Financial Mathematics

Clicker review session, Midterm 02

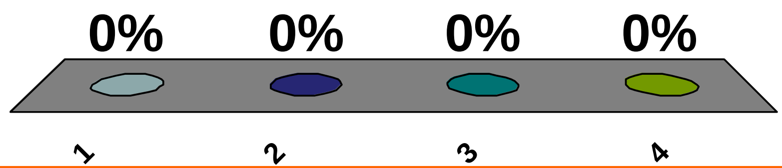
$$\Pr[X = 2] = 0.4$$
$$\Pr[X = 4] = 0.6$$
$$E[X] =$$

(a) $(0.4)(2) + (0.6)(4)$

(b) $\dfrac{(0.4)(2) + (0.6)(4)}{2}$

(c) $(0.4)(0.6)(4)(2)$

(d) none of the above



0 of 5

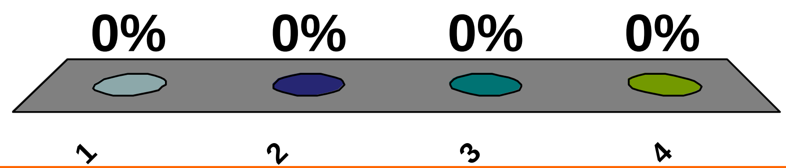
$$\Pr[X = 1] = 0.8$$
$$\Pr[X = 2] = 0.2$$
$$E[X] =$$

(a) $(0.8)(0.2)(1)(2)$

(b) $(0.8)(1) + (0.2)(2)$

(c) $(0.8)(2) + (0.2)(1)$

(d) none of the above



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$$\Pr[X = 8] = 0.1$$
$$\Pr[X = 4] = 0.9$$

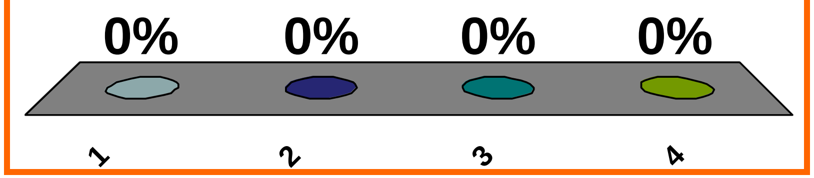
$$E[X] =$$

(a) $(8)(4) + (0.1)(0.9)$

(b) $(0.1)(8) + (0.9)(4)$

(c) $(0.1)(4) + (0.9)(8)$

(d) none of the above



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$$\Pr[X = 2] = 0.4$$
$$\Pr[X = 4] = 0.6$$
$$\mathrm{Var}[X] =$$

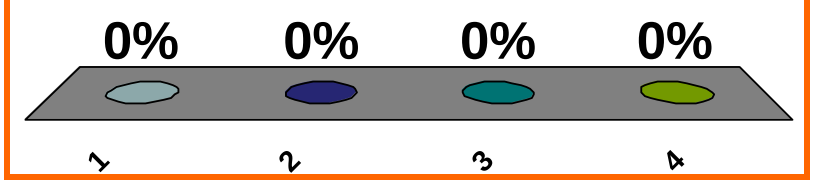
(a) $(0.4)(2) + (0.6)(4)$

(b) $(0.4)(0.6)(4 - 2)$

(c) $\sqrt{(0.4)(0.6)}(4 - 2)$

(d) none of the above

$$(0.4)(0.6)(4 - 2)^2$$



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$$\Pr[X = 1] = 0.8$$
$$\Pr[X = 2] = 0.2$$

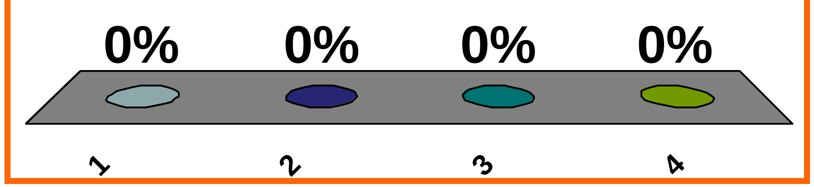
$$\text{Var}[X] =$$

(a) $(0.8)(0.2)(2-1)^2$

(b) $(1)(2)(0.8-0.2)^2$

(c) $(0.8)(2)(1-0.2)^2$

(d) none of the above



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$$\Pr[X = 8] = 0.1$$
$$\Pr[X = 4] = 0.9$$

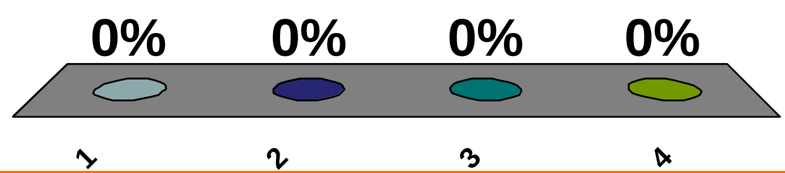
$$\mathrm{Var}[X] =$$

(a) $(8)(4)(0.1 - 0.9)^2$

(b) $(0.1)(8) + (0.9)(4)$

(c) $\boxed{(0.1)(0.9)(8 - 4)^2}$

(d) none of the above



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$$Z_n := (C_1 + \cdots + C_n)/\sqrt{n}$$

$$\lim_{n\to\infty} \mathsf{E}[e^{3Z_n+5}] = ??$$

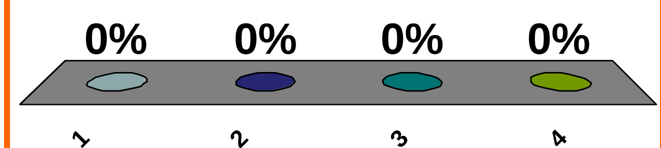
a. $\int_{-\infty}^{\infty} e^{3x-5}\, dx$

b. $\int_{-\infty}^{\infty} \left[e^{3x-5}\right]\left[e^{-x^2/2}\right]\, dx$

c. 0

d. none of the above

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \left[e^{3x-5}\right]\left[e^{-x^2/2}\right]\, dx$$



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$$Z_n := (C_1 + \cdots + C_n)/\sqrt{n}$$

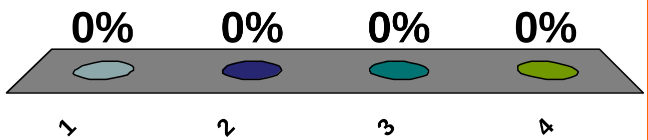
$$\lim_{n\to\infty} \mathrm{E}[\cos(2Z_n^2 - 8)] = ??$$

a. $\int_0^1 \left[\cos(2x^2 - 8)\right] \left[e^{-x^2/2}\right] \, dx$

b. $\frac{1}{\sqrt{2\pi}} \int_0^1 \left[\cos(2x^2 - 8)\right] \left[e^{-x^2/2}\right] \, dx$

c. $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[\cos(2x^2 - 8)\right] \left[e^{-x^2/2}\right] \, dx$

d. none of the above



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$$Z_n := (C_1 + \cdots + C_n)/\sqrt{n}$$

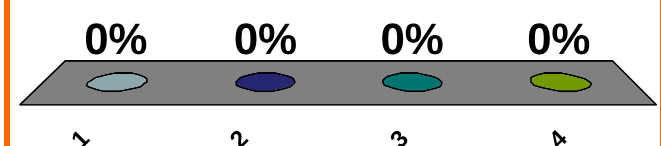
$$\lim_{n\to\infty} \mathsf{E}[e^{3Z_n - 7}] = ??$$

a. $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^1 \left[e^{3x-7}\right]\left[e^{-x^2/2}\right]\, dx$

b. $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^\infty \left[e^{3x-7}\right]\left[e^{-x^2/2}\right]\, dx$

c. $\boxed{\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^\infty \left[e^{3x-7}\right]\left[e^{-x^2/2}\right]\, dx}$

d. none of the above



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$$Z_n := (C_1 + \cdots + C_n)/\sqrt{n}$$

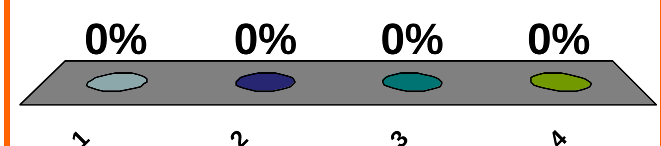
$$\lim_{n\to\infty} \mathsf{E}\left[\left(e^{Z_n} - 4\right)_+\right] = ??$$

a. $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} [e^x - 4] \left[e^{-x^2/2}\right] dx$

b. $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{\ln 4}^{\infty} [e^x - 4] \left[e^{-x^2/2}\right] dx$

c. $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\ln 4}^{\infty} [e^x - 4] \left[e^{-x^2/2}\right] dx$

d. none of the above



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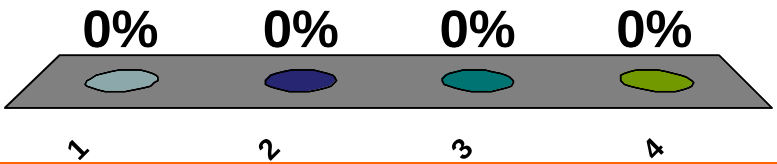
$$\int_{-\infty}^{\infty} x^3 e^{-x^2/2}\, dx$$

(a) 0

(b) 1

(c) -1

(d) none of the above



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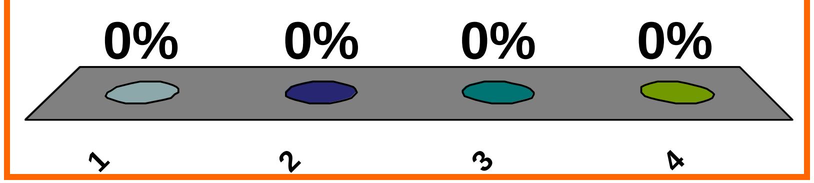
$$\int_{-\infty}^{\infty} x^5 e^{-x^2/2}\, dx$$

(a) 0

(b) 1

(c) -1

(d) none of the above



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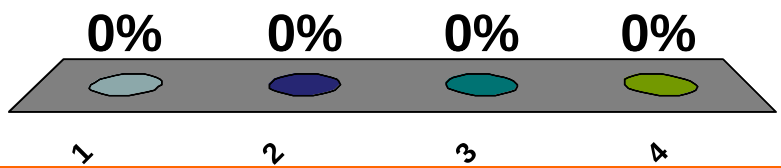
$$\int_{-\infty}^{\infty} x^{99} e^{-x^2/2}\, dx$$

(a) 0

(b) 1

(c) -1

(d) none of the above



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$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx$$

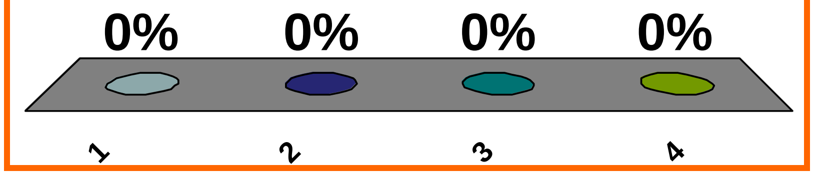
(a) 0

(b) 1

(c) -1

(d) none of the above

$\sqrt{2\pi}$



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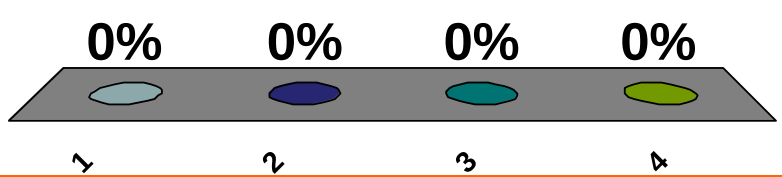
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x^2 e^{-x^2/2}\,dx$$

(a) 0

(b) 1

(c) -1

(d) none of the above



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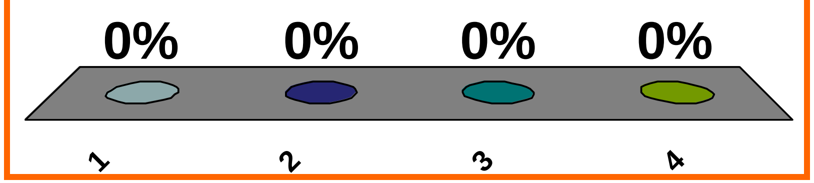
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x^4 e^{-x^2/2}\,dx$$

(a) 1

(b) 2

(c) 3

(d) none of the above



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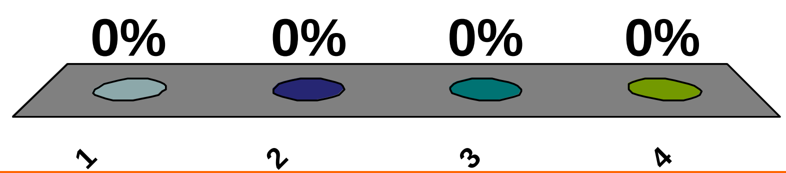
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x^{100} e^{-x^2/2}\,dx$$

(a) 99!

(b) $\dfrac{99!}{(2^{49})(49!)}$

(c) $\dfrac{99!}{49!}$

(d) none of the above



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$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^{102} e^{-x^2/2}\, dx$$

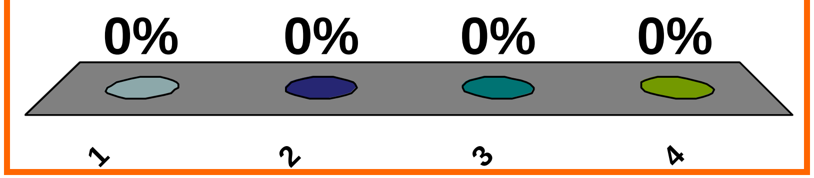
(a) $\dfrac{100!}{(2^{50})(50!)}$

(b) $\dfrac{100!}{(2^{49})(50!)}$

(c) $\dfrac{101!}{(2^{49})(50!)}$

(d) none of the above

$$\frac{101!}{(2^{50})(50!)}$$



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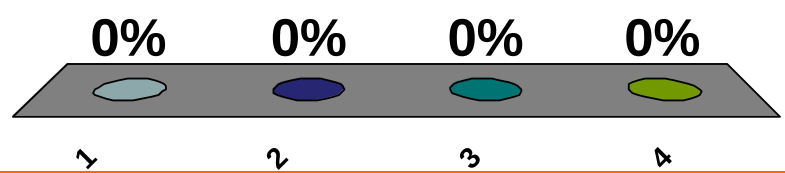
$$\int x^3 e^{-x^2/2}\,dx$$

(a) $$-x^2 e^{-x^2/2} + 2\int x e^{-x^2/2}\,dx$$

(b) $$-x^3 e^{-x^2/2} + 3\int x^2 e^{-x^2/2}\,dx$$

(c) $$-(x^4/4) e^{-x^2/2} + \int x^3 e^{-x^2/2}\,dx$$

(d) none of the above



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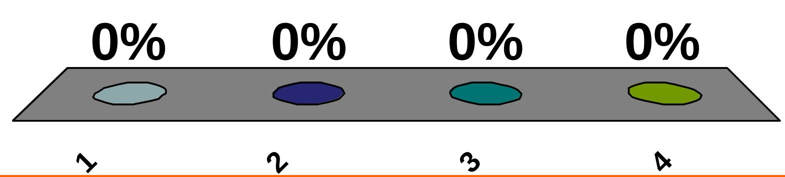
$$\int x^6 e^{-x^2/2}\, dx$$

(a) $-x^6 e^{-x^2/2} + 5\int x^4 e^{-x^2/2}\, dx$

(b) $-x^6 e^{-x^2/2} + 6\int x^5 e^{-x^2/2}\, dx$

(c) $-x^5 e^{-x^2/2} + 5\int x^4 e^{-x^2/2}\, dx$

(d) none of the above



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$$\int x^8 e^{-x^2/2}\,dx$$

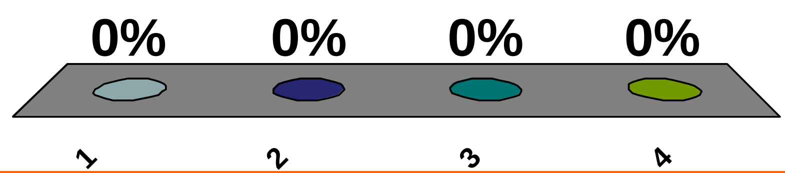
(a) $-x^8 e^{-x^2/2}$
$$+7\int x^6 e^{-x^2/2}\,dx$$

(b) $-x^8 e^{-x^2/2}$
$$+8\int x^7 e^{-x^2/2}\,dx$$

(c) $-x^7 e^{-x^2/2}$
$$-7\int x^6 e^{-x^2/2}\,dx$$

$\boxed{\text{(d)}}$ none of the above

$-x^7 e^{-x^2/2}$
$$+7\int x^6 e^{-x^2/2}\,dx$$



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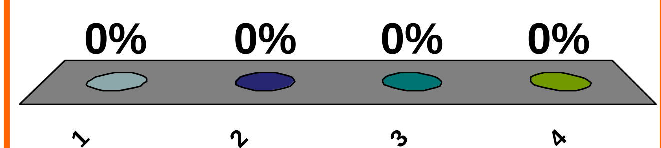
Compute $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(4x^6+2x^5)e^{-x^2/2}\,dx$

(a) $4(6!)+2(5!)$

(b) 0

(c) $4(1)(3)(5)$

(d) none of the above



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Compute $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(x^9+2x^7-4x^4)e^{-x^2/2}\,dx$

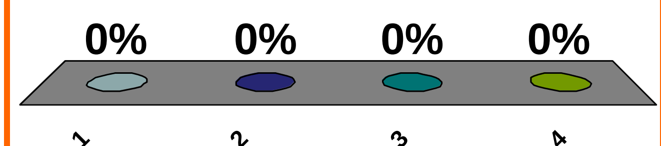
(a) $9+2(7)-4(4)$

(b) $(9!)+2(7!)-4(4!)$

(c) 0

(d) none of the above

$-4(1)(3)$



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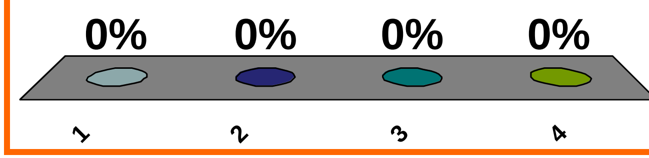
Compute $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x^9 + 2x^7 - 4x^3) e^{-x^2/2}\, dx$

(a) $(9!) + 2(7!) - 4(3!)$

(b) 0

(c) $9 + 2(7) - 4(3)$

(d) none of the above



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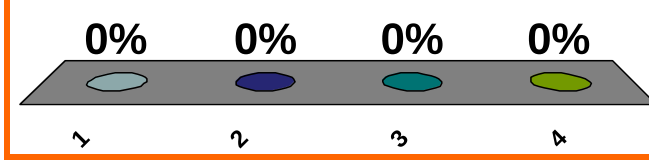
Compute $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(x^8+2x^6-4x^2)e^{-x^2/2}\,dx$

(a) $(3)(5)(7)+2(3)(5)-4$

(b) $8(3)(5)(7)+2(3)(5)$

(c) $2(1)(3)(5)-4(1)$

(d) none of the above



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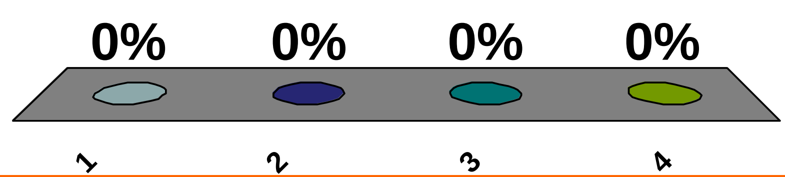
$$\int_{-\infty}^{\infty} (e^x - e^7)_+ e^{-x^2/2}\, dx$$

(a) $\int_{-7}^{\infty} (e^x - e^7) e^{-x^2/2}\, dx$

(b) $\int_{\ln 7}^{\infty} (e^x - e^7) e^{-x^2/2}\, dx$

(c) $\int_{7}^{\infty} (e^x - e^7) e^{-x^2/2}\, dx$

(d) none of the above



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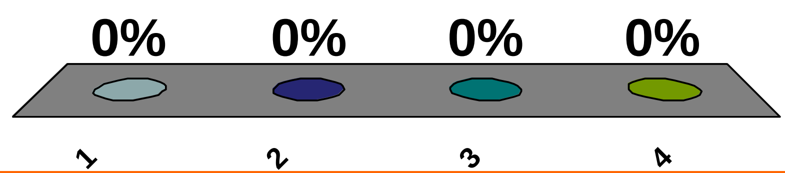
$$\int_{-\infty}^{\infty} (e^x - 7)_+ e^{-x^2/2}\, dx$$

(a) $\int_{-7}^{\infty} (e^x - 7) e^{-x^2/2}\, dx$

(b) $\int_{\ln 7}^{\infty} (e^x - 7) e^{-x^2/2}\, dx$

(c) $\int_{7}^{\infty} (e^x - 7) e^{-x^2/2}\, dx$

(d) none of the above



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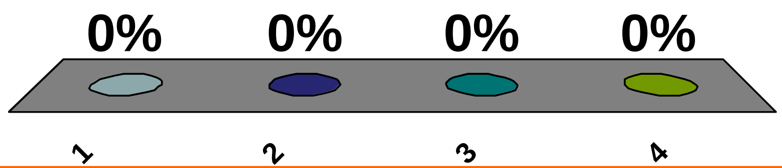
$$\int_{-\infty}^{\infty} (e^x - e^{-7})_+ e^{-x^2/2}\, dx$$

(a) $\boxed{\int_{-7}^{\infty} (e^x - e^{-7}) e^{-x^2/2}\, dx}$

(b) $\int_{\ln 7}^{\infty} (e^x - e^{-7}) e^{-x^2/2}\, dx$

(c) $\int_{7}^{\infty} (e^x - e^{-7}) e^{-x^2/2}\, dx$

(d) none of the above



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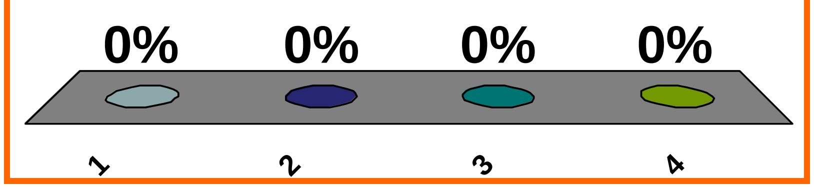
$$\int_7^\infty e^x e^{-x^2/2}\,dx$$

(a) $e^{1/2}\int_6^\infty e^{-x^2/2}\,dx$

(b) $e^{1/2}\int_7^\infty e^{-x^2/2}\,dx$

(c) $e^{1/2}\int_8^\infty e^{-x^2/2}\,dx$

(d) none of the above



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$$\int_{\ln 7}^{\infty} e^{x} e^{-x^2/2}\,dx$$

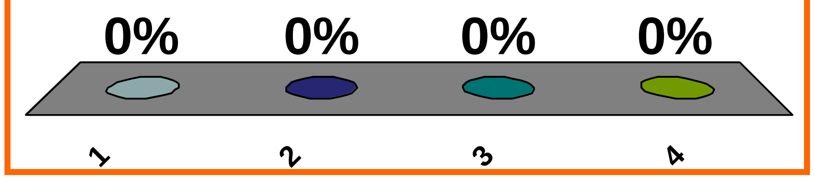
(a) $e^{1/2}\int_{\ln 6}^{\infty} e^{-x^2/2}\,dx$

(b) $e^{1/2}\int_{\ln 7}^{\infty} e^{-x^2/2}\,dx$

(c) $e^{1/2}\int_{\ln 8}^{\infty} e^{-x^2/2}\,dx$

$$e^{1/2}\int_{-1+\ln 7}^{\infty} e^{-x^2/2}\,dx$$

(d) $\boxed{\text{none of the above}}$



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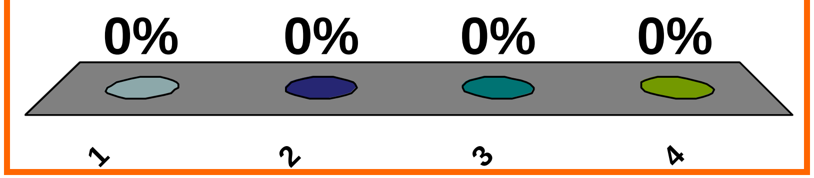
$$\int_{-7}^{\infty} e^{x} e^{-x^2/2}\, dx$$

(a) $e^{1/2} \int_{-6}^{\infty} e^{-x^2/2}\, dx$

(b) $e^{1/2} \int_{-7}^{\infty} e^{-x^2/2}\, dx$

(c) $e^{1/2} \int_{-8}^{\infty} e^{-x^2/2}\, dx$

(d) none of the above



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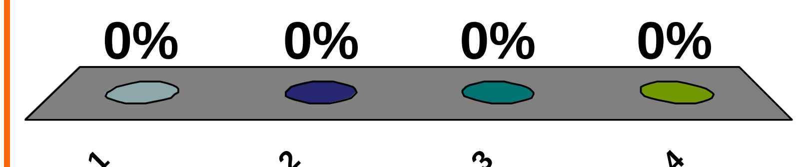
$$\frac{1}{\sqrt{2\pi}}\left[\int_{8}^{\infty} e^{-x^2/2}\,dx\right]$$

(a) $\Phi(8)$

(b) $\boxed{\Phi(-8)}$

(c) 0

(d) none of the above



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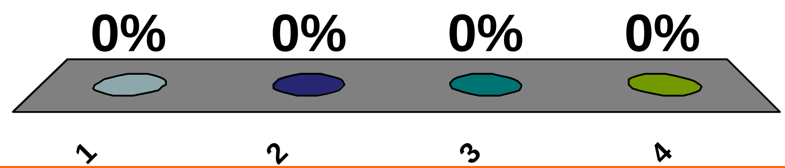
$$\int_{-1+\ln 7}^{\infty} e^{-x^2/2}\,dx$$

(a) $[\sqrt{2\pi}][\Phi(-1+\ln 7)]$

(b) $[\sqrt{2\pi}][\Phi(1-\ln 7)]$

(c) 0

(d) none of the above



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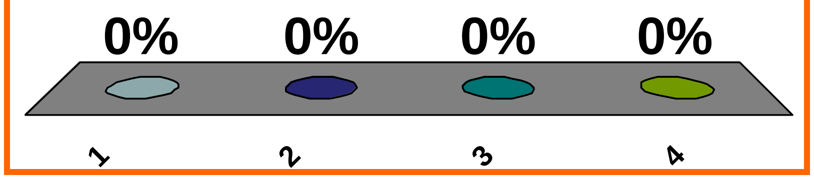
$$\int_{-8}^{\infty} e^{-x^2/2}\, dx$$

(a) $[\sqrt{2\pi}][\Phi(8)]$

(b) $[\sqrt{2\pi}][\Phi(-8)]$

(c) 0

(d) none of the above



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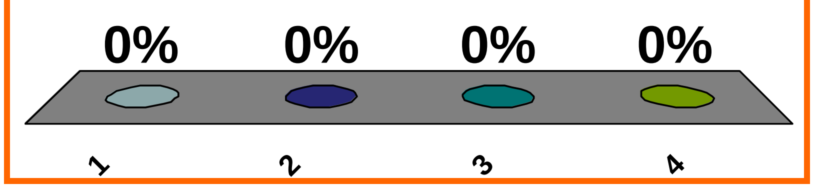
$$\int_{-\infty}^{8} e^{-x^2/2}\,dx$$

(a) $[\sqrt{2\pi}][\Phi(8)]$

(b) $[\sqrt{2\pi}][\Phi(-8)]$

(c) 0

(d) none of the above



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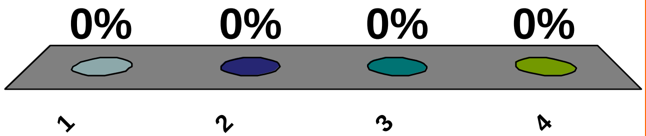
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{4x} e^{-x^2/2}\,dx$$

(a) e^{4^2}

(b) $e^{4^2/2}/\sqrt{2\pi}$

(c) $e^{4^2}/\sqrt{2\pi}$

(d) none of the above



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

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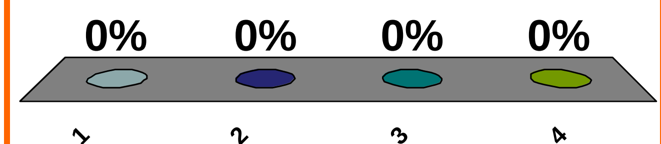
$$\int_{-\infty}^{\infty} e^{8x} e^{-x^2/2}\, dx$$

(a) e^{8^2}

(b) $e^{8^2/2}\sqrt{2\pi}$

(c) $e^{8^2/2}/\sqrt{2\pi}$

(d) none of the above



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0 of 5

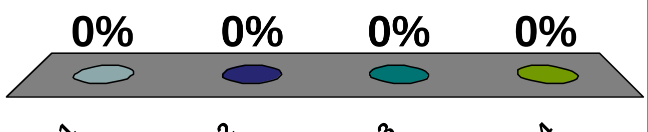
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-4x}e^{-x^2/2}\,dx$$

$\boxed{\text{(a) } e^{(-4)^2/2}}$

(b) $e^{(-4)^2}$

(c) $e^{(-4)^2/2}\sqrt{2\pi}$

(d) none of the above



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

(a) $\displaystyle\int_{-5}^{\infty} e^{(-4)^2/2} e^{-x^2/2}\, dx$

$\displaystyle\int_{\boxed{-7}}^{\infty} e^{-4x} e^{-x^2/2}\, dx$

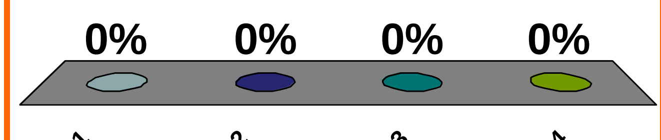
(b) $\displaystyle\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-4x} e^{-x^2/2}\, dx$

(c) $\displaystyle\frac{1}{\sqrt{2\pi}} \int_{-9}^{\infty} e^{(-4)^2/2} e^{-x^2/2}\, dx$

(d) none of the above

Correct answer:

$$\int_{-3}^{\infty} e^{(-4)^2/2} e^{-x^2/2}\, dx$$



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0 of 5

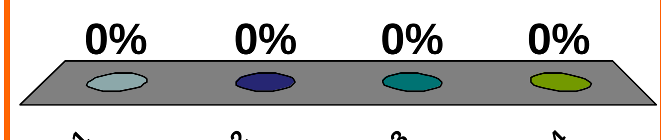
$$\int_5^{\infty} e^{3x} e^{-x^2/2}\, dx$$

(a) $\int_8^{\infty} e^{3^2/2} e^{-x^2/2}\, dx$

(b) $\int_{-\infty}^{\infty} e^{3^2/2} e^{-x^2/2}\, dx$

(c) $\boxed{\int_2^{\infty} e^{3^2/2} e^{-x^2/2}\, dx}$

(d) none of the above



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0 of 5

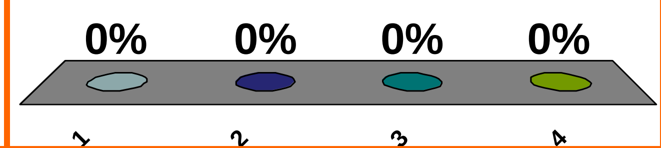
$$\int_{13}^{\infty} e^{\pi x} e^{-x^2/2}\, dx$$

(a) $\displaystyle \boxed{\int_{13-\pi}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx}$

(b) $\displaystyle \int_{13-(\pi/2)}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx$

(c) $\displaystyle \int_{13+\pi}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx$

(d) none of the above



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

$$\int_8^{\infty} e^{8x} e^{-x^2/2}\,dx$$

(a) $\displaystyle\int_5^{\infty} e^{8^2/2} e^{-x^2/2}\,dx$

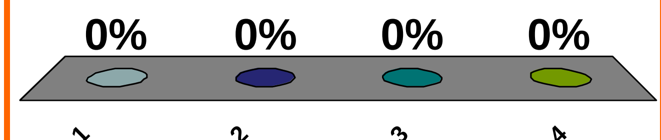
(b) $\displaystyle\int_4^{\infty} e^{8^2/2} e^{-x^2/2}\,dx$

(c) $\displaystyle\int_{12}^{\infty} e^{8^2/2} e^{-x^2/2}\,dx$

(d) none of the above

Correct answer:

$$\int_0^{\infty} e^{8^2/2} e^{-x^2/2}\,dx$$



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

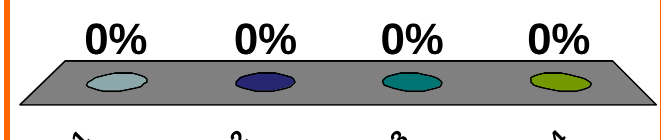
$$\frac{1}{\sqrt{2\pi}}\int_{-5}^{\infty} e^4 e^{-x^2/2}\,dx$$

(a) $e^4(\Phi(5))$

(b) $e^4(\Phi(5))\sqrt{2\pi}$

(c) $e^4(\Phi(-5))$

(d) none of the above



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

$$\int_{10}^{\infty} e^{7/3} e^{-x^2/2}\, dx$$

(a) $e^{7/3}(\Phi(10))$

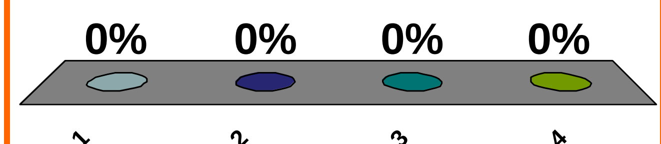
(b) $e^{7/3}(\Phi(10))\sqrt{2\pi}$

(c) $e^{7/3}(\Phi(-10))$

(d) none of the above

Correct answer:

$$e^{7/3}(\Phi(-10))\sqrt{2\pi}$$



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

Assume $\Pr[X = 3] = 0.4$ and $\Pr[X = 8] = 0.6$.
Find $\mathrm{E}[X]$.

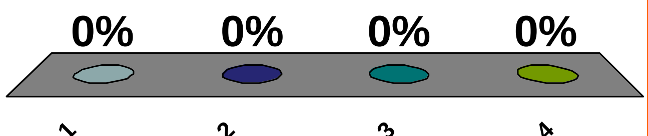
(a) $(0.4)(0.6)(8-3)$

(b) $(0.4)(0.6)(8-3)^2$

(c) $\sqrt{(0.4)(0.6)}(8-3)$

(d) none of the above

Correct answer: $(0.4)(3) + (0.6)(8)$



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

Assume $\Pr[X = 2] = 0.1$, $\Pr[X = -2] = 0.9$.

Find $\mathrm{E}[X]$.

(a) $(0.1)(0.9)(2 - (-2))$

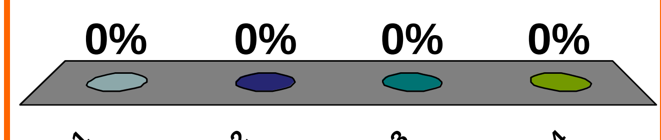
(b) $(0.1)(0.9)(2 - (-2))^2$

(c) $\sqrt{(0.1)(0.9)}(2 - (-2))$

(d) none of the above

Correct answer:

$$(0.1)(2) + (0.9)(-2)$$



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

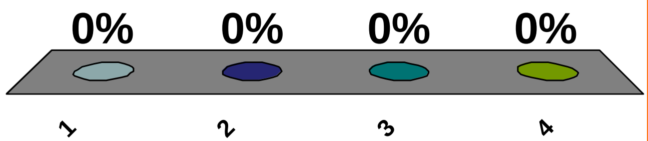
Assume $\Pr[X = 3] = 0.4$ and $\Pr[X = 8] = 0.6$.
Find $\mathrm{Var}[X]$.

(a) $(0.4)(0.6)(8-3)$

$\boxed{\text{(b) } (0.4)(0.6)(8-3)^2}$

(c) $\sqrt{(0.4)(0.6)}(8-3)$

(d) none of the above



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0 of 5

Assume $\Pr[X = 2] = 0.1$, $\Pr[X = -2] = 0.9$.

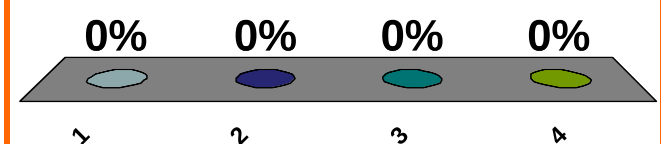
Find $\text{Var}[X]$.

(a) $(0.1)(0.9)(2 - (-2))$

(b) $(0.1)(0.9)(2 - (-2))^2$

(c) $\sqrt{(0.1)(0.9)}(2 - (-2))$

(d) none of the above



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

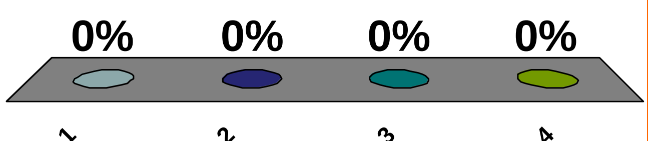
Assume $\Pr[X = 3] = 0.4$ and $\Pr[X = 8] = 0.6$. Find $\mathrm{SD}[X]$.

(a) $(0.4)(0.6)(8-3)$

(b) $(0.4)(0.6)(8-3)^2$

$\boxed{\text{(c) } \sqrt{(0.4)(0.6)}(8-3)}$

(d) none of the above



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0 of 5

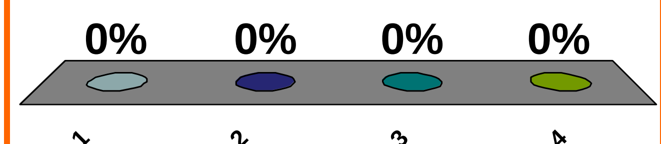
Assume $\Pr[X = 2] = 0.1$, $\Pr[X = -2] = 0.9$. Find $\mathrm{SD}[X]$.

(a) $(0.1)(0.9)(2 - (-2))$

(b) $(0.1)(0.9)(2 - (-2))^2$

(c) $\sqrt{(0.1)(0.9)}(2 - (-2))$

(d) none of the above



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

Assume $X = \begin{cases} 1, & \text{on } [0, 0.3] \\ 2, & \text{on } (0.3, 1] \end{cases}$

and $Y = \begin{cases} 10^7, & \text{on } [0, 0.3) \\ 0, & \text{on } [0.3, 1] \end{cases}$

Find $\mathrm{Corr}[X, Y]$.

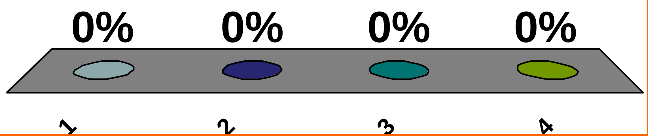
(a) $(0.3)(0.7)$

(b) 0

(c) 1

$\boxed{\text{(d) none of the above}}$

Correct answer: -1



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

Assume $X = \begin{cases} 10^{10^{100}}, & \text{on } [0, 0.7] \\ -2, & \text{on } (0.7, 1] \end{cases}$

and $Y = \begin{cases} 2, & \text{on } [0, 0.7) \\ 0, & \text{on } [0.7, 1] \end{cases}$

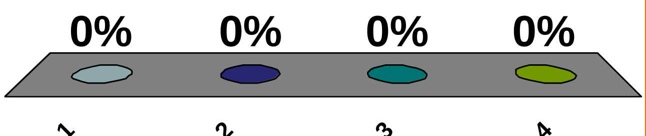
Find $\mathrm{Corr}[X, Y]$.

(a) $(0.3)(0.7)$

(b) 0

(c) 1

(d) none of the above



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

Assume $X = \begin{cases} -10^{10^{100}}, & \text{on } [0, 0.7] \\ -2, & \text{on } (0.7, .1] \end{cases}$

and $Y = \begin{cases} 2, & \text{on } [0, 0.7) \\ 5, & \text{on } [0.7, .1] \end{cases}$

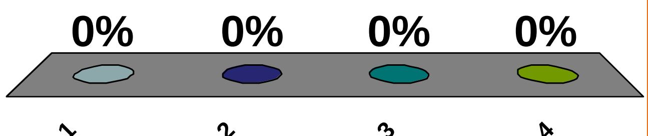
Find $\mathrm{Corr}[X, Y]$.

(a) $(0.3)(0.7)$

(b) 0

(c) 1

(d) none of the above



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

$$A = \begin{bmatrix} s & t \\ 0 & u \end{bmatrix}, \; A^t = \begin{bmatrix} s & 0 \\ t & u \end{bmatrix}, \; M = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix}.$$

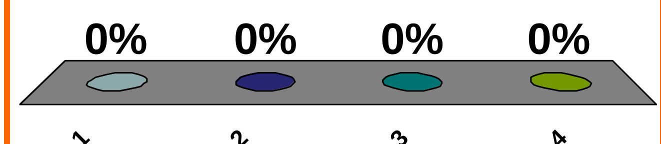
Find (upper triangular) A s.t. $AA^t = M$.

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

(d) none of the above

Correct answer: $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$



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0 of 5

$$B = \begin{bmatrix} s & 0 \\ t & u \end{bmatrix}, \quad B^t = \begin{bmatrix} s & t \\ 0 & u \end{bmatrix}, \quad M = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix}.$$

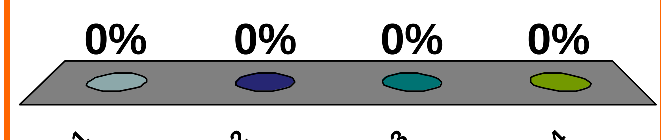
Find (lower triangular) B s.t. $B^tB = M$.

(a) $\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$\boxed{\text{(d) none of the above}}$

Correct answer: $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

$$A = \begin{bmatrix} s & t \\ 0 & u \end{bmatrix}, \qquad \begin{aligned} X_1 &= sZ_1 + tZ_2, \\ X_2 &= uZ_2, \end{aligned} \qquad M = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix},$$

Find (lower triangular) A s.t.
var/covar matrix of X_1, X_2 is M.

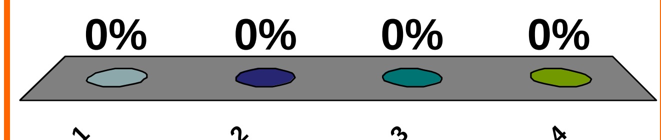
(a) $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

(d) none of the above

Correct answer: $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$



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21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

0 of 5

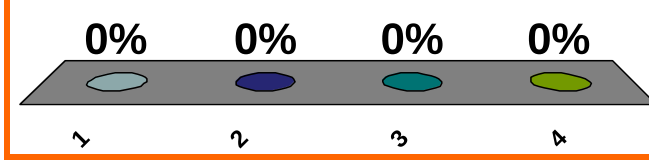
$$X_1, \ldots, X_{25} \text{ iid}$$
$$\mathsf{E}[X_1 + \cdots + X_{25}] = 50$$
$$\mathsf{E}[X_1] = ??$$

(a) 50

(b) 10

(c) 2

(d) none of the above



0 of 5

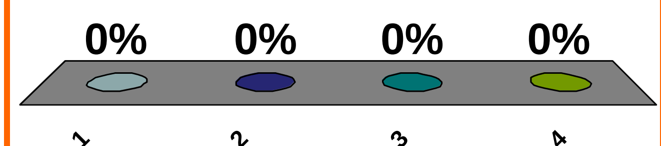
$$X_1, \ldots, X_{25} \text{ iid}$$
$$\mathrm{SD}[X_1 + \cdots + X_{25}] = 50$$
$$\mathrm{SD}[X_1] = ??$$

(a) 2

(b) $50/(25^2)$

$\boxed{\text{(c) } 10}$

(d) none of the above



0 of 5

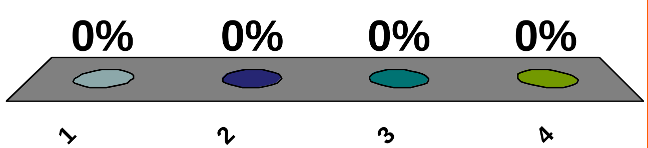
$$(4xy\,dx + 2y\,dy) \wedge (3x\,dx - 7e^{xy}\,dy) =$$

a. $(-28xye^{xy} - 6xy)\,dx \wedge dy$

b. $(28xye^{xy} - 6xy)\,dx \wedge dy$

c. $(28xye^{xy} + 6xy)\,dx \wedge dy$

d. none of the above



0 of 5

$$d(4xy\,dx + 2y\,dy) =$$

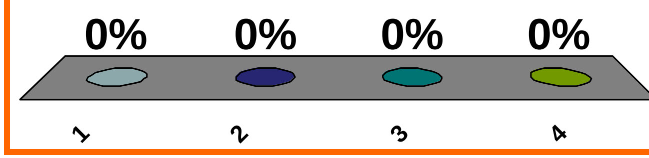
a. $(4x + 2)\,dx \wedge dy$

b. $(-4x + 2)\,dx \wedge dy$

c. $(-4x - 2)\,dx \wedge dy$

d. none of the above

$$-4x\,dx \wedge dy$$



0 of 5

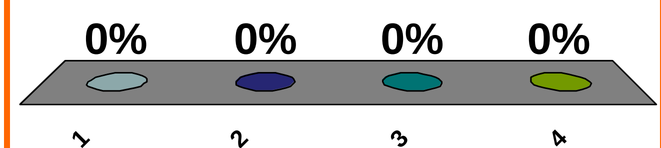
Compute $\displaystyle\int_{(2,3)}^{(4,7)} e^{-xy}\,dx + xy\,dy.$

(a)

(b)

(c)

(d)



0 of 5

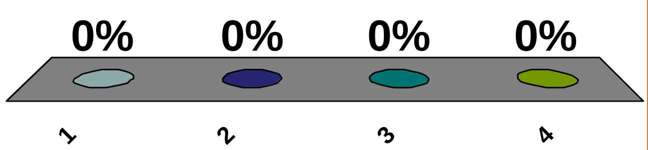
Compute $\int_{(8,3)}^{(4,7)} y\,dx - x\,dy.$

(a)

(b)

(c)

(d)



0 of 5

$$R = (2,8) \times (1,3)$$

Compute $\displaystyle\int_R dy \wedge dx$.

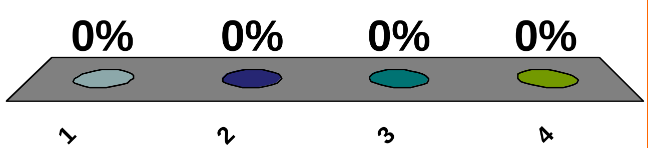
(a) 24

(b) 12

(c) 0

(d) none of the above

-12



0 of 5

$p = (8,1)$, $q = (8,3)$, $r = (2,3)$, $s = (2,1)$

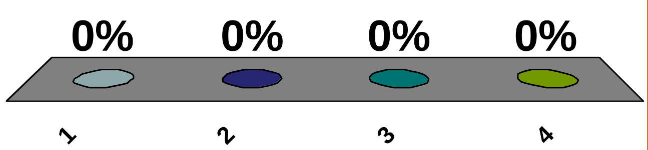
Compute $\left(\int_p^q + \int_q^r + \int_r^s + \int_s^t\right) x\,dy - y\,dx.$

(a) 24

(b) 12

(c) 0

(d) none of the above



0 of 5

$$p = (8,1),\ q = (8,3),\ r = (2,3),\ s = (2,1)$$

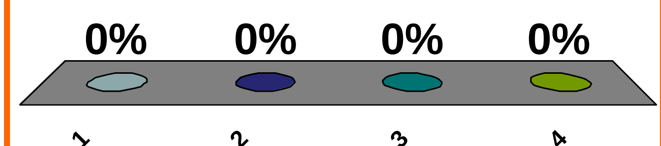
Compute $\left(\int_p^q + \int_q^r + \int_r^s + \int_s^t\right) y\,dx + x\,dy.$

(a) -1

(b) 0

(c) 1

(d) none of the above



0 of 5

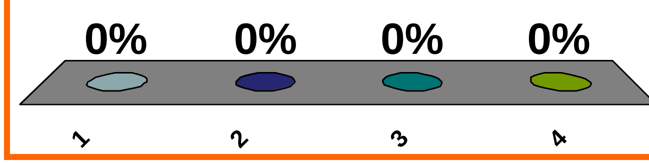
Compute $\displaystyle\int_{1+i}^{2+i} e^{-z}\,dz$.

(a) $e^{2+i} - e^{1+i}$

(b) $[-e^{2+i}] - [-e^{1+i}]$

$\boxed{\text{(c) } -e^{-(2+i)} + e^{-(1+i)}}$

(d) $e^{-(2+i)} - e^{-(1+i)}$



0 of 5

Compute $\left(\int_{1+i}^{2+4i} + \int_{2+4i}^{3-7i} + \int_{3-7i}^{1+i}\right) e^{-z}\,dz.$

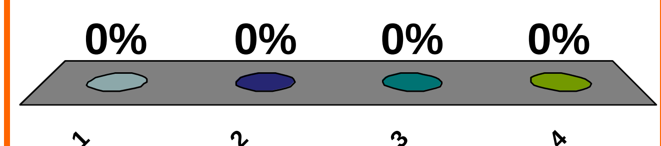
(a) $e^{1+i} + e^{2+4i} + e^{3-7i}$

(b) $-e^{1+i} - e^{2+4i} - e^{3-7i}$

(c) $-e^{-(1+i)} - e^{-(2+4i)} - e^{-(3-7i)}$

(d) none of the above

0



0 of 5

Compute $\left(\int_{1+i}^{2+4i} + \int_{2+4i}^{3-7i} + \int_{3-7i}^{1+i}\right) \frac{e^{-z^2/2}}{\sqrt{2\pi}}\,dz.$

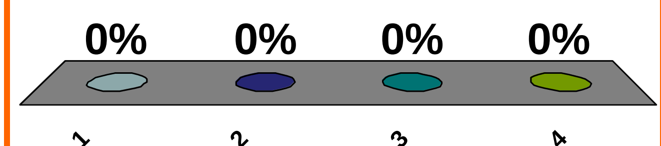
(a) $\Phi(1+i) + \Phi(2+4i)$
$\Phi(3-7i)$

(b) $-\Phi(1+i) - \Phi(2+4i)$
$-\Phi(3-7i)$

(c) $-\Phi(-1-i) - \Phi(-2-4i)$
$-\Phi(-3+7i)$

(d) none of the above

0



0 of 5

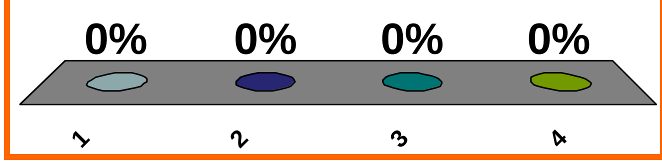
Compute $\int_{1+i}^{2+i} \bar{z}\, dz$.

$\boxed{\text{(a) } \frac{2^2-1}{2} - i}$

(b) $\frac{2^2-1}{2} + i$

(c) $2^2 - 1 - i$

(d) $2^2 - 1 + i$



0 of 5

$$k \in \mathbb{R}^{n\times 1}, \qquad \frac{dz}{dt} = k, \qquad [z]_{t:\to 0} = 0 \in \mathbb{R}^{n\times 1},$$

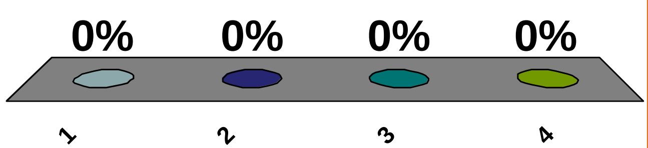
$$z = ?$$

a. e^{tM}

b. $e^{tM}k$

c. ke^{tM}

d. kt



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$$M \in \mathbb{R}^{n\times n}, \qquad \frac{dz}{dt} = Mz, \qquad [z]_{t=0} = k \in \mathbb{R}^{n\times 1},$$

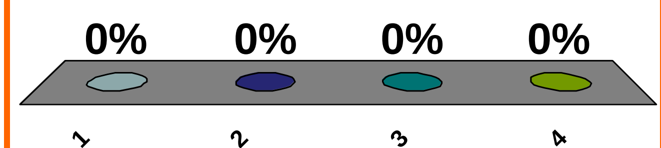
$$z = ?$$

a. e^{tM}

b. $\boxed{e^{tM}k}$

c. ke^{tM}

d. kt



0 of 5

$$M \in \mathbb{R}^{n\times n}, \qquad C^{-1}MC = D,$$

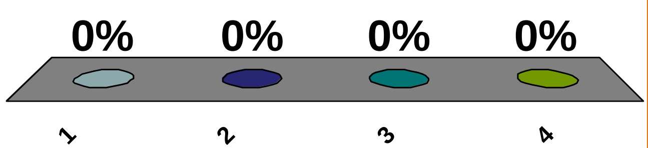
$$e^{tM} = ?$$

a. $C^{-1}e^{tD}C$

b. $\boxed{Ce^{tD}C^{-1}}$

c. e^{tD}

d. none of the above



0 of 5

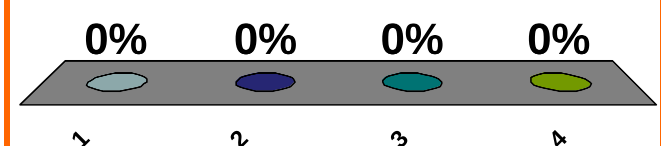
$$\frac{dz}{dt} = Mz, \qquad [z]_{t=0} = c \in \mathbb{R}^{2\times 1}, \qquad z = ?$$

a. e^{tM}

b. $e^{tM}c$

c. ce^{tM}

d. vt



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Financial Mathematics

Regular review session, Midterm 02

Review definitions:

PCRV
expectation, mean
variance
distribution, joint distribution
covariance, correlation

Skills:

Mean, variance and std dev. of a PCRV
esp. for binary PCRVs

e.g., say $\Pr[X = 4] = 0.3$ and $\Pr[X = 8] = 0.7$

Minimize $\mathrm{SD}[A + xB]$,
given SD of A, B and correlation

Discussion:

Expectation $\mathrm{E}[X] = \int_0^1 X(\omega)\, d\omega$

Variance $\mathrm{Var}[X] = \mathrm{E}[(X - \mathrm{E}[X])^2]$

Standard deviation $\mathrm{SD}[X] = \sqrt{\mathrm{Var[X]}}$

Covariance $\mathrm{Cov}[X, Y]$

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] + 2(\mathrm{Cov}[X,Y])$$

$$\mathrm{Cov}[X,Y] = (\mathrm{E}[XY]) - (\mathrm{E}[X])(\mathrm{E}[Y])$$

Correlation $\mathrm{Corr}[X, Y] = \cdots$

standard PCRV $= \cdots$

Discussion:

Conditional probability
of one event, given another

Independence of events

Independence of PCRVs

Discussion: Find a lower triangular matrix B

$$\text{s.t. } B^t B = \begin{bmatrix} s^2 & t \\ t & u^2 \end{bmatrix}.$$

Find an upper triangular matrix A

$$\text{s.t. } AA^t = \begin{bmatrix} s^2 & t \\ t & u^2 \end{bmatrix}.$$

Assume

the variance-covariance matrix of

$$\begin{matrix} Y \\ Z \end{matrix} \quad \text{is} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Find a, b, c, d s.t.

the variance-covariance matrix of

$$\begin{matrix} aY + bZ \\ cY + dZ \end{matrix} \quad \text{is} \quad \begin{bmatrix} s^2 & t \\ t & u^2 \end{bmatrix}.$$

1. True or False:
There exists a 2×2 real matrix A s.t.

$$AA^t = \begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix}$$

t. True

$\boxed{\text{f. False}}$

$$\forall A, v, \quad AA^t v \cdot v = A^t v \cdot A^t v \geq 0$$

Then $\forall A$, AA^t is pos. semidef.

so $\det(AA^t) \geq 0$.

$$\det \begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix} = 21 - 25 < 0$$

Homework#(02/20)-1

(02/20)-1-1. Let $X := \begin{bmatrix} 1 & 3 & -2 \\ 3 & 10 & -7 \\ -2 & -7 & 8 \end{bmatrix}$

a. Find a 3×3 matrix A s.t. $AA^t = X$.
and s.t. A is lower triangular.

c. Find a 3×3 matrix C s.t. $C^tC = X$.
and s.t. C is upper triangular.

e. Does there exist a *symmetric*
3×3 matrix S s.t. $\widehat{}$
(You must explain why or why not,
but you needn't write out such an S.)

Discussion:

Example of two different PCRVs with the same distribution.

Do they have have the same mean?
variance?
standard deviation?

Is there a formula for the variance of their sum?

What implies what?

X is independent of Y

Y is independent of X

$\mathrm{Cov}[X, Y] = 0$

$\mathrm{Corr}[X, Y] = 0$

$\mathrm{E}[X|X] = \cdots$

$\mathrm{Corr}[X, X] = \cdots$

$\mathrm{Corr}[X, 2X] = \cdots$

Discussion:

Polarization of

$$\text{Var}[X] = \text{E}[\,(\,X - \text{E}[X]\,)^2\,]$$

$\text{Var}[X]$ in terms of the first two raw moments, $\text{E}[X]$ and $\text{E}[X^2]$, of X.

Polarization of that formula.

$$\int_{-\infty}^{\infty} e^{ax} e^{-x^2/2}\, dx \qquad\qquad \int_{-c}^{\infty} e^{ax} e^{-x^2/2}\, dx$$

$$\int_{-\infty}^{\infty} e^{ax} e^{-x^2}\, dx \qquad\qquad \int_{-\infty}^{\infty} x^n e^{-x^2/2}\, dx$$

$\forall S, T \subseteq \mathbb{R}$,

$$\boxed{\Pr[V \in S \,|\, W \in T]} = \frac{\Pr[(V \in S) \ \& \ (W \in T)]}{\Pr[(W \in T)]}$$

Definition:

We say PCRVs V and W are **independent** if

$\forall S, T \subseteq \mathbb{R}$, $\qquad \Pr[(V \in S) \ \& \ (W \in T)] =$

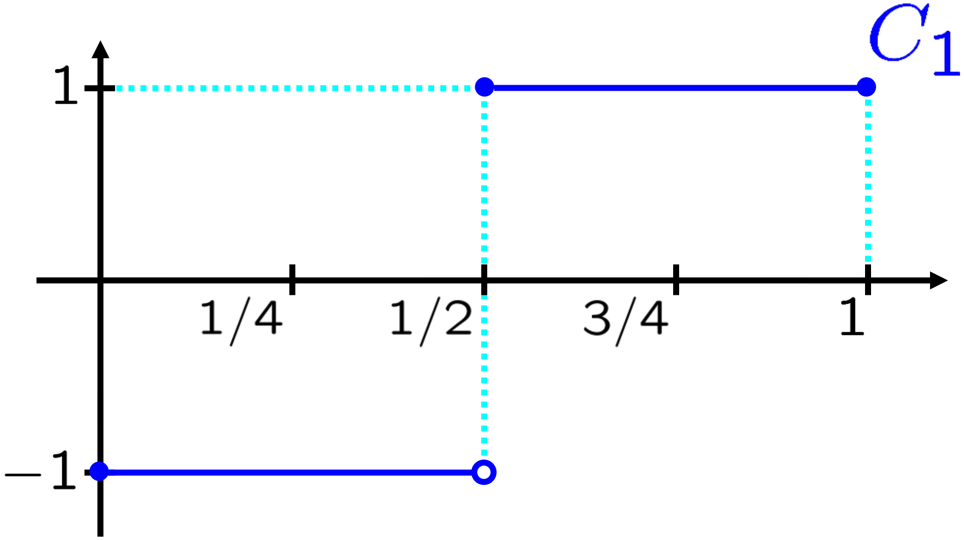
$$(\Pr[V \in S])(\Pr[W \in T]).$$

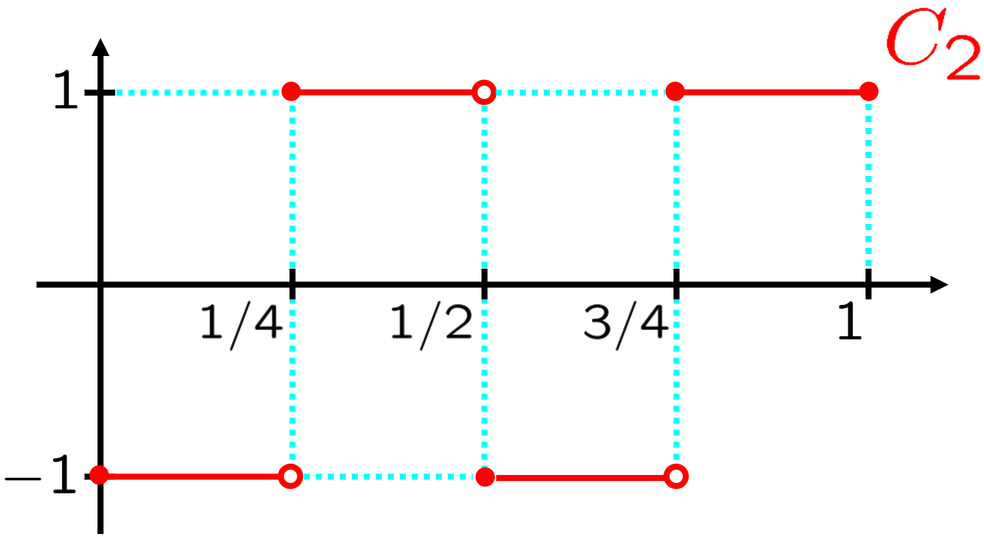
$$\boxed{\text{equivalently: } \Pr[V \in S \,|\, W \in T] = \Pr[V \in S]}$$

The idea:

Finding out that $W \in T$ has no effect on the probability that $V \in S$.

Finding out something about W is never of any help in guessing anything about V.





Note: C_1 and C_2 are independent.

Fact:

Let X and Y be independent PCRVs.

Then, for any functions $f, g : \mathbb{R} \to \mathbb{R}$,

$$f(X) \text{ and } g(Y) \text{ are independent.}$$

coin has $+1$ and -1 instead of H and T.

The idea:

Flip a $\boxed{\pm 1}$ fair coin twice.

If I tell you the first flip,
you get no useful info about the second.

If I tell you $3 \times (\text{the first flip}) + 7$,
you get no useful info about
$5 \times (\text{the second flip}) - 1$.

Let $W := 2X + 3Y$.

$$\begin{aligned}\mathrm{Corr}[2X + 3Y, 4X + 6Y] &= \mathrm{Corr}[W, 2W] \\ &= \mathrm{Corr}[W, W] \\ &= 1\end{aligned}$$

2. Say X, Y independent, std PCRVs.
Compute $\mathrm{Corr}[2X + 3Y, 4X + 6Y]$.

a. -1

$\boxed{\text{c. } 1}$

b. 0

d. 0.6

e. None of the above

1. Suppose $\Pr[B|A] = 6\%$,

$\Pr[B] = 4\%$ and $\Pr[A] = 2\%$.

Find $\Pr[A|B]$.

a. 6%

b. 12%

c. 1%

d. $\boxed{3\%}$

e. None of the above

$$\Pr[A|B] = \Pr[B|A] \cdot \frac{\Pr[A]}{\Pr[B]}$$

$$= (6\%) \cdot \frac{2\%}{4\%} = 3\%$$

Homework#(02/20)-1

(02/20)-1-2.

b. Compute $\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-5}^{7} xe^{-x^2/2}\,dx$.

c. Compute $\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty} x^4e^{-x^2/2}\,dx$.

d. Compute $\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty} x^5e^{-x^2/2}\,dx$.

Homework#(02/20)-1

(02/20)-1-3.

d. Compute $\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty}(e^{3x+4}-5)_+ e^{-x^2/2}\,dx.$

Homework#(02/13)-1

(02/13)-1-2.

Let X be a PCRV.

a. What is $\mathrm{Corr}[X, X]$?

b. Let $v := \mathrm{Var}[X]$.

What is $\mathrm{Cov}[X, X]$?

Your answer will be a (very simple) formula involving v.

Homework#(02/13)-1

(02/13)-1-6. Let $X := \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 7 \end{bmatrix}$

a. Find a 3×3 matrix A s.t. $AA^t = X$.

b. Does there exist a *symmetric* 3×3 matrix S s.t. $S^2 = X$?

(You must explain why or why not, but you needn't write out such an S.)

Homework#(02/06)-1

(02/06)-1-1. Let X be a binary PCRV
s.t. $\mathrm{Pr}[X = 4] = 0.65$
and s.t. $\mathrm{Pr}[X = 10] = 0.35$.
Find the mean, the variance and the standard deviation of X.

(02/06)-1-2. Let X and Y be a PCRVs
s.t. $\mathrm{Pr}[X = 4] = 0.65$,
s.t. $\mathrm{Pr}[(X = 4)\&(Y = 7)] = 0.45$
and s.t. $\mathrm{Pr}[(X = 4)\&(Y = 3)] = 0.2$.

(02/06)-1-2a. Find $\mathrm{Pr}[(Y = 3)|(X = 4)]$.

(02/06)-1-2b. Find $\mathrm{E}[Y|(X = 4)]$.

Homework#(02/06)-1

(02/06)-1-3. Let A and B be events

s.t. Pr$[A|B] = 0.3$,

s.t. Pr$[A] = 0.4$

and s.t. Pr$[B] = 0.8$.

Find Pr$[B|A]$.

(02/06)-1-4. Let A, B and C be events

s.t. Odds$[A] = 3/2$,

s.t. Pr$[B|A] = 0.4$,

s.t. Pr$[B|(\text{not } A)] = 0.4$,

s.t. Pr$[C|(B\&A)] = 0.2$,

and s.t. Pr$[C|(B\&(\text{not } A))] = 0.1$.

Find Odds$[A|(B\&C)]$.

Discussion: the partition of a PCRV

$\mathrm{E}[X]$, $\mathrm{Var}[X]$, $\mathrm{Cov}[X, Y]$

$X = Y$ a.s.

$\mathrm{E}[X|A]$, A an event of positive measure

$\mathrm{E}[X|\mathcal{P}]$, $\mathcal{P}$ a partition by fUofIs
of positive measure

$\mathrm{E}[X|Y]$, Y a PCRV
whose partition has events
of positive measure

independent events

independent PCRVs — independent partitions

$\mathcal{P}$-measurable PCRV

measurability of one PCRV w.r.t. another

$\mathrm{E}[X|Y]$, when X is indep. of Y

$\mathrm{E}[X|Y]$, when X is msbl w.r.t. Y

distribution of a PCRV — Fourier transform

to what extent is a PCRV det'd by its dist.?

e^x

$$\int xe^x\,dx = xe^x - \underline{\int e^x\;dx} = xe^x - (e^x + C)$$

1

$$= xe^x - e^x + C$$

2. $\displaystyle\int xe^x\,dx =$

a. $(x^2/2)e^x + C$

b. $-(x^2/2)e^x + C$

$\boxed{\text{c. } xe^x - e^x + C}$

d. $xe^x + e^x + C$

e. None of the above

$$\int x^n e^{-x^2/2}\,dx$$

$$\int_{-\infty}^{\infty} x^n e^{-x^2/2}\,dx$$

$$\int e^{ax+b} e^{-x^2/2}\,dx$$

$$\int_{-\infty}^{\infty} (e^{ax+b} - k)_+ e^{-x^2/2}\,dx$$

$$\int \underset{\substack{\downarrow\\ 1}}{x}\,\overset{\substack{e^x\\ \uparrow}}{e^x}\,dx = xe^x - \underline{\int e^x\;dx} = xe^x - (e^x + C)$$

$$= xe^x - e^x + C$$

2. $\int xe^x\,dx =$

a. $(x^2/2)e^x + C$

b. $-(x^2/2)e^x + C$

$\boxed{\text{c. } xe^x - e^x + C}$

d. $xe^x + e^x + C$

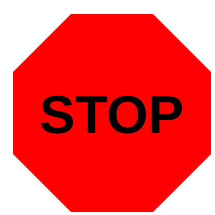
e. None of the above

$$\int x^n e^{-x^2/2}\,dx$$

$$\int_{-\infty}^{\infty} x^n e^{-x^2/2}\,dx$$

$$\int e^{ax+b} e^{-x^2/2}\,dx$$

$$\int_{-\infty}^{\infty} (e^{ax+b} - k)_+ e^{-x^2/2}\,dx$$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