Financial Mathematics

Clicker review session, Midterm 01

$$k \in \mathbb{R}^{n\times 1}, \qquad \frac{dz}{dt} = k, \qquad [z]_{t:\to 0} = 0 \in \mathbb{R}^{n\times 1},$$

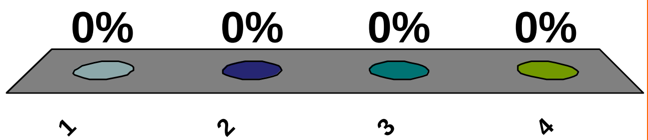
$$z = ?$$

a. e^{tM}

b. $e^{tM}k$

c. ke^{tM}

d. kt



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$$M \in \mathbb{R}^{n\times n}, \qquad \frac{dz}{dt} = Mz, \qquad [z]_{t=0} = k \in \mathbb{R}^{n\times 1},$$

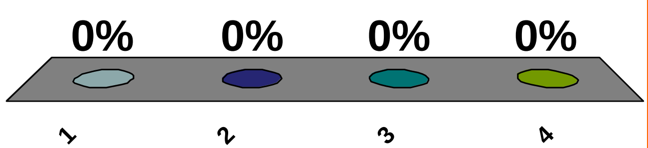
$$z = ?$$

a. e^{tM}

b. $e^{tM}k$

c. ke^{tM}

d. kt



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$$M \in \mathbb{R}^{n\times n}, \qquad C^{-1}MC = D,$$

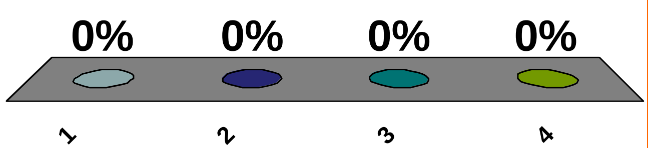
$$e^{tM} = ?$$

a. $C^{-1}e^{tD}C$

b. $Ce^{tD}C^{-1}$

c. e^{tD}

d. none of the above



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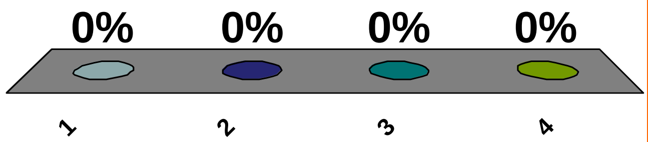
$$\frac{dz}{dt} = Mz, \qquad [z]_{t=0} = c \in \mathbb{R}^{2\times 1}, \qquad z = ?$$

a. e^{tM}

b. $e^{tM}c$

c. ce^{tM}

d. vt



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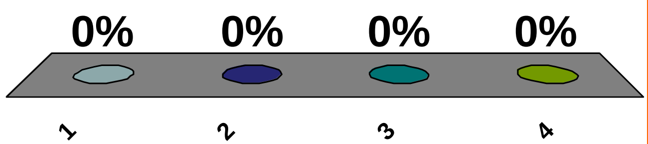
$$(4xy\,dx + 2y\,dy) \wedge (3x\,dx - 7e^{xy}\,dy) =$$

a. $(-28xye^{xy} - 6xy)\,dx \wedge dy$

b. $(28xye^{xy} - 6xy)\,dx \wedge dy$

c. $(28xye^{xy} + 6xy)\,dx \wedge dy$

d. none of the above



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$$d(4xy\,dx + 2y\,dy) =$$

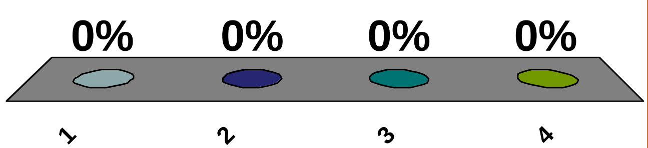
a. $(4x + 2)\,dx \wedge dy$

b. $(-4x + 2)\,dx \wedge dy$

c. $(-4x - 2)\,dx \wedge dy$

d. none of the above

$$-4x\,dx \wedge dy$$



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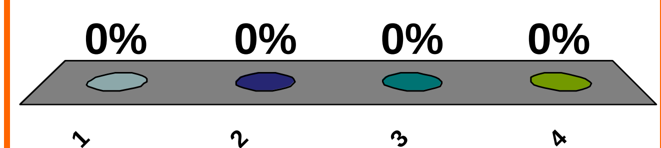
How many terms in an inhomogeneous polynomial of degree ≤ 5 in seven variables?

(a) $\binom{7}{5}$

(b) $\binom{5}{7}$

(c) $\binom{12}{7}$

(d) $\binom{7}{12}$



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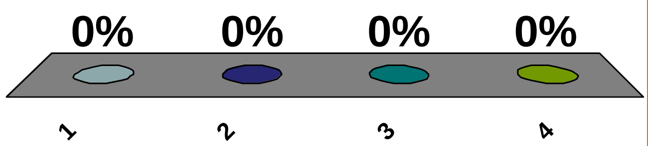
How many terms in a homogeneous of degree $= 8$ in six variables?

(a) $\binom{14}{8}$

(b) $\binom{13}{6}$

(c) $\binom{13}{8}$

(d) none of the above



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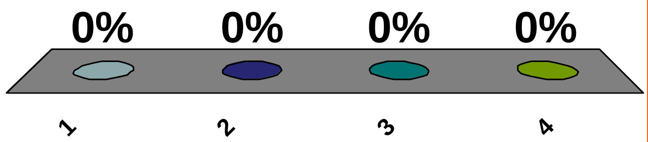
How many terms in a 5th order Macl. approx. to a fn with seven variables?

(a) $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$

(b) $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$

(c) $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$

(d) $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$



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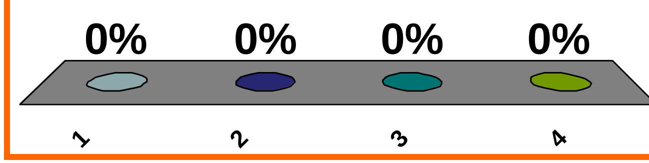
Compute $\int_{(2,3)}^{(4,7)} e^{-xy}\,dx + xy\,dy.$

(a)

(b)

(c)

(d)



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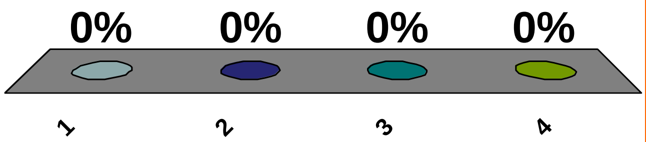
Compute $\int_{(8,3)}^{(4,7)} y\,dx - x\,dy$.

(a)

(b)

(c)

(d)



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$$R = (2,8) \times (1,3)$$

Compute $\int_R dy \wedge dx$.

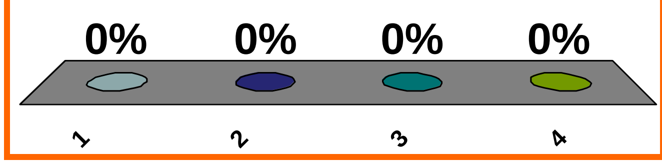
(a) 24

(b) 12

(c) 0

(d) none of the above

-12



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$p = (8,1)$, $q = (8,3)$, $r = (2,3)$, $s = (2,1)$

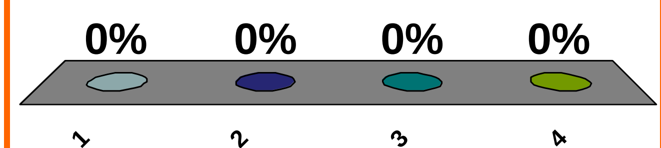
Compute $\left(\int_p^q + \int_q^r + \int_r^s + \int_s^t \right) x\,dy - y\,dx$.

(a) 24

(b) 12

(c) 0

(d) none of the above



0 of 5

$$p = (8,1),\ q = (8,3),\ r = (2,3),\ s = (2,1)$$

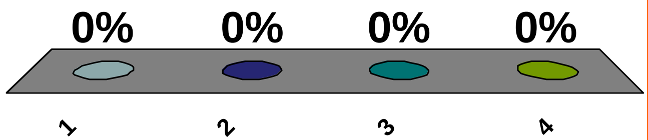
Compute $\displaystyle \left(\int_p^q + \int_q^r + \int_r^s + \int_s^t \right) y\,dx + x\,dy.$

(a) -1

(b) 0

(c) 1

(d) none of the above



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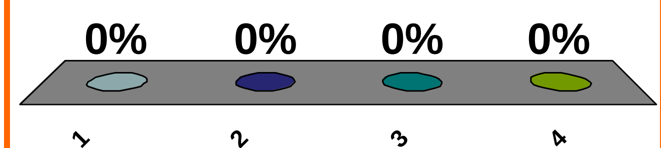
Compute $\int_{1+i}^{2+i} e^{-z}\,dz$.

(a) $e^{2+i} - e^{1+i}$

(b) $[-e^{2+i}] - [-e^{1+i}]$

(c) $-e^{-(2+i)} + e^{-(1+i)}$

(d) $e^{-(2+i)} - e^{-(1+i)}$



0 of 5

Compute $\left(\int_{1+i}^{2+4i} + \int_{2+4i}^{3-7i} + \int_{3-7i}^{1+i}\right) e^{-z}\,dz.$

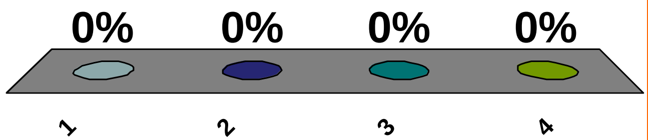
(a) $e^{1+i} + e^{2+4i} + e^{3-7i}$

(b) $-e^{1+i} - e^{2+4i} - e^{3-7i}$

(c) $-e^{-(1+i)} - e^{-(2+4i)} - e^{-(3-7i)}$

(d) none of the above

0



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Compute $\left(\int_{1+i}^{2+4i} + \int_{2+4i}^{3-7i} + \int_{3-7i}^{1+i}\right) \frac{e^{-z^2/2}}{\sqrt{2\pi}}\, dz.$

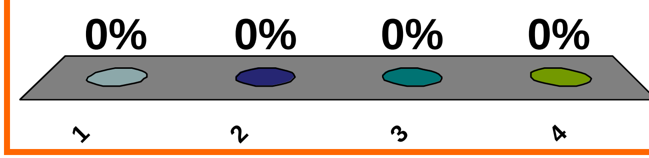
(a) $\Phi(1+i) + \Phi(2+4i)$
$\Phi(3-7i)$

(b) $-\Phi(1+i) - \Phi(2+4i)$
$-\Phi(3-7i)$

(c) $-\Phi(-1-i) - \Phi(-2-4i)$
$-\Phi(-3+7i)$

(d) none of the above

0



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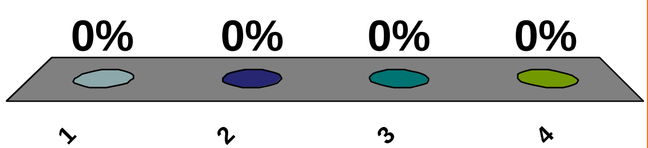
Compute $\int_{1+i}^{2+i} \bar{z}\, dz$.

(a) $\frac{2^2 - 1}{2} - i$

(b) $\frac{2^2 - 1}{2} + i$

(c) $2^2 - 1 - i$

(d) $2^2 - 1 + i$



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Financial Mathematics

Regular review session, Midterm 01

Discussion:

for one-variable functionals:

- Maclaurin approximation
- Maclaurin expansion
- k-jet
- increasing, decreasing on I
- increasing vs. positive derivative

for multi-variable functionals:

- Maclaurin approximation
- Maclaurin expansion
- k-jet
- gradient
- Hessian
- Laplacian $=$ trace of the Hessian

$f'(a)$ $(\nabla f)(a)$ $f''(a)$ $(\Delta f)(a)$

$$f''(a) = (Hf)(a) = (\text{Hess } f)(a) = (\nabla\nabla f)(a) = (\nabla^2 f)(a)$$

Discussion:

for multi-variable functions $\mathbb{R}^n \to \mathbb{R}^m$:

gradient

Hessian

$f'(a)$ ~~$f''(a)$~~

for multi-variable functions $\mathbb{R} \xrightarrow{f} \mathbb{R} \xrightarrow{g} \mathbb{R}$:

chain rule:

$$(g \circ f)'(p) = [\, g'(f(p))\,][f'(p)\,]$$

for multi-variable functions $\mathbb{R}^l \xrightarrow{f} \mathbb{R}^m \xrightarrow{g} \mathbb{R}^n$,

chain rule:

$$(g \circ f)'(p) = [\, g'(f(p))\,][f'(p)\,]$$

$n \times l$ $n \times m$ $m \times l$

Discussion:

for multi-variable functions $\mathbb{R} \xrightarrow{f} \mathbb{R}^m \xrightarrow{g} \mathbb{R}$,

chain rule:

$$(g \circ f)'(p) = [\nabla g(f(p))] \bullet [f'(p)]$$

scalar in $\mathbb{R}$ — vector in $\mathbb{R}^n$ — vector in $\mathbb{R}^n$

for multi-variable functions $\mathbb{R}^l \xrightarrow{f} \mathbb{R}^m \xrightarrow{g} \mathbb{R}^n$,

chain rule:

$$(g \circ f)'(p) = [g'(f(p))][f'(p)]$$

$n \times l$ — $n \times m$ — $m \times l$

Discussion: vector field

flowline = integral curve

"footed at" existence? uniqueness?

compute: flowline footed at p of L_M

inhomogeneous linear?

integration of a vector field through a point

integration of a vector field

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time-dependent vector fields

$$V : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$$

$\pi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$
first coordinates projection

$W : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n \times \mathbb{R}$
$W(p,t) = (V(p,t), 1)$

to integrate V through p,
integrate W through $(p, 0)$,
then project to $\mathbb{R}^n$.

Discussion:

derivative of e^x w.r.t. x

derivative of $e^x + 1$ w.r.t. x

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gradient of $x^2 + y^2 + z^3$ w.r.t. (x, y, z)

gradient of $x^2 + y^2 + z^3 + 1$ w.r.t. (x, y, z)

reverse gradient   reverse gradient flow

---

integrate $V(x, y) = (x, y)$ through $(2, 3)$

Say $L_M(v) = 5v$.

integrate $V(x, y) = L_M(x, y)$ through v

Say $L_M(w) = 8v$.

integrate $V(x, y) = L_M(x, y)$ through w

integrate $V(x, y) = L_M(x, y)$ through $2v - 7w$

Discussion:

Cauchy-Riemann equations

$U + iV$ is complex differentiable iff

$$\partial_x U = \partial_y V \text{ and } \partial_y U = -\partial_x V$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \partial_x U \\ \partial_y U \end{bmatrix} = \begin{bmatrix} \partial_x V \\ \partial_y V \end{bmatrix}$$

counter-clockwise rot'n by 90°	∇U	∇V

Discussion:

critical point of $f : \mathbb{R}^n \to \mathbb{R}$

second derivative test for $f : \mathbb{R}^n \to \mathbb{R}$ at a critical point

$f, g : \mathbb{R}^n \to \mathbb{R}$

critical point of f on $g^{-1}(k)$

non-smooth point of $g^{-1}(k)$

---

contraction mapping $\mathbb{R}^2 \to \mathbb{R}^2$
contraction factor

fixpoints for contractions?

Discussion:

Make the change

$$(x, y) :\to (3s + 4t - 8, 2s - t + 15)$$

in $\displaystyle\int\int_{(2,3)\times(4,5)} e^{xy}\,dx\,dy$

Discussion:

directed line segment in $\mathbb{R}^2$

1-chain in $\mathbb{R}^2$

directed line segment in $\mathbb{C}$

1-chain in $\mathbb{C}$

zero-form, one-form, two-form

exterior derivative, wedge product

Discussion: Two variables: x and y

C a 1-chain, ω a 1-form	$\int_C \omega :=$
R a 2-rectangle	$\int_R f(x,y)\,dx\,dy :=$
R a 2-rectangle, ω a 2-form	$\int_R \omega :=$
ω, η both 1-forms	$\omega \wedge \eta$
ω a 0-form	$d\omega$
ω a 1-form	$d\omega$

$$d[(f(x)\,dx) + (f(y)\,dy)] =$$

Counterclockwise boundary of R

Eigenvalues: 3, 2 and 0

Symmetric? Yes.

2. T or F: The matrix

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is positive semi-definite.

T. True F. False

Homework#(05/07)-1 ungraded

(05/07)-1-5. Maximize $2x - 5y$ subject to the constraint $x^4 + y^4 = 1$.

(05/07)-1-6. Minimize $7x - y$ subject to the constraint $x^6 + y^6 = 1$.

(05/07)-1-7a. For every integer $n \geq 1$, maximize $x + y$ subject to the constraint $x^{2n} + y^{2n} = 1$.

(05/07)-1-7b. Let (a_n, b_n) denote the answer to (05/07)-1-7a. Compute $\lim_{n\to\infty} (a_n, b_n)$.

Homework#(04/30)-1

(04/30)-1-3.

Compute $d(ze^{xy})$,
the exterior derivative of

$$ze^{xy},$$

with respect to x, y, z.

(04/30)-1-4.

Compute $d(xy\,dx + ye^z\,dy - \sin(xz)\,dz)$,
the exterior derivative of

$$xy\,dx + ye^z\,dy - \sin(xz)\,dz,$$

with respect to x, y, z.

Homework#(04/30)-1

(04/30)-1-5.
Compute $\displaystyle\int_{(1,2,3)}^{(4,5,6)} x\,dx + y\,dy + xe^z\,dz$,

i.e., compute $\displaystyle\int_L x\,dx + y\,dy + xe^z\,dz$,

where L is the directed line segment from $(1,2,3)$ to $(4,5,6)$.

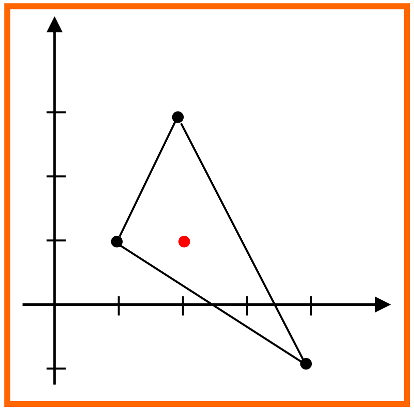
(04/30)-1-6. Let $R := (1,2) \times (3,4)$.

Compute $\displaystyle\int_R [e^{x+y}]\,dy \wedge dx$.

Homework#(03/05)-1

(03/05)-1-1c. Compute $\int_{1+i}^{4-i} e^x \, dx$.

(03/05)-1-1d. Show that the complex limit



$$\lim_{h\to 0} \frac{e^{2+i+h} - e^{2+i}}{h} \text{ exists,}$$

and is equal to e^{2+i}.

Homework#(03/05)-1

(03/05)-1-2c. Compute $\displaystyle\int_i^{3i+2} \overline{x}\,dx$.

(03/05)-1-1d. Show that the complex limit

$$\lim_{h\to 0} \frac{\overline{1+2i+h} - \overline{1+2i}}{h}$$

does not exist.

1. Compute $\int_{2+2i}^{4+6i} e^{x/2}\, dx$.

a. $e^{(2+3i)-(1+i)}$

c. $2(e^{2+3i} - e^{1+i})$

b. $e^{2+3i} - e^{1+i}$

d. $(1/2)(e^{2+3i} - e^{1+i})$

e. None of the above

Antiderivative for $e^{x/2}$: $\dfrac{e^{x/2}}{1/2} = 2(e^{x/2})$

$$\int_{2+2i}^{4+6i} e^{x/2}\, dx = [2(e^{x/2})]_{x=2+2i}^{x=4+6i}$$

$$= [2(e^{2+3i} - e^{1+i})]$$


STOP

IRREGULARITIES: