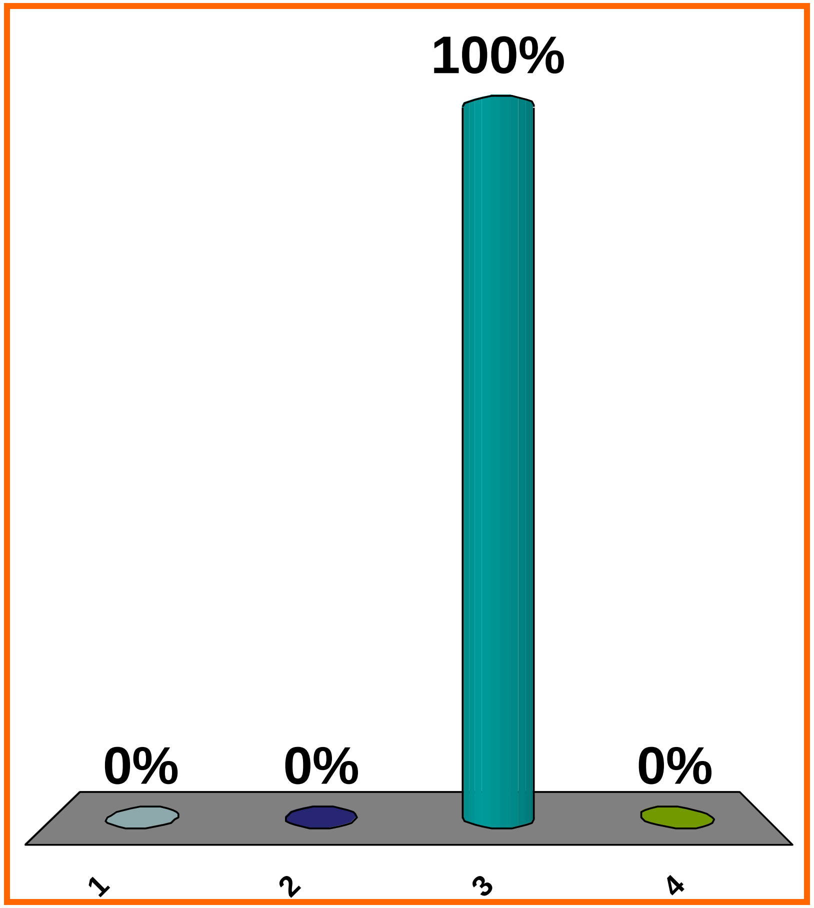
$$A \oplus B =$$

(a) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 4 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 4 \\ -1 & 1 \end{bmatrix}$

(c) $\boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -1 & 0 \end{bmatrix}}$

(d) none of the above



$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

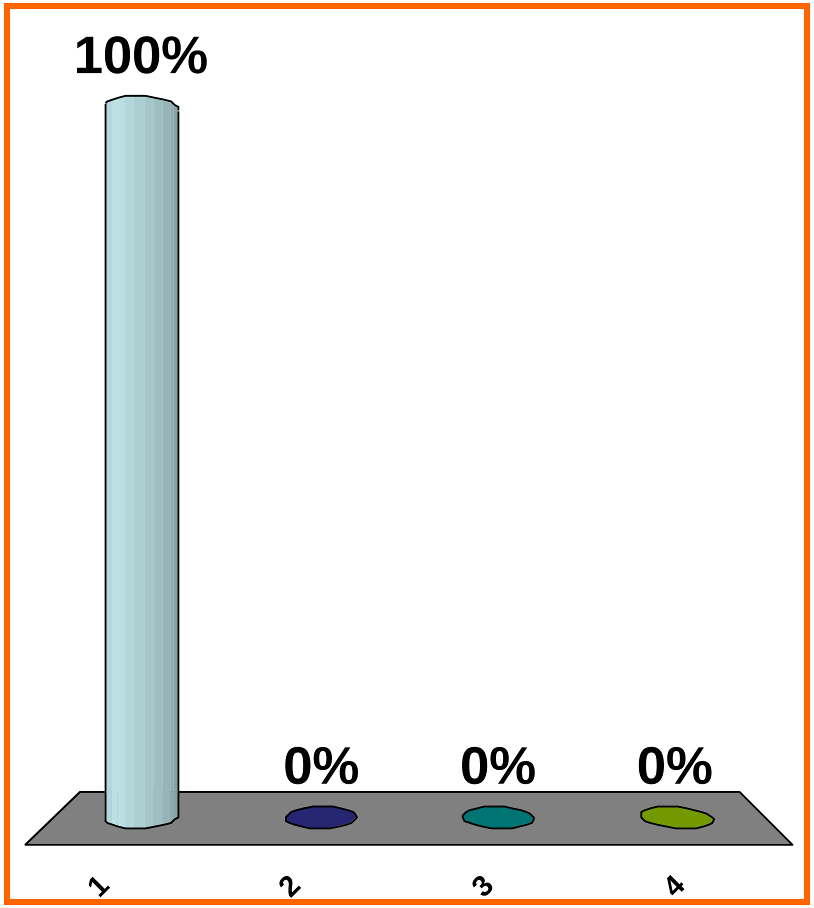
$$A \oplus B =$$

(a) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \\ 5 & 6 & 5 & 6 \\ 7 & 8 & 7 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 6 & 0 & 0 \\ 7 & 8 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{bmatrix}$

(d) none of the above



$$\begin{array}{c} \\ M := \end{array} \begin{array}{c} \begin{array}{ccc} \color{blue}{1} & \color{blue}{0} & \color{blue}{1} \end{array} \\ \left[\begin{array}{ccc} 1 & 0 & 3 \\ -1 & 2 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ \color{blue}{-1} \\ \color{blue}{2} \end{array} \qquad \begin{array}{l} v := (1,0,1), \\ w := (-1,2) \end{array}$$

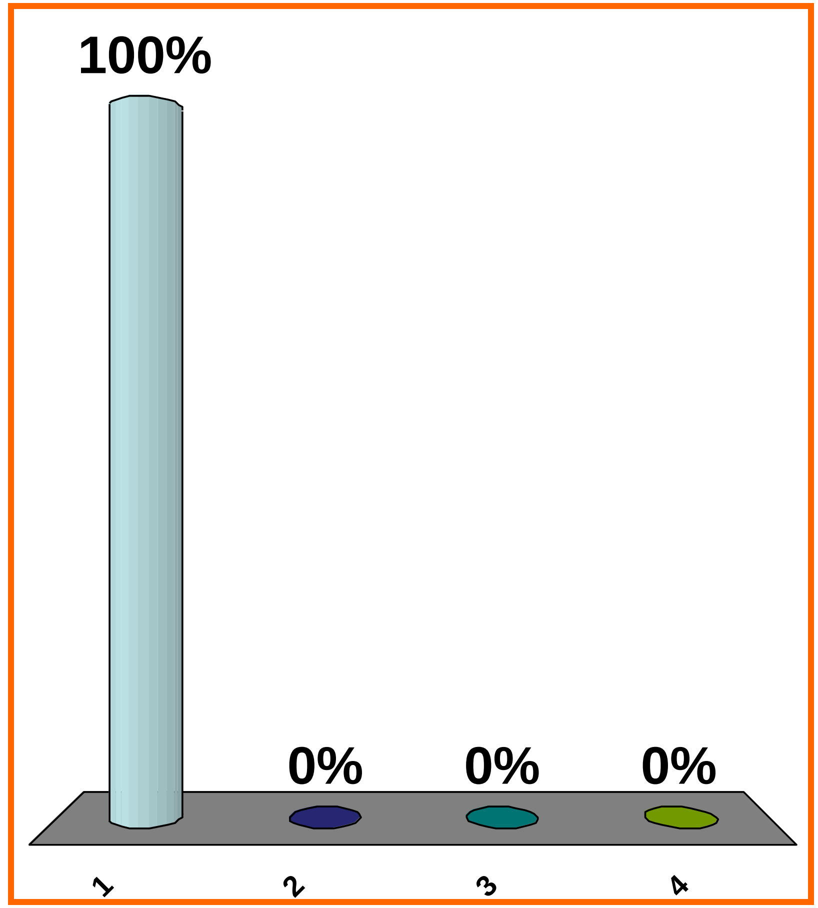
$$(L_M(v)) \cdot w = \color{blue}{??}$$

(a) $2+6+1$

(b) 0

(c) $\boxed{-1-3-2}$

(d) none of the above



$$v := (1,0,1),$$
$$w := (-1,2)$$

$$M := \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \end{bmatrix}$$

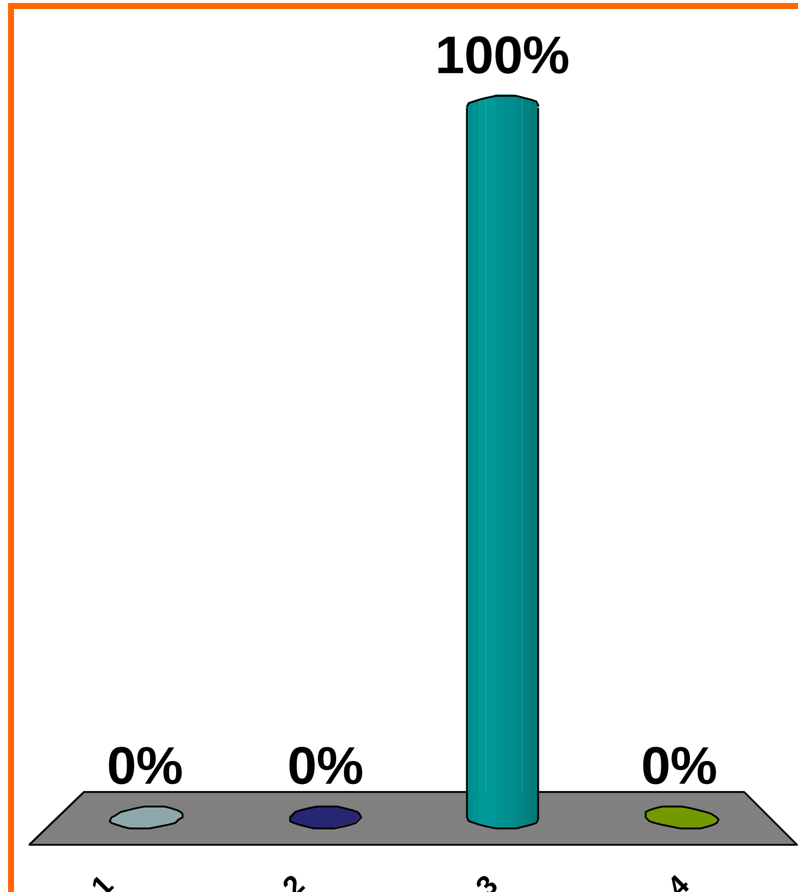
$$v \cdot (L_{M^t}(w)) = ??$$

(a) $2+6+1$

(b) 0

(c) $\boxed{-1-3-2}$

(d) none of the above



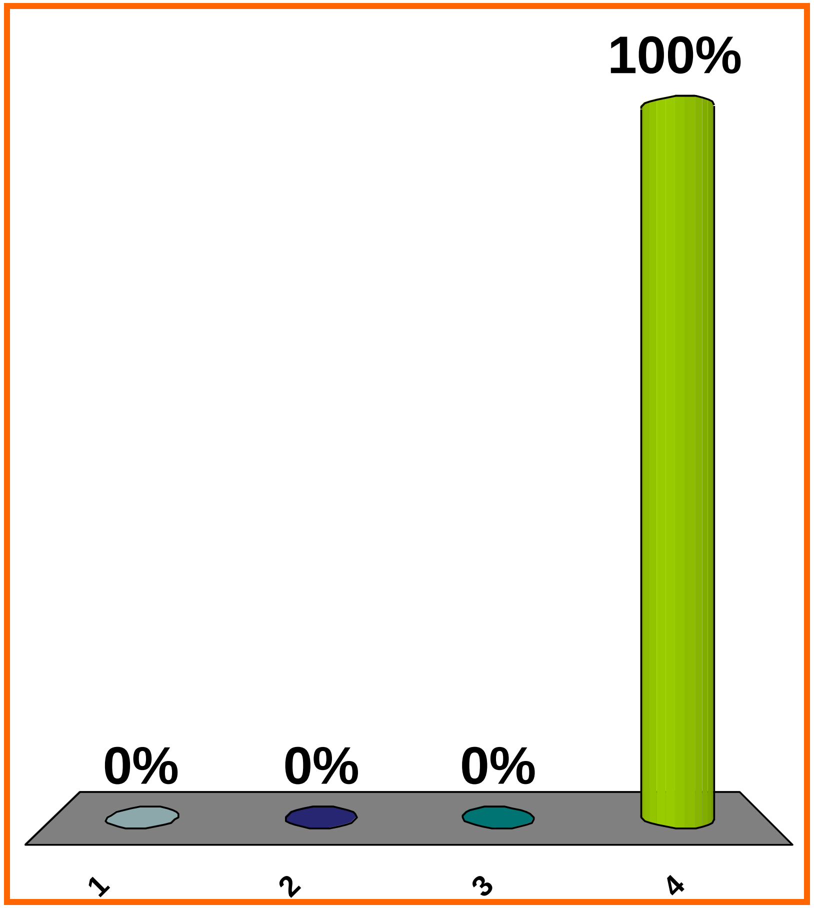
Which is standard nilpotent?

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(d) none of the above



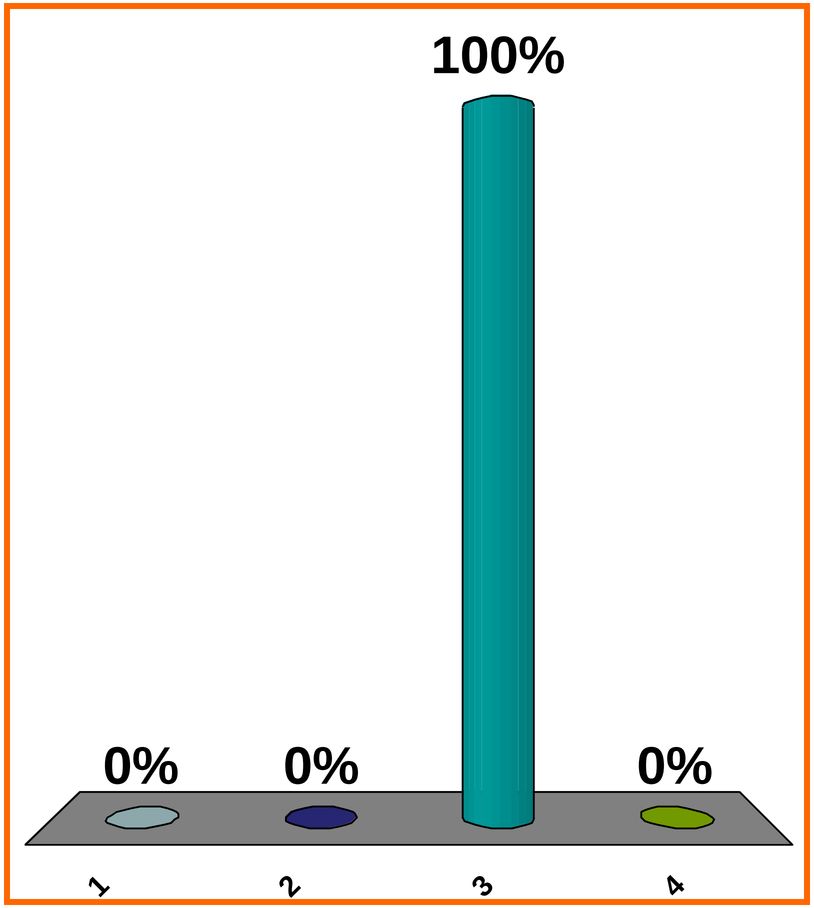
Which is standard nilpotent?

(a) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) none of the above



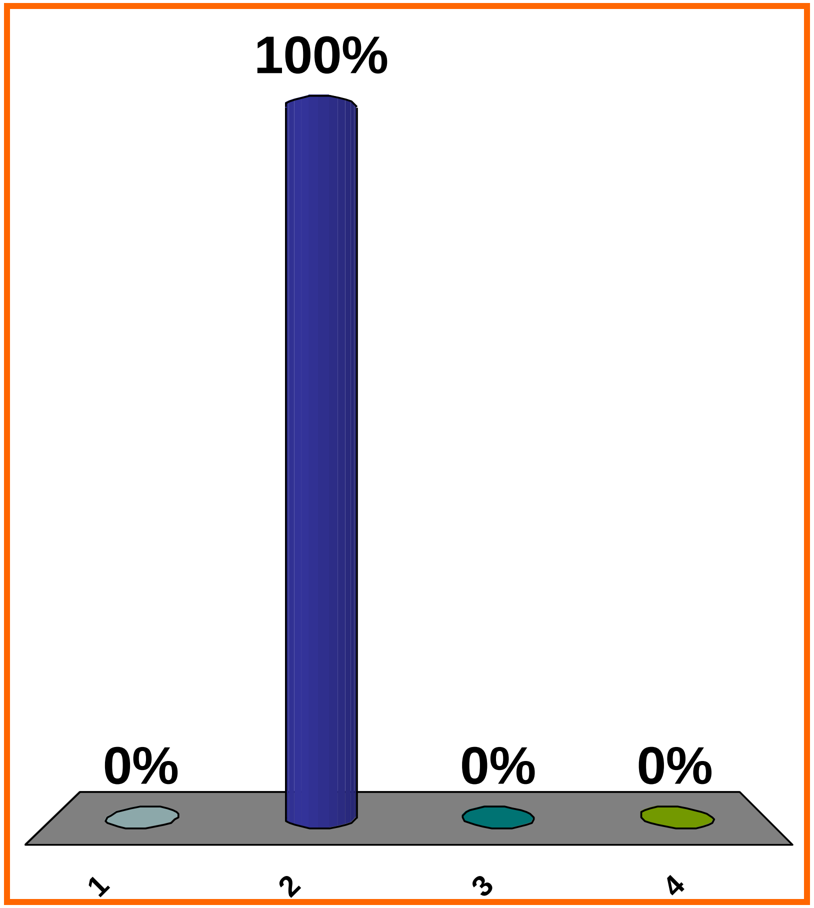
Which is scalar?

(a) $\begin{bmatrix} 8 & 1 & 0 & 0 & 0 \\ 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(b) $\boxed{\begin{bmatrix} 8 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}}$

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) none of the above



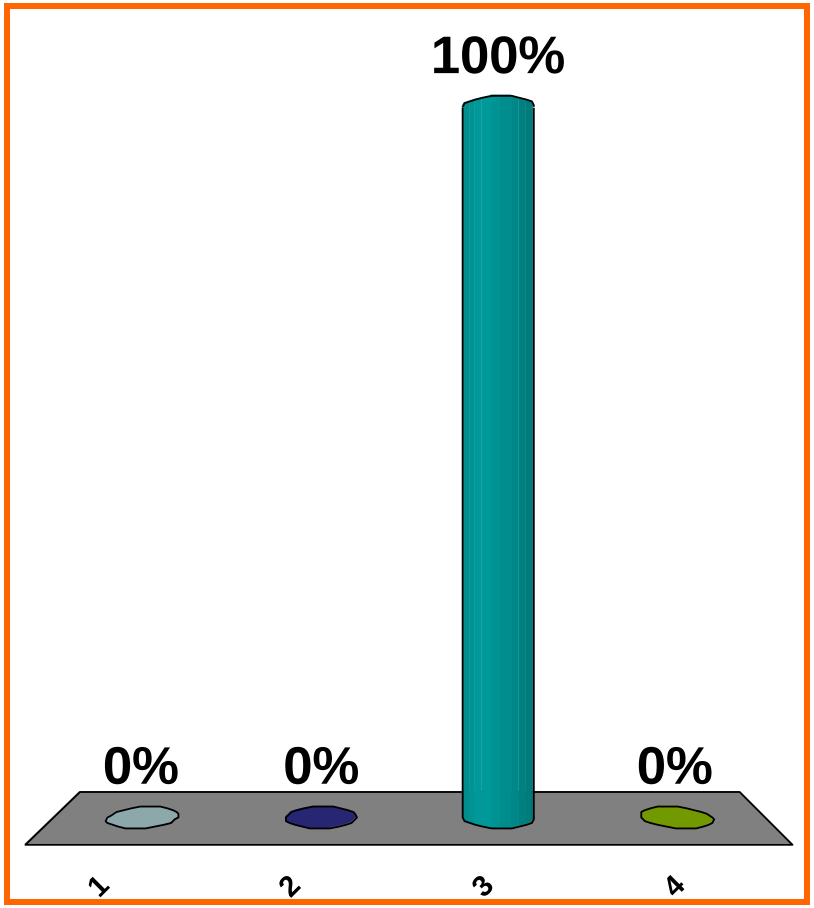
Which is scalar?

(a) $\begin{bmatrix} 8 & 0 & 0 & 0 & 1 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 1 & 0 & 0 & 0 \\ 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(c) $\boxed{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$

(d) none of the above



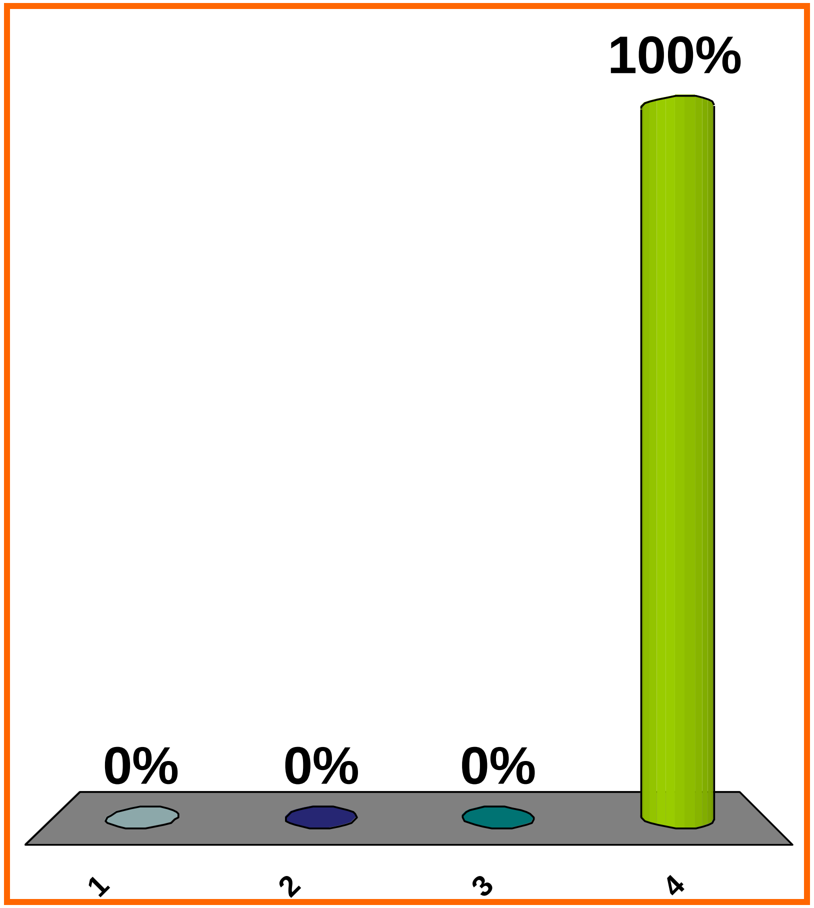
Which is scalar?

(a) $\begin{bmatrix} 8 & 0 & 0 & 0 & 1 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 1 & 0 & 0 & 0 \\ 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) none of the above



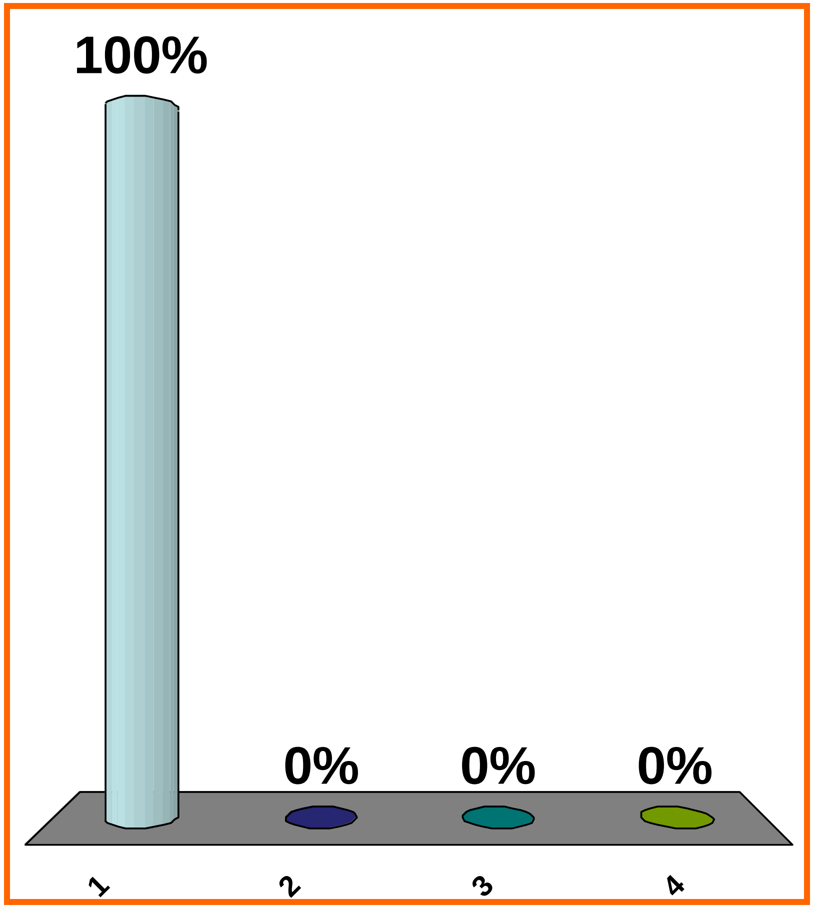
Which is a Jordan block?

(a) $\begin{bmatrix} 8 & 1 & 0 & 0 & 0 \\ 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$

(d) none of the above



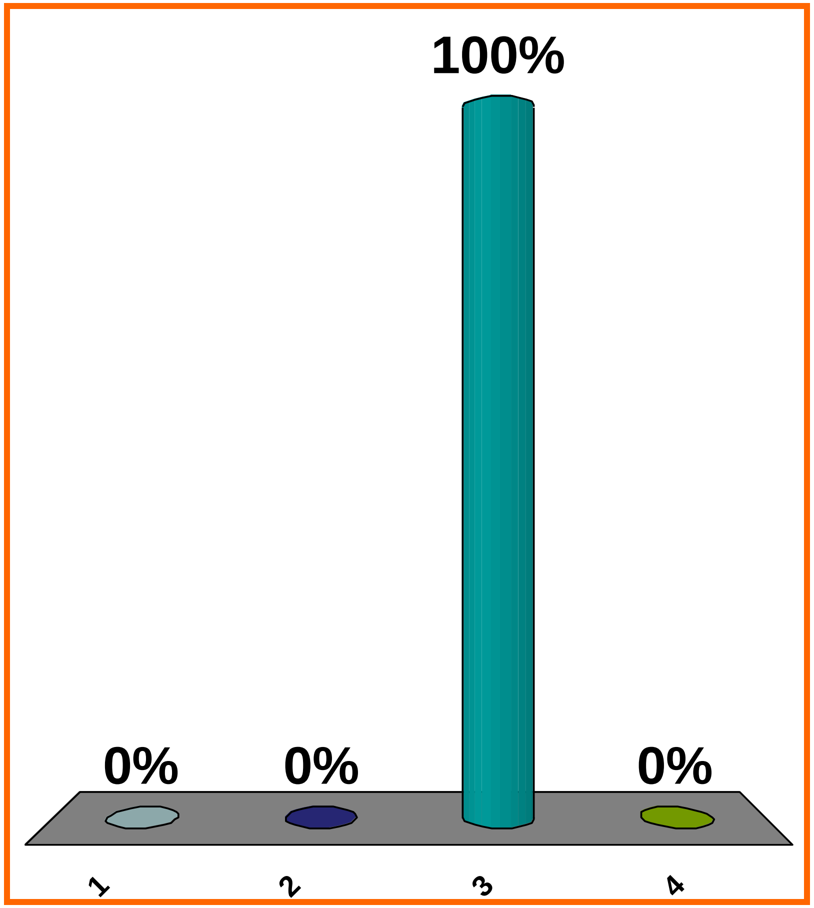
Which is a Jordan block?

(a)

(b)

(c)
$$\begin{bmatrix} -4 & 1 & 0 & 0 & 0 \\ 0 & -4 & 1 & 0 & 0 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{bmatrix}$$

(d) none of the above



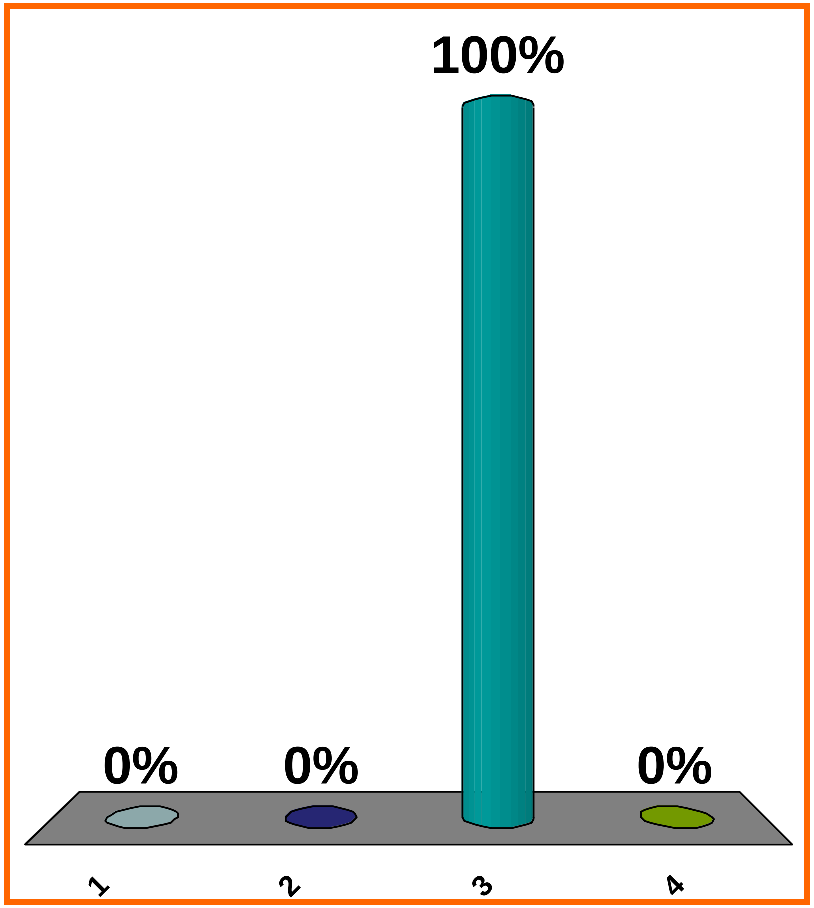
Which is a Jordan block?

(a) $\begin{bmatrix} 5 & 0 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 & 1 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$

$\boxed{\text{(c) } [5]}$

(d) none of the above



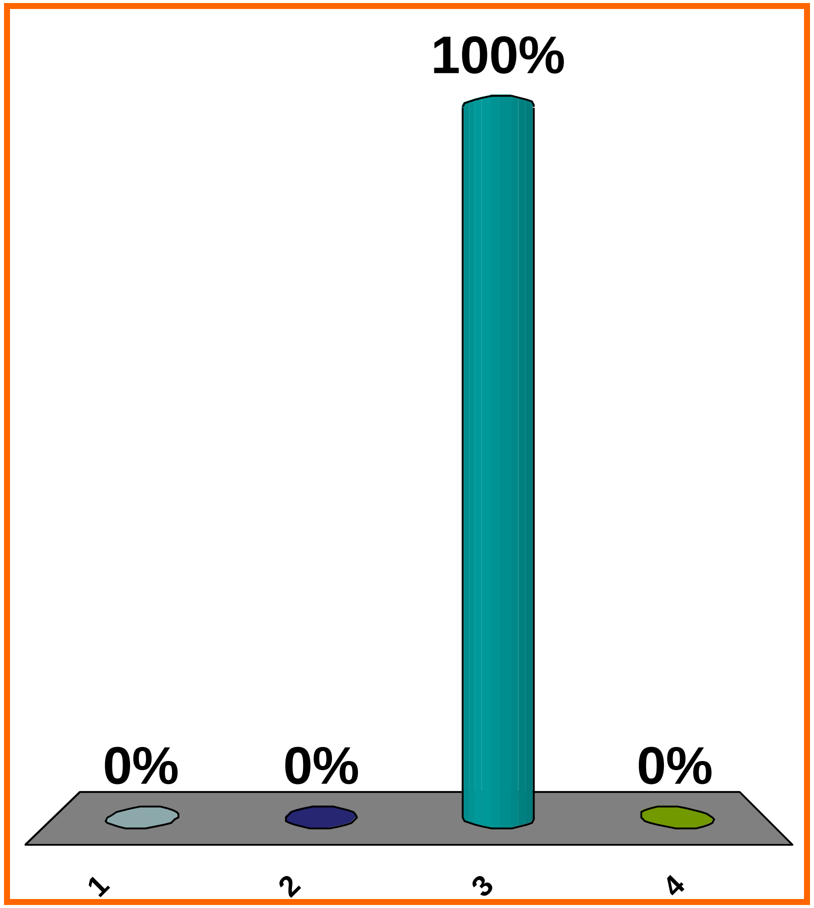
$$\exp \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} e^2 & 1 & 1 \\ 1 & e^3 & 1 \\ 1 & 1 & e^{-4} \end{bmatrix}$

(c) $\boxed{\begin{bmatrix} e^2 & 0 & 0 \\ 0 & e^3 & 0 \\ 0 & 0 & e^{-4} \end{bmatrix}}$

(d) none of the above



$$\exp\begin{bmatrix} 6 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

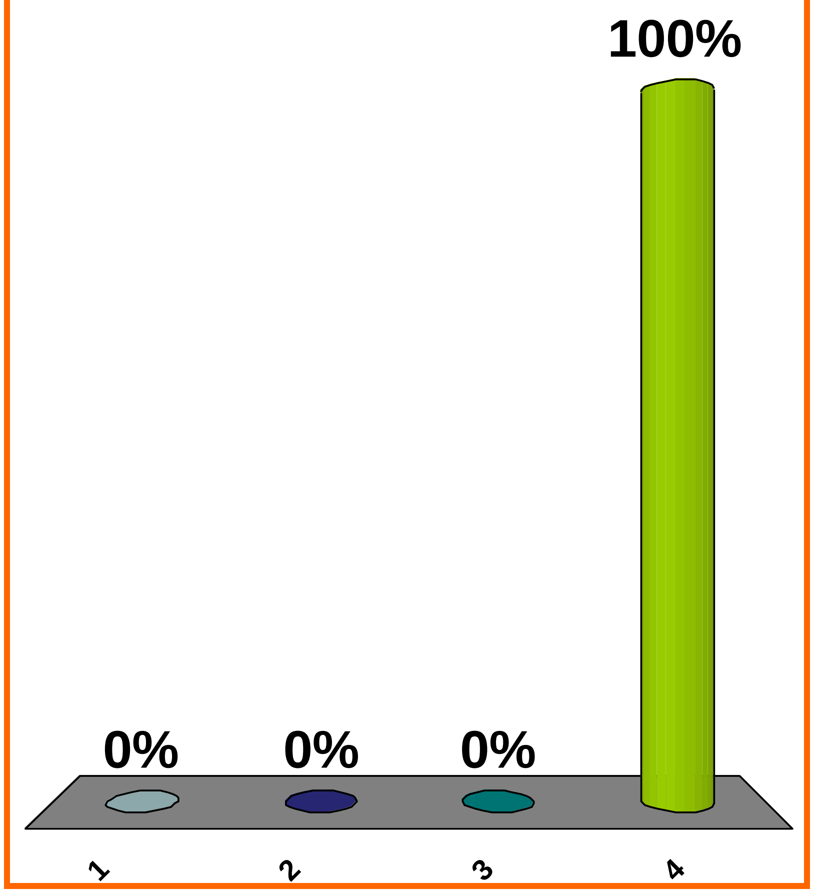
(a) $\begin{bmatrix} 6 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} e^6 & 1 & 1 \\ 1 & e^{-1} & 1 \\ 1 & 1 & e^2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\begin{bmatrix} e^6 & 0 & 0 \\ 0 & e^{-1} & 0 \\ 0 & 0 & e^2 \end{bmatrix}$$

(d) none of the above



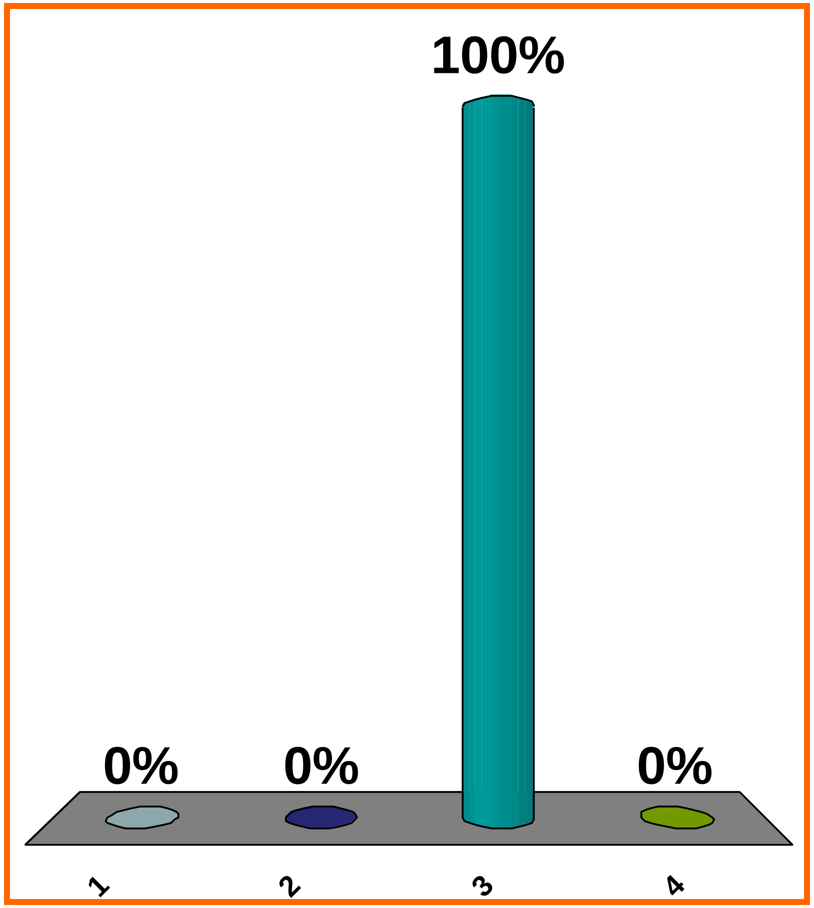
$$\exp \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

(a) $\begin{bmatrix} e^{-1} & 1 & 1 \\ 1 & e^{-2} & 1 \\ 1 & 1 & e^{-3} \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} e^{-1} & 0 & 0 \\ 0 & e^{-2} & 0 \\ 0 & 0 & e^{-3} \end{bmatrix}$

(d) none of the above



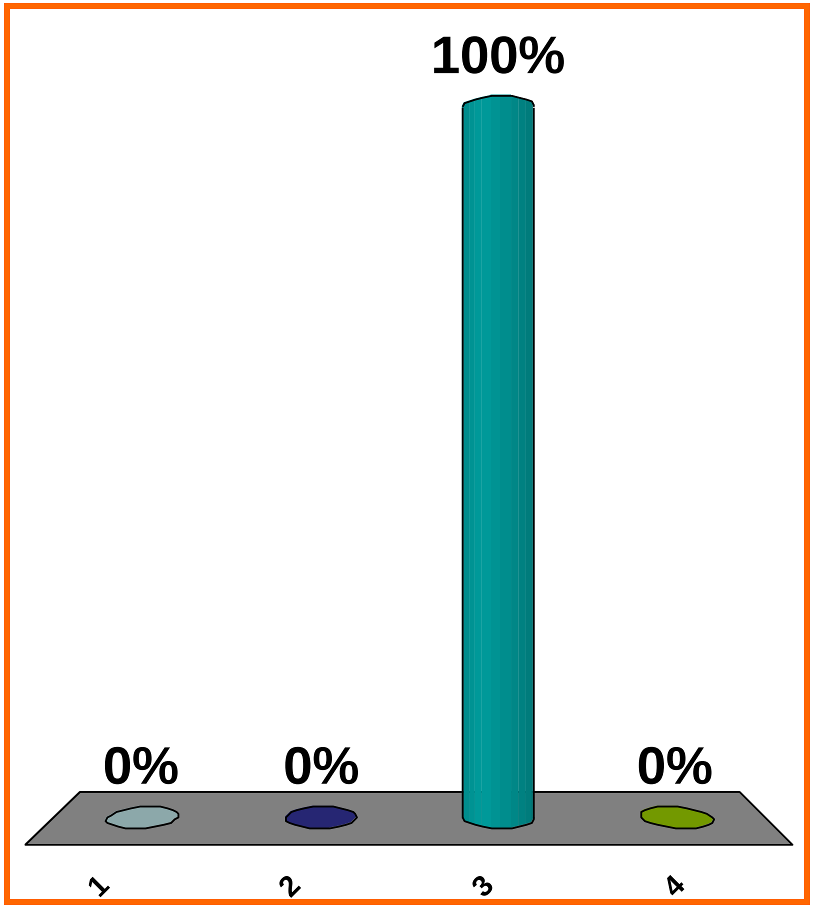
$$\exp \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & e \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) none of the above



$$\exp\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

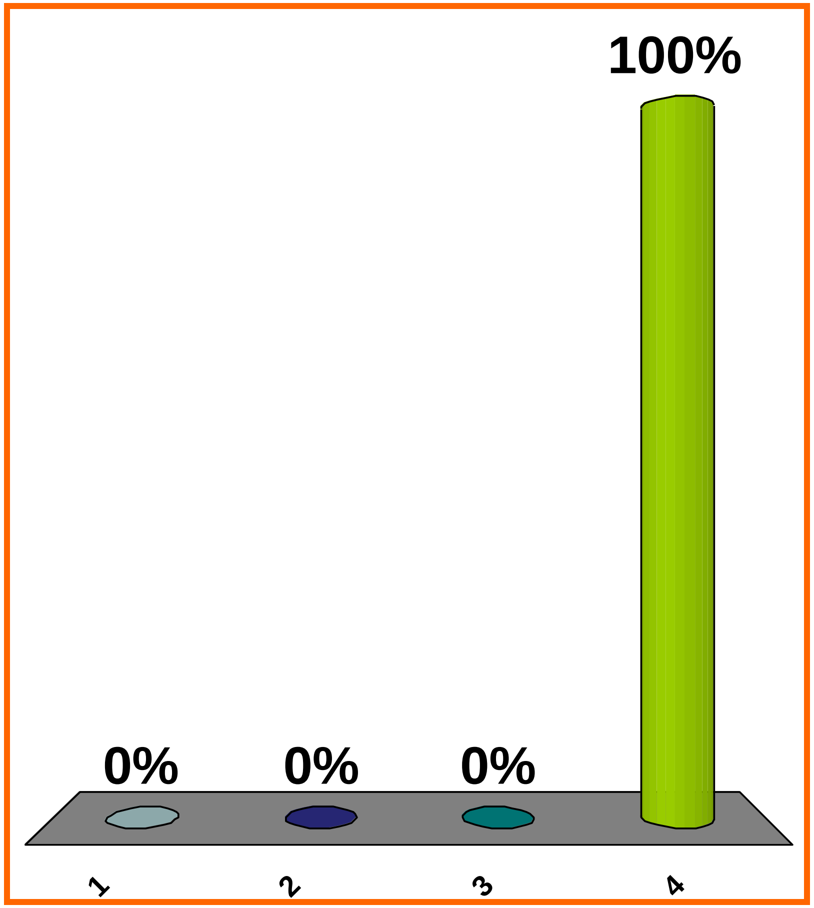
(a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & e & 1 \\ 1 & 1 & e \\ 1 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\text{(d) none of the above}}$



$$\exp\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

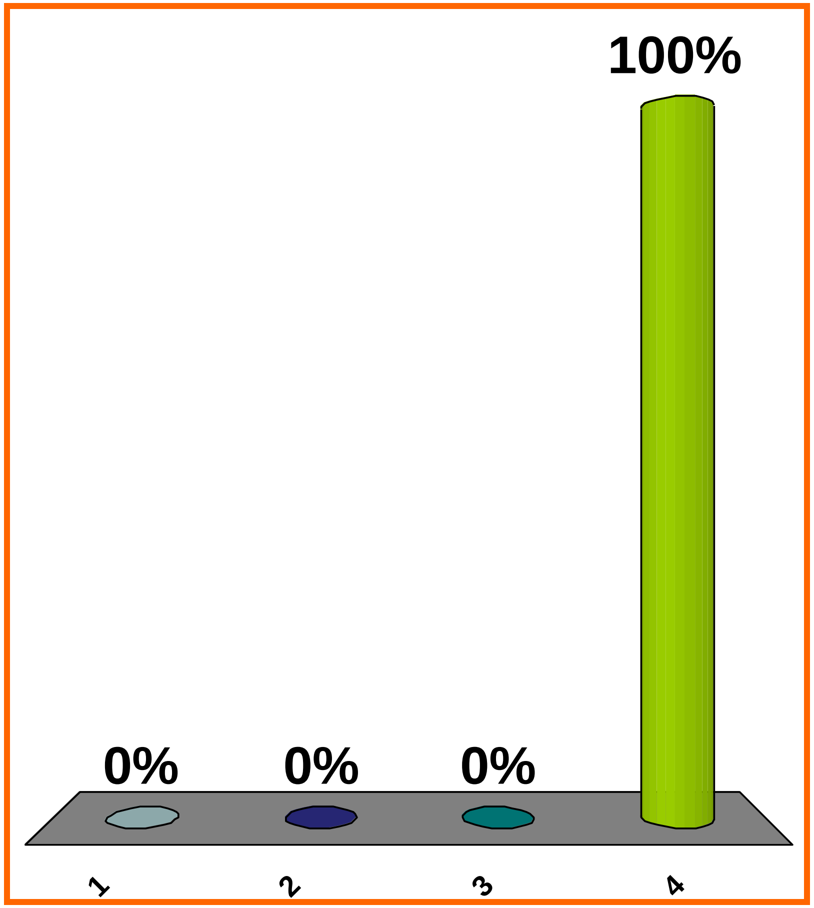
(a) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & e & e^2 \\ 1 & 1 & e \\ 1 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 5/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\text{(d) none of the above}}$



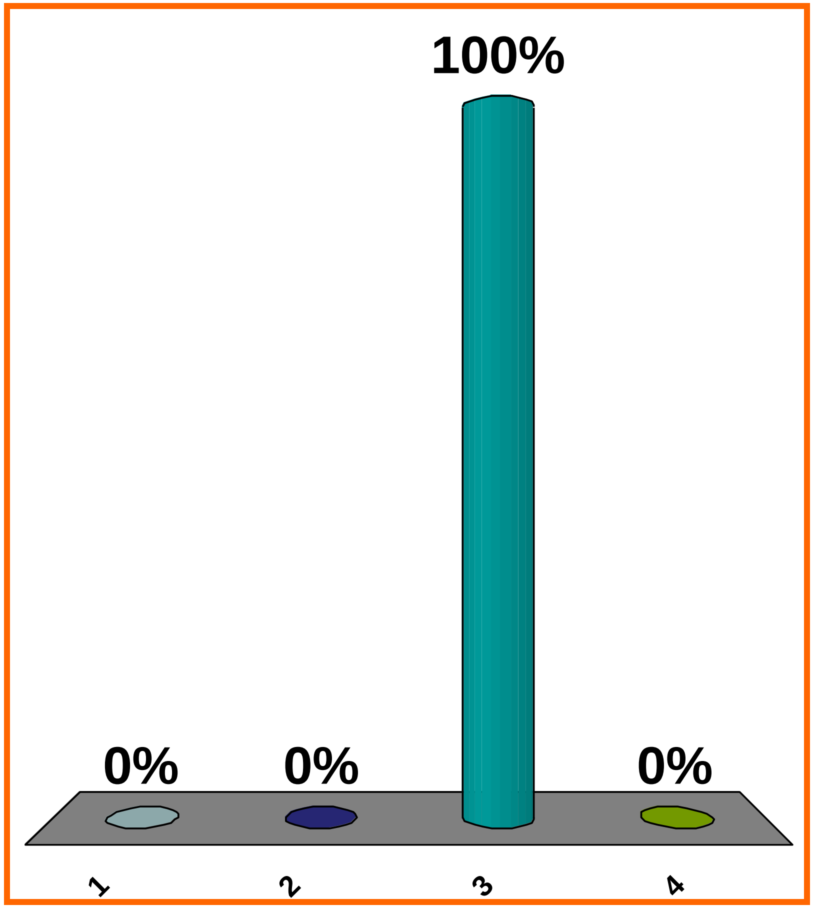
$$\exp\begin{bmatrix} 0 & 1 & -4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) $\begin{bmatrix} 0 & 1 & -4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & e & e^{-4} \\ 1 & 1 & e \\ 1 & 1 & 1 \end{bmatrix}$

(c) $\boxed{\begin{bmatrix} 1 & 1 & -7/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$

(d) none of the above



$$A := \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\exp A = ??$$

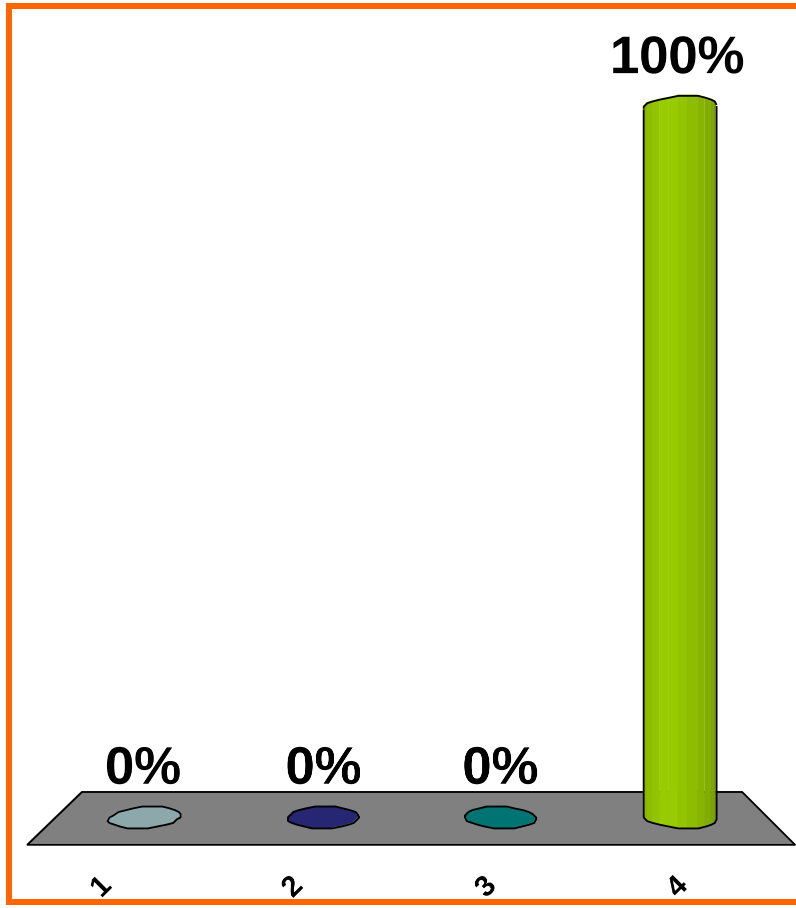
(a) $\begin{bmatrix} e^2 & e \\ 1 & e^2 \end{bmatrix}$

(b) $\begin{bmatrix} e^2 & e \\ 0 & e^2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3/2 \\ 0 & 3 \end{bmatrix}$

$$\begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$\boxed{\text{(d) none of the above}}$



$$A := \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

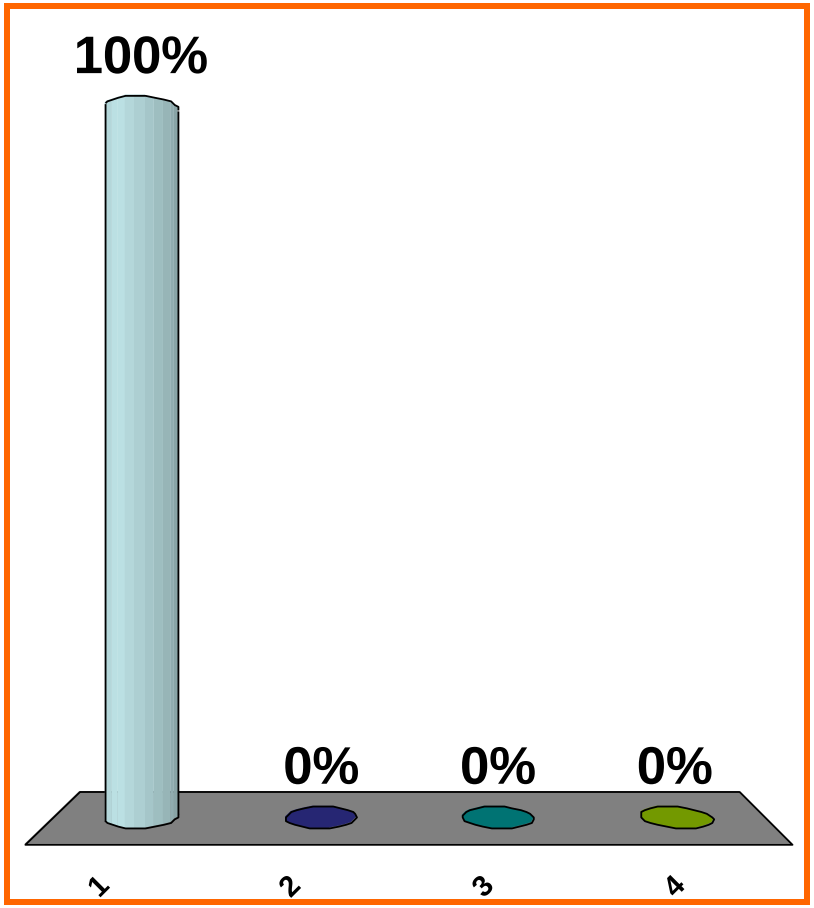
$$\exp A = ??$$

(a) $\begin{bmatrix} e^{-1} & 0 & 0 \\ 0 & e^{-1} & 0 \\ 0 & 0 & e^{-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} e^{-1} & 1 & 1/2 \\ 0 & e^{-1} & 1 \\ 0 & 0 & e^{-1} \end{bmatrix}$

(d) none of the above



$$A := \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, B := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

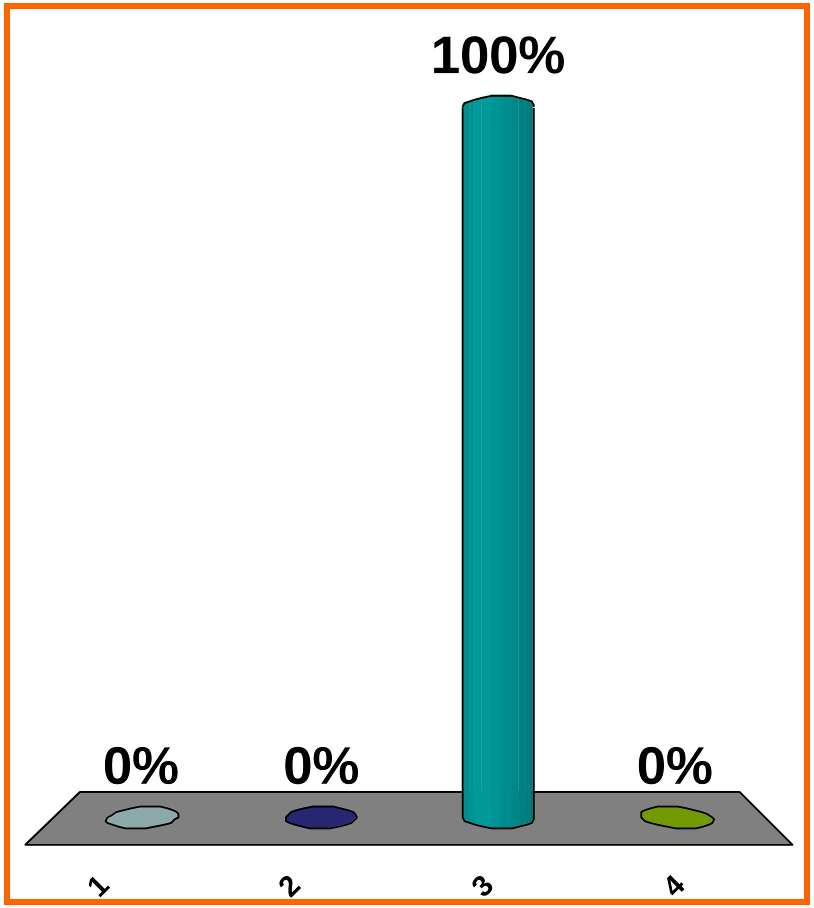
$$\exp(BAB^{-1}) = ??$$

(a) $[\exp(B)][\exp(A)][\exp(B^{-1})]$

(b) $[\exp(B)][\exp(A)][(\exp(B))^{-1}]$

(c) $\boxed{[B][\exp(A)][B^{-1}]}$

(d) none of the above



(a) This image cannot currently be displayed.

(b) This image cannot currently be displayed.

(c) This image cannot currently be displayed.

(d) none of the above

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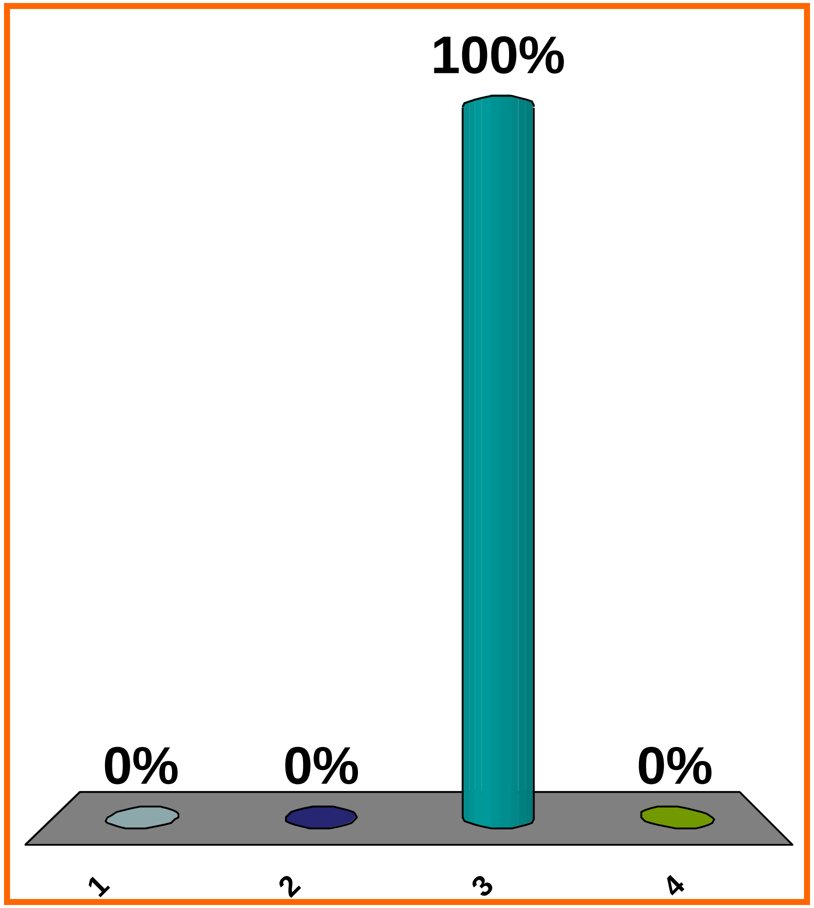
$$\exp(BAB^{-1}) = ??$$

(a) $[\exp(B)][\exp(A)][\exp(B^{-1})]$

(b) $[\exp(B)][\exp(A)][(\exp(B))^{-1}]$

(c) $[B][\exp(A)][B^{-1}]$

(d) none of the above



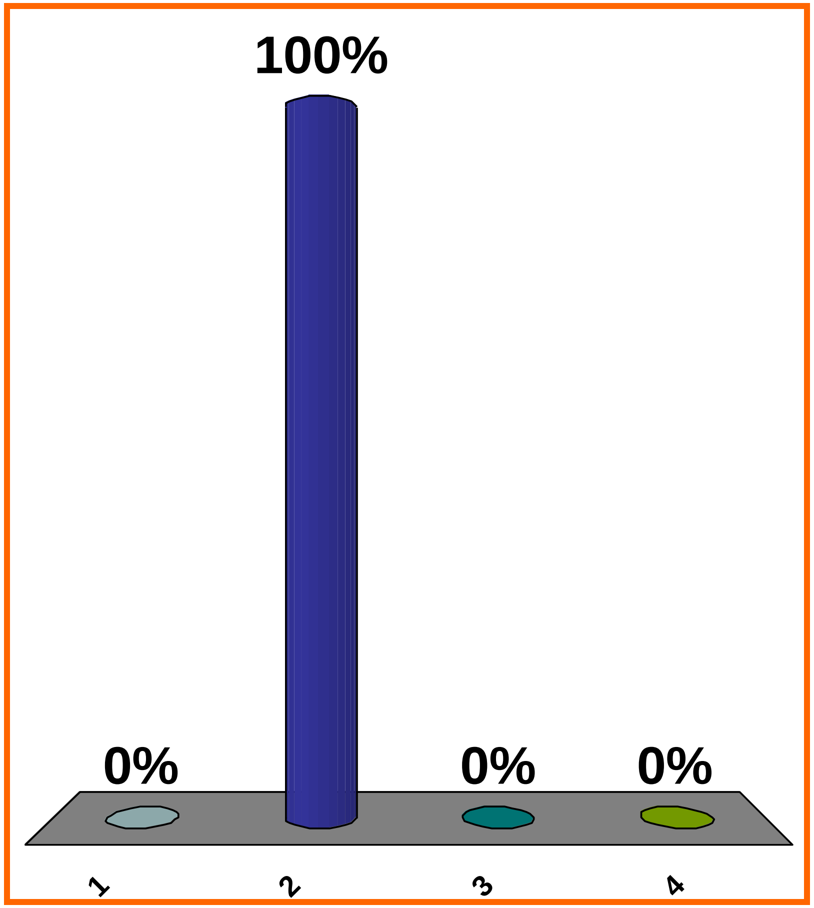
$$\dim \ker \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

(a) 1

(b) 2

(c) $\boxed{3}$

(d) none of the above



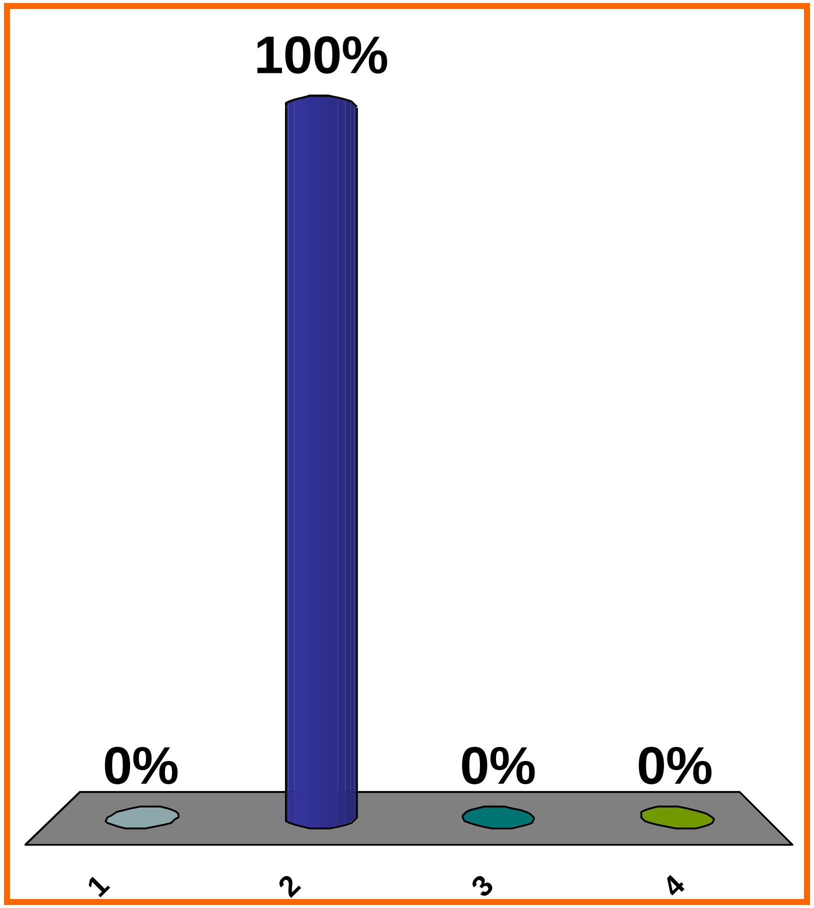
$$\dim \operatorname{im} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

$\boxed{\text{(a) } 1}$

(b) 2

(c) 3

(d) none of the above



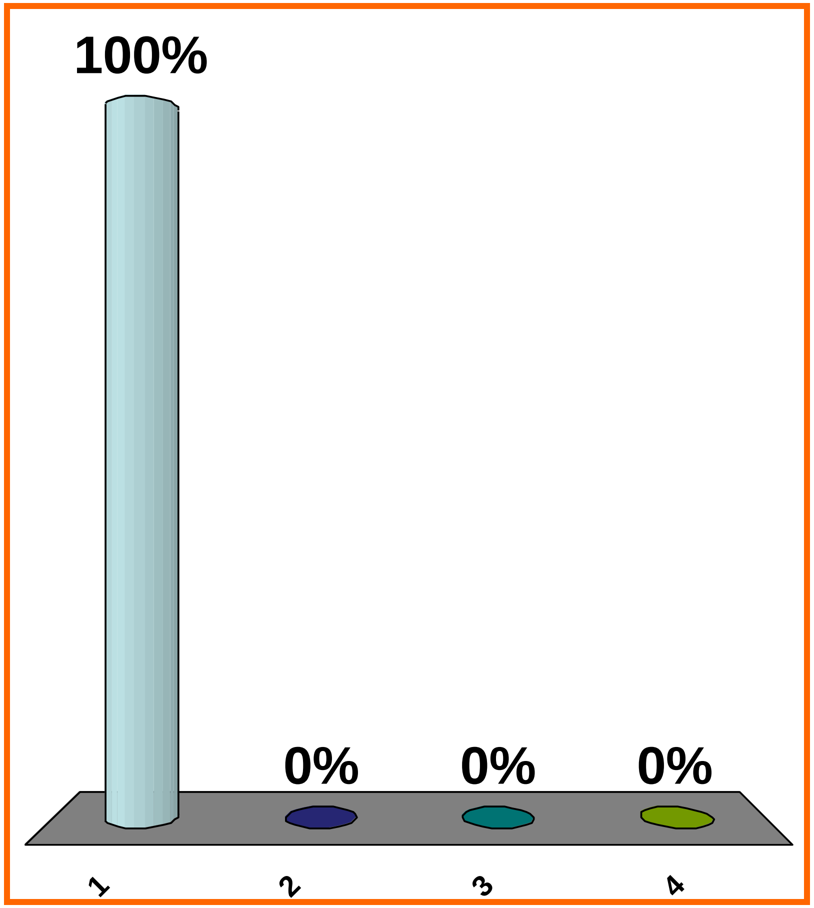
$$\dim \ker \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 9 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

(a) 1

$\boxed{\text{(b) } 2}$

(c) 3

(d) none of the above



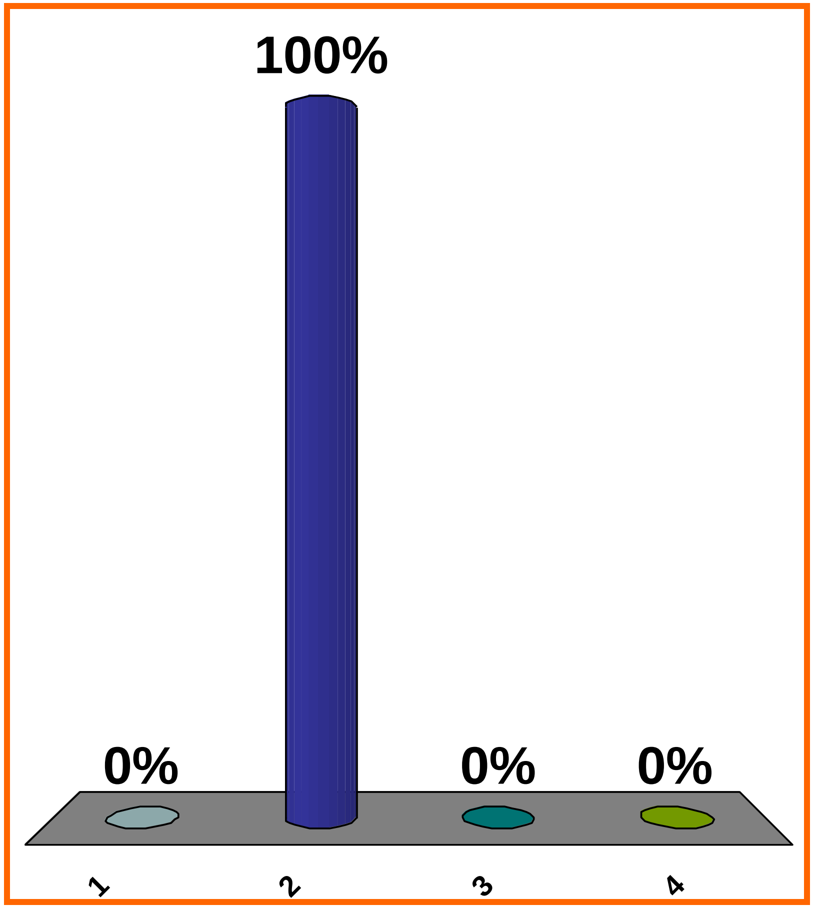
$$\dim \operatorname{im} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 9 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

(a) 1

(b) 2

(c) 3

(d) none of the above



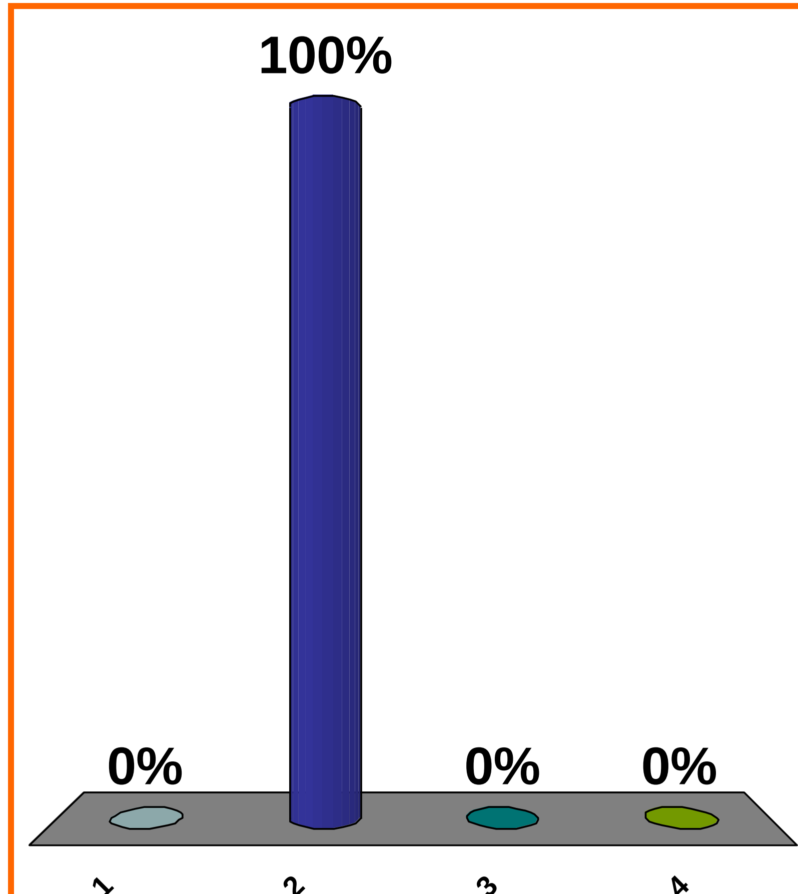
inverse of $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

(a) $\dfrac{1}{-2}\begin{bmatrix} 6 & 5 \\ 4 & 3 \end{bmatrix}$

(b) $\dfrac{1}{-2}\begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$

(c) $\dfrac{1}{-2}\begin{bmatrix} 6 & 4 \\ 5 & 3 \end{bmatrix}$

(d) none of the above



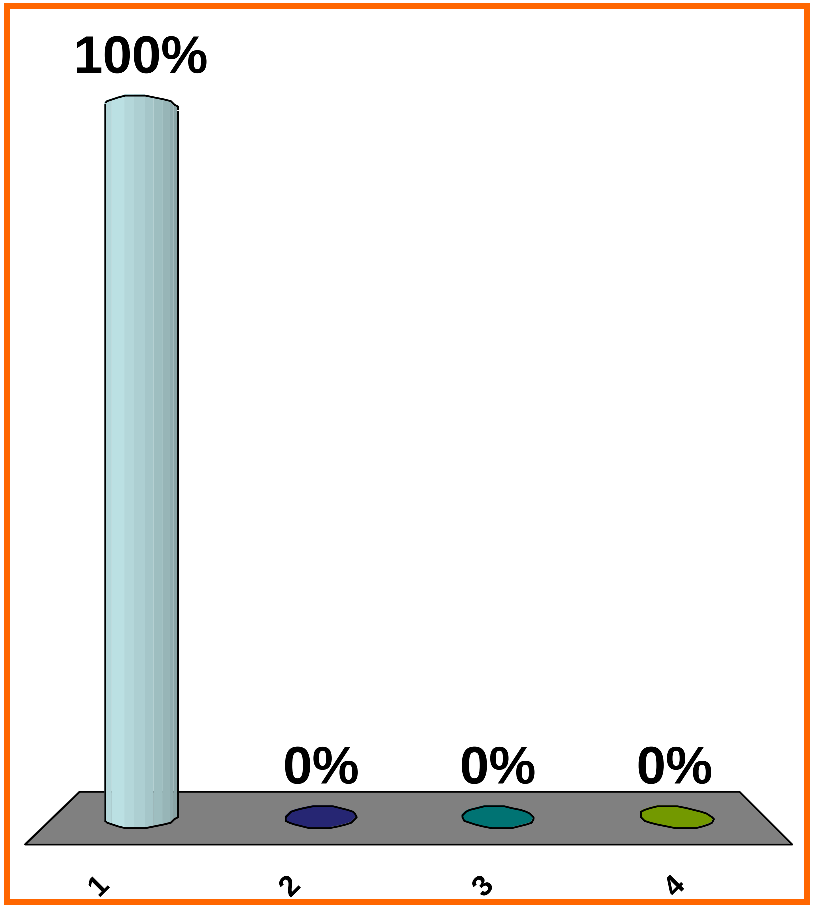
inverse of $\begin{bmatrix} -5 & 1 \\ 6 & -8 \end{bmatrix}$

(a) $\boxed{\dfrac{1}{34}\begin{bmatrix} -8 & -1 \\ -6 & -5 \end{bmatrix}}$

(b) $\dfrac{1}{34}\begin{bmatrix} -8 & 6 \\ 1 & -5 \end{bmatrix}$

(c) $\dfrac{1}{34}\begin{bmatrix} -8 & -6 \\ -1 & -5 \end{bmatrix}$

(d) none of the above



inverse of $\begin{bmatrix} 8 & 7 \\ 6 & 5 \end{bmatrix}$

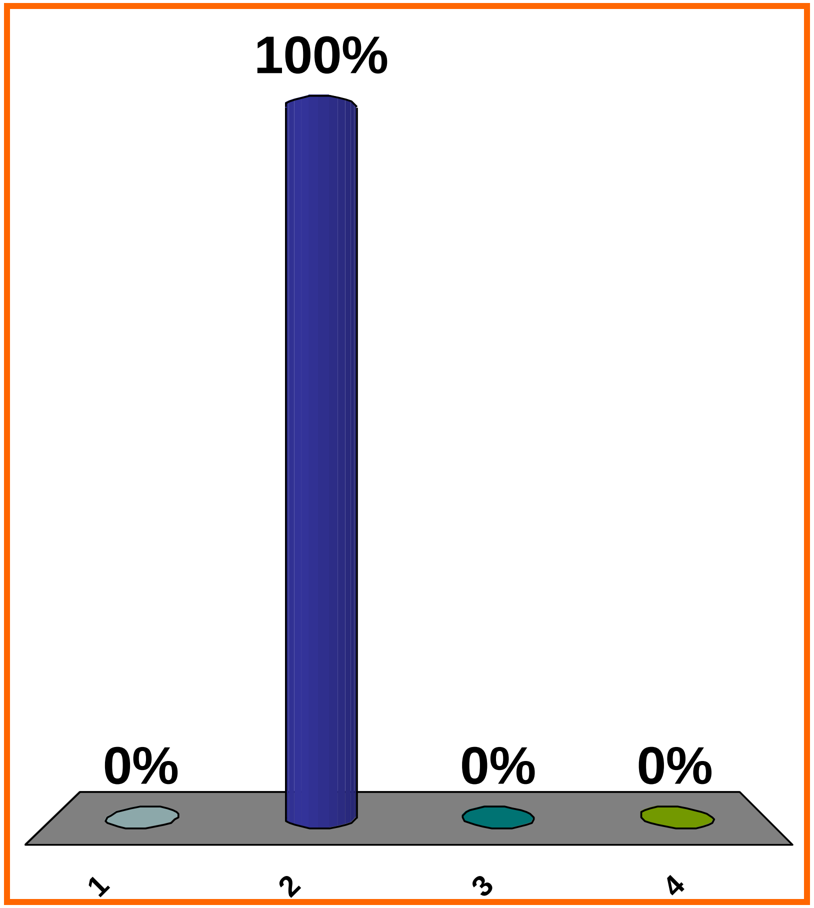
(a) $\frac{1}{2}\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$

(b) $\frac{1}{2}\begin{bmatrix} 5 & -7 \\ -6 & 8 \end{bmatrix}$

(c) $\frac{1}{2}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$\frac{1}{-2}\begin{bmatrix} 5 & -7 \\ -6 & 8 \end{bmatrix}$

$\boxed{\text{(d) none of the above}}$



Financial Mathematics

Regular review session, Midterm 02

Discussion:

What is a linear combination? l.i.? l.d.?
What is a subspace?
What is the span of a subset?
What is a spanning set of a subspace?
What is a basis of a subspace?
What is a linear transformation?
What is a matrix?

Types of matrices:

strictly upper triangular
nilpotent
diagonal
standard nilpotent
Jordan block

Q: Which of these types are invariant under conjugation?

Discussion:

What is an elementary matrix? Invertible?

What is an shearing matrix? Invertible?

Diagonalizable and nilpotent?

Surjection $\mathbb{R}^p \to \mathbb{R}^q$?

Injection $\mathbb{R}^p \to \mathbb{R}^q$?

Bijection $\mathbb{R}^p \to \mathbb{R}^q$?

$AB = I$, shape of A and B?

$$e^M = ?$$

$$e^{CDC^{-1}} = ?$$

Inversion algorithm?

$$A := \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & -1 \end{bmatrix}$$

$$\begin{aligned} L_A(x,y,z) = (&2x + 3y + 4z, \\ &5x + 6y + 7z, \\ &8x + 9y - z) \\ =(&(2,3,4)\cdot(x,y,z), \\ &(5,6,7)\cdot(x,y,z), \\ &(8,9,-1)\cdot(x,y,z)) \end{aligned}$$

Is $L(x,y,z) = (2x + 4y, x^2 - 2y + z)$ linear?

The kernel and image of a linear function

Let V, W be subspaces of $\mathbb{R}^n$, $\mathbb{R}^k$, resp.
Let $L : V \to W$ be a linear function.

The kernel of L is

$$\boxed{\ker(L)} := \{v \in V \mid L(v) = 0\}.$$

Fact:
$L : V \to W$ is one-to-one iff $\ker(L) = \{0\}$.

The image of L is

$$\boxed{L(V)} := \{L(v) \in W \mid v \in V\}.$$

Observation:
$L : V \to W$ is onto iff $L(V) = W$.

Fact: Kernels and images are subspaces.

Ordered bases $\leftrightarrow$ isomorphisms w/ Eucl. space

Fact:

Let $(v_1, \ldots, v_d) \in V^d$ be an ordered basis of a subspace V of some Euclidean space.

Then the function $F : \mathbb{R}^d \to V$ defined by

$$F(a_1, \ldots, a_d) = a_1 v_1 + \cdots + a_d v_d$$

is a vector space isomorphism.

e.g.: Find an isomorphism from $\mathbb{R}^3$ to $\langle (1,3,5,7,9), (2,3,4,5,6), (5,1,1,1,1) \rangle$.

Review definitions:

injective, surjective, bijective
basis of a subspace
dimension of a subspace
linear transformation
image, kernel of a linear transformation
matrix of a linear transformation
symmetric, antisymmetric

Review definitions:

left conjugate, right conjugate

oriented n-parallelpiped,

oriented parallelogram

standard oriented n-parallelpiped,

standard oriented parallelogram

Fact:

left inverse iff right inverse
iff injective iff surjective
iff nonzero determinant

Questions:

Can a nonsquare matrix be invertible?

Can every matrix be made diagonal through row operations?

Questions:

product of direct sums
of square matrices

inverse of direct sum
of square matrices

Exercise: $\begin{bmatrix} 10 & 3 \\ -3 & -1 \end{bmatrix}^{-1} = ??$

Skills:

Row canonical, column canonical forms
Fully canonical form
Finding kernels and images
Finding dimensions of kernels and images
Finding cofactors and inverses
Finding characteristic polynomial
Exponentiating a matrix

How many fully canonical $p \times q$ matrices?

Discussion:

What are

$$A + B,\ A \oplus B,\ AB,\ A \otimes B,\ A \otimes [B],\ e^A?$$

$$A := \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & -1 \end{bmatrix}$$

$$\begin{aligned} L_A(x, y, z) = (&2x + 3y + 4z, \\ &5x + 6y + 7z, \\ &8x + 9y - z) \\ = (&(2, 3, 4) \cdot (x, y, z), \\ &(5, 6, 7) \cdot (x, y, z), \\ &(8, 9, -1) \cdot (x, y, z)) \end{aligned}$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x + 3y + 4z \\ 5x + 6y + 7z \\ 8x + 9y - z \end{bmatrix}$$

Diagonal matrices

Definition:

A matrix is said to be **diagonal** if it's a square matrix such that all of its entries off the diagonal are zero.

$$\begin{bmatrix} a & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & e \end{bmatrix}$$

Triangular matrices

Definition:

A matrix is **upper triangluar** if it's a square matrix such that all of its entries strictly below the diagonal are zero.

$$\begin{bmatrix} a & b & c & d & e \\ 0 & f & g & h & i \\ 0 & 0 & j & k & l \\ 0 & 0 & 0 & m & n \\ 0 & 0 & 0 & 0 & o \end{bmatrix}$$

Exercise: Write out the only reasonable definitions of **strictly upper triangular**,
lower triangular,
strictly lower triangular.

Symmetric matrices

Definition:

A matrix M is **symmetric**

if $M^t = M$.

$$\begin{bmatrix} a & b & c & d & e \\ b & f & g & h & i \\ c & g & j & k & l \\ d & h & k & m & n \\ e & i & l & n & o \end{bmatrix}$$

Anti-symmetric matrices

Definition:

A matrix M is **anti-symmetric**

if $M^t = -M$.

$$\begin{bmatrix} 0 & b & c & d & e \\ -b & 0 & g & h & i \\ -c & -g & 0 & k & l \\ -d & -h & -k & 0 & n \\ -e & -i & -l & -n & 0 \end{bmatrix}$$

Nilpotent matrices

Definition:

A (square) matrix is **nilpotent** if some power of it is zero.

Examples?

Scalar matrices

$$\underbrace{\begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}}_{9I}$$

Definition: A matrix is **scalar** if it's a scalar multiple of the identity.

e.g.: $9I$

scalar identity

Standard nilpotent matrices

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\substack{\| \\ \because \\ N}}$$

Definition: A matrix is **standard nilpotent** if it's square and has 1s just above the diagonal and 0s everywhere else.

e.g.: N is the 4×4 standard nilpotent.
$[0]$ is the 1×1 standard nilpotent.

Jordan blocks

$$\underbrace{\begin{bmatrix} 9 & 1 & 0 & 0 \\ 0 & 9 & 1 & 0 \\ 0 & 0 & 9 & 1 \\ 0 & 0 & 0 & 9 \end{bmatrix}}_{=:\, B} = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition: A matrix is a **Jordan block** if it's the sum of a scalar matrix and a standard nilpotent matrix.

e.g.: B, the 1×1 matrix $[4]$

Q: $e^{A+B} = e^A e^B$?

Q: products of diagonal matrices are diagonal?

Q: powers of diagonal matrices are diagonal?

Q: products of nilpotent matrices are nilpotent?

Q: powers of nilpotent matrices are nilpotent?

Q: $\det(A \oplus B) = ?$

Q: $\det(AB) = ?$

Q: $\det(A^{-1}) = ?$

Q: $\det(A^t) = ?$

$[\mathrm{det}(A)][\mathrm{sv}(P)] = \mathrm{sv}(AP)$

$T : V \to W$ linear transformation

$\mathrm{ker}(T) = T^{-1}(0) = \{v \in V \mid T(v) = 0\}$

$\mathrm{im}(T) = T(V) = \{T(v) \mid v \in V\}$

What is a **basis of** S?

What is the **dimension of** S?

How to compute $\mathrm{sv}(P)$?

Let M be a square matrix.

TFAE:

M has a left inverse

M has a right inverse

M is invertible (has a two-sided inverse)

$\det(M) \neq 0$

the fully canonical form of M is the identity

Say $A \in \mathbb{R}^{5\times 3}$ and $B \in \mathbb{R}^{3\times 5}$.

Is it possible that AB is the 5×5 identity?

Is it possible that BA is the 3×3 identity?

How to find the dimension of the kernel and image of a matrix?

Find the determinant of

$$S := \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 6 & 0 & 0 \\ 0 & 0 & 7 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

and find the determinant of S^t, too.

Definition:

A matrix is a **column-padded identity** if it's obtained from the identity by adding in columns, but never putting in a nonzero entry to the left of a 1 from the original identity matrix.

e.g.:

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 6 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Row canonical form:

$$\begin{bmatrix} \boxed{\text{column-padded identity}} \\ \boxed{0} \end{bmatrix}$$

all zeroes

Fully canonical form:

$$\begin{bmatrix} \boxed{I} & \boxed{0} \\ \boxed{0} & \boxed{0} \end{bmatrix}$$

Fully canonical form:

Identity in the upper left
and zeroes elsewhere.

Direct sum of
identity matrix (square)
and
zero matrix (not necessarily square)

Q: Given k vectors in $\mathbb{R}^n$,
how can we extract
a basis of their span?

Q: Given a matrix $M \in \mathbb{R}^{n \times k}$,
how can we find the dimensions of
$\ker(L_M)$ and $\mathrm{im}(L_M)$?

Q: Given an invertible matrix $M \in \mathbb{R}^{n \times n}$,
how can we find its inverse?

Q: Given a matrix $M \in \mathbb{R}^{n \times n}$,
how can we determine
whether M is invertible?

The kernel and image of a linear function

Let V, W be subspaces of $\mathbb{R}^n$, $\mathbb{R}^k$, resp.
Let $L : V \to W$ be a linear function.

The **kernel** **of** L is

$$\boxed{\ker(L)} := \{v \in V \,|\, L(v) = 0\}.$$

Fact:
$L : V \to W$ is one-to-one iff $\ker(L) = \{0\}$.

The **image** **of** L is

$$\boxed{L(V)} := \{L(v) \in W \,|\, v \in V\}.$$

Observation:
$L : V \to W$ is onto iff $L(V) = W$.

Fact: Kernels and images are subspaces.

Ordered bases $\leftrightarrow$ isomorphisms w/ Eucl. space

Fact:

Let $(v_1, \ldots, v_d) \in V^d$ be an ordered basis of a subspace V of some Euclidean space.
Then the function $F : \mathbb{R}^d \to V$ defined by

$$F(a_1, \ldots, a_d) = a_1 v_1 + \cdots + a_d v_d$$

is a vector space isomorphism.

Q: $e^{A+B} = e^A e^B$?


STOP

IRREGULARITIES: