Financial Mathematics

Clicker review session A, Final

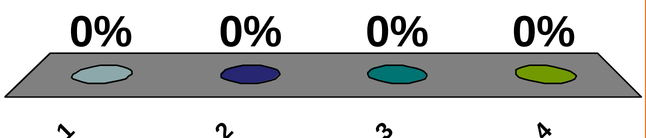
$$(\nabla f)(x,y) = (2x, 5y)$$
$$(\nabla g)(x,y) = (7, 3y)$$
$$(\nabla g)(x,y) = \lambda[(\nabla f(x,y)] \Leftrightarrow ?$$

(a) $(2x)(7) = (5y)(3y)$

(b) $(2x)(7) + (5y)(3y) = 0$

(c) $(2x)(3y) = (5y)(7)$

(d) none of the above



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0 of 5

$\forall n,\ 0.01 < p_n, q_n < 0.99$

$$X_n \in \sum^n \mathcal{B}_{q_n}^{p_n}$$

$\mathrm{E}[X_n] \to 1$

$\mathrm{Var}[X_n] \to 4$

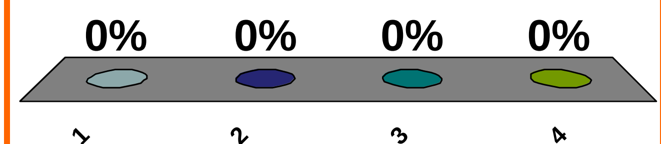
$X_n \to ?$ in dist.

(a) $4Z + 1$

(b) $2Z + 1$

(c) $Z + 4$

(d) none of the above



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0 of 5

$\forall n, \; 0.01 < p_n, q_n < 0.99$

$X_n \in \sum^n \mathcal{B}_{q_n}^{p_n}$

$$h(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

$\mathsf{E}[X_n] \to 1$

$\mathsf{Var}[X_n] \to 4$

$X_n \to 2Z + 1$

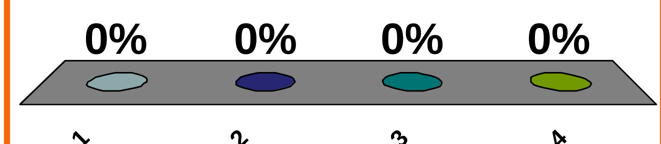
$\mathsf{E}[\cos X_n] \to ?$

(a) $\displaystyle\int_{-\infty}^{\infty} [\cos(2x+1)][h(x)]\,dx$

(b) $\displaystyle\int_{-\infty}^{\infty} [\cos(x)][h(2x+1)]\,dx$

(c) $\displaystyle\int_{-\infty}^{\infty} [\cos((x-1)/2)][h(x)]\,dx$

(d) none of the above



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0 of 5

$$k \in \mathbb{R}^{n\times 1}, \qquad \frac{dz}{dt} = k, \qquad [z]_{t:\to 0} = 0 \in \mathbb{R}^{n\times 1},$$

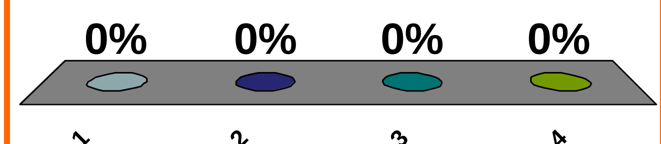
$$z = ?$$

a. e^{tM}

b. $e^{tM}k$

c. ke^{tM}

d. kt



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$$M \in \mathbb{R}^{n\times n}, \qquad \frac{dz}{dt} = Mz, \qquad [z]_{t=0} = k \in \mathbb{R}^{n\times 1},$$

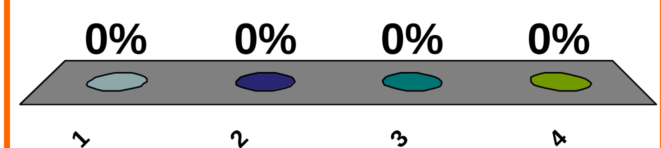
$$z = ?$$

a. e^{tM}

b. $e^{tM}k$

c. ke^{tM}

d. kt



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$$M \in \mathbb{R}^{n\times n}, \qquad C^{-1}MC = D,$$

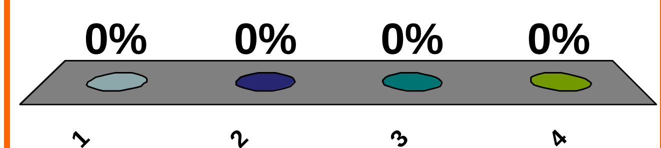
$$e^{tM} = ?$$

a. $C^{-1}e^{tD}C$

b. $\boxed{Ce^{tD}C^{-1}}$

c. e^{tD}

d. none of the above



0 of 5

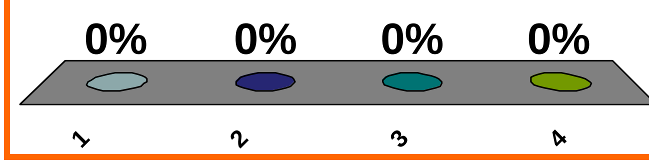
$$\frac{dz}{dt} = Mz, \qquad [z]_{t=0} = c \in \mathbb{R}^{2\times 1}, \qquad z = ?$$

a. e^{tM}

b. $e^{tM}c$

c. ce^{tM}

d. vt



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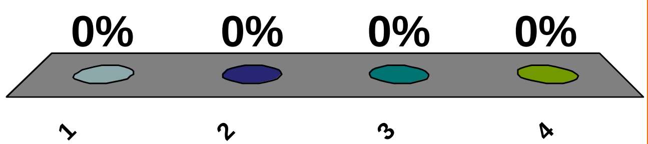
$$(4xy\,dx + 2y\,dy) \wedge (3x\,dx - 7e^{xy}\,dy) =$$

a. $(-28xye^{xy} - 6xy)\,dx \wedge dy$

b. $(28xye^{xy} - 6xy)\,dx \wedge dy$

c. $(28xye^{xy} + 6xy)\,dx \wedge dy$

d. none of the above



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$$d(4xy\,dx + 2y\,dy) =$$

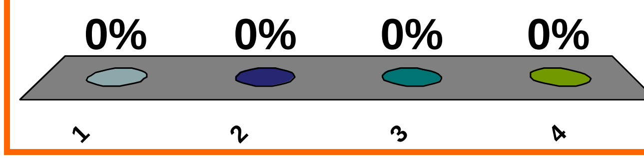
a. $(4x + 2)\,dx \wedge dy$

b. $(-4x + 2)\,dx \wedge dy$

c. $(-4x - 2)\,dx \wedge dy$

d. none of the above

$$-4x\,dx \wedge dy$$



0 of 5

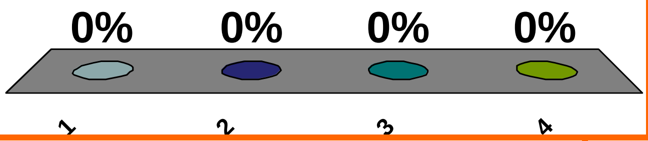
$$d(6y\,dx + 2e^{xy}\,dy) =$$

(a) $(-6 + 2ye^{xy})dx \wedge dy$

(b) $(-6 + 2xe^{xy})dx \wedge dy$

(c) $(6 + 2ye^{xy})dx \wedge dy$

(d) none of the above



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0 of 5

$$f(x,y) = 2x^2 + 5xy$$

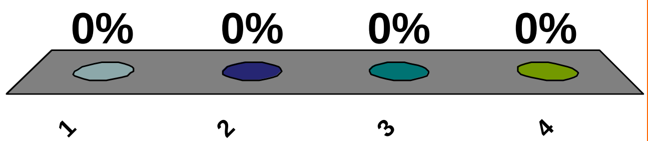
$$(\nabla f)(x,y) =$$

(a) $(4x, 5y)$

(b) $(4x + 5y, 5x)$

(c) $(2x, 5x)$

(d) none of the above



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0 of 5

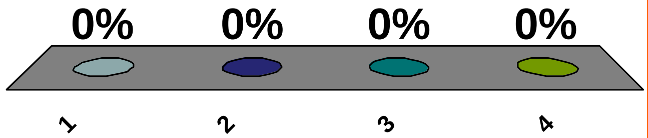
$$f(x,y) = 2x^2 + 5xy$$
$$(\nabla f)(x,y) = (4x + 5y, 5x)$$
$$(Hf)(x,y) =$$

(a) $\begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 5 \\ 5 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 5/2 \\ 5/2 & 0 \end{bmatrix}$

(d) none of the above



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0 of 5

$f(0,0) = 2 \qquad (\nabla f)(0,0) = (0,0)$

$$(Hf)(x,y) = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$$

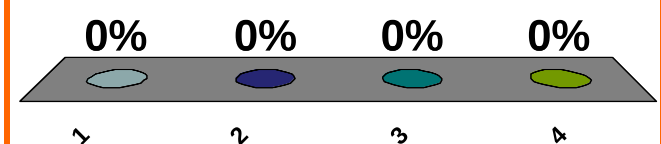
2nd order Macl.?

(a) $2 - 2x^2 + 2xy$

(b) $\boxed{2 - x^2 + xy}$

(c) $-2x^2 + 2xy$

(d) none of the above



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0 of 5

$$f(0,0) = 2 \qquad (\nabla f)(0,0) = (0,0)$$

$$(Hf)(x,y) = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$$

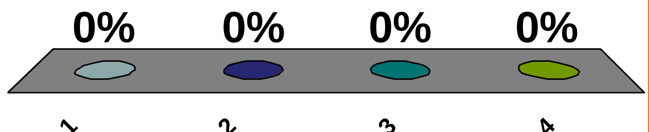
local max/min at $(0,0)$?

(a) local max

(b) local min

(c) neither

(d) none of the above



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0 of 5

$$f(0,0) = 2 \qquad (\nabla f)(0,0) = (0,0)$$

$$(Hf)(x,y) = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}$$

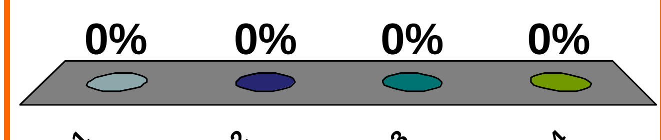
local max/min at $(0,0)$?

(a) local max

(b) local min

(c) neither

(d) none of the above



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0 of 5

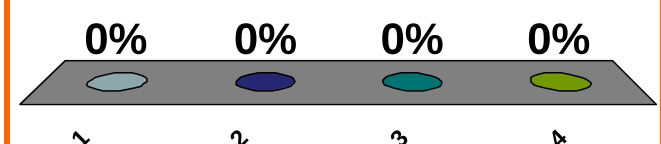
$$d(6x\,dx + 2e^{xy}\,dy) =$$

(a) $(-6 + 2ye^{xy})dx \wedge dy$

(b) $(6 + 2xe^{xy})dx \wedge dy$

(c) $(2ye^{xy})dx \wedge dy$

(d) none of the above



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0 of 5

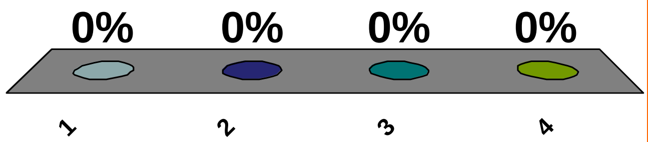
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{4x}e^{-x^2/2}\,dx$$

a. e^{4^2}

b. $\boxed{e^{4^2/2}/\sqrt{2\pi}}$

c. $e^{4^2}/\sqrt{2\pi}$

d. $e^{4^2}/2$



0 of 5

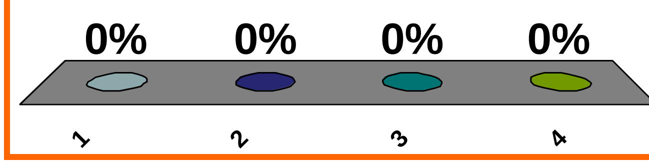
$$\int_{-\infty}^{\infty} e^{8x} e^{-x^2/2}\, dx$$

a. e^{8^2}

b. $e^{8^2/2}\sqrt{2\pi}$

c. $e^{8^2/2}/\sqrt{2\pi}$

d. $e^{8^2/2}$



0 of 5

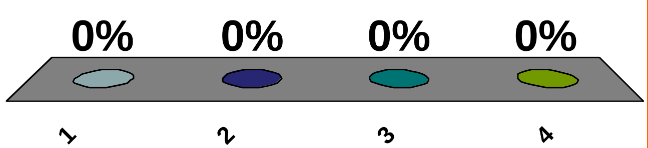
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-4x} e^{-x^2/2}\, dx$$

a. $e^{(-4)^2}$

b. $e^{(-4)^2/2}\sqrt{2\pi}$

c. $e^{(-4)^2/2}/\sqrt{2\pi}$

d. $e^{(-4)^2/2}$



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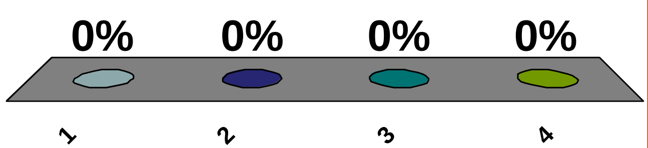
$$\int_{\boxed{-7}}^{\infty} e^{-4x} e^{-x^2/2}\, dx$$

a. $\displaystyle \int_{-5}^{\infty} e^{(-4)^2/2} e^{-x^2/2}\, dx$

b. $\displaystyle \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-4x} e^{-x^2/2}\, dx$

c. $\displaystyle \frac{1}{\sqrt{2\pi}} \int_{-9}^{\infty} e^{(-4)^2/2} e^{-x^2/2}\, dx$

d. $\displaystyle \int_{-3}^{\infty} e^{(-4)^2/2} e^{-x^2/2}\, dx$



0 of 5

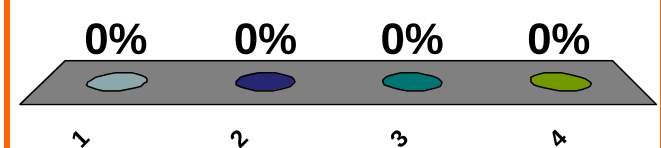
$$\int_5^\infty e^{3x} e^{-x^2/2}\, dx$$

a. $\int_8^\infty e^{3^2/2} e^{-x^2/2}\, dx$

b. $\int_{-\infty}^\infty e^{3^2/2} e^{-x^2/2}\, dx$

c. $\int_2^\infty e^{3^2/2} e^{-x^2/2}\, dx$

d. $\int_0^\infty e^{3^2/2} e^{-x^2/2}\, dx$



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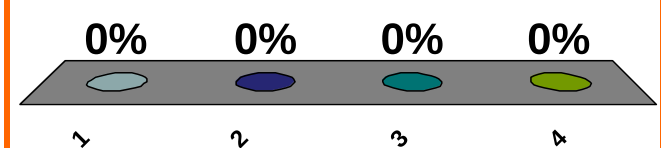
$$\int_{13}^{\infty} e^{\pi x} e^{-x^2/2}\, dx$$

a. $\displaystyle\int_{13-\pi}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx$

b. $\displaystyle\int_{13-(\pi/2)}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx$

c. $\displaystyle\int_{13+\pi}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx$

d. $\displaystyle\int_{0}^{\infty} e^{\pi^2/2} e^{-x^2/2}\, dx$



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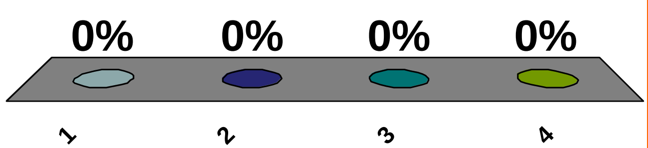
$$\int_3^\infty e^{7^2/2} e^{-x^2/2}\,dx$$

a. $e^{7^2/2}(\Phi(3))$

b. $e^{7^2/2}(\Phi(3))\sqrt{2\pi}$

c. $e^{7^2/2}(\Phi(-3))$

d. $\boxed{e^{7^2/2}(\Phi(-3))\sqrt{2\pi}}$



0 of 5

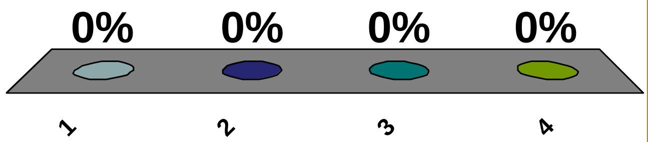
$$\frac{1}{\sqrt{2\pi}}\int_{-5}^{\infty} e^4 e^{-x^2/2}\,dx$$

a. $\boxed{e^4(\Phi(5))}$

b. $e^4(\Phi(5))\sqrt{2\pi}$

c. $e^4(\Phi(-5))$

d. $e^4(\Phi(-5))\sqrt{2\pi}$



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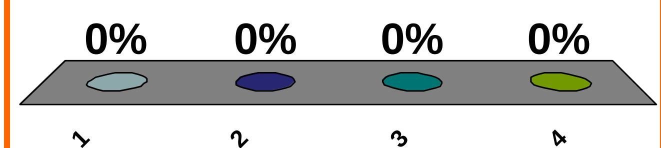
$$\int_{10}^{\infty} e^{7/3} e^{-x^2/2}\, dx$$

a. $e^{7/3}(\Phi(10))$

b. $e^{7/3}(\Phi(10))\sqrt{2\pi}$

c. $e^{7/3}(\Phi(-10))$

d. $e^{7/3}(\Phi(-10))\sqrt{2\pi}$



0 of 5

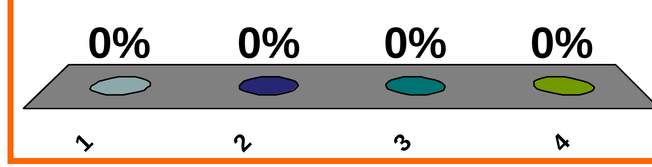
$$\int_{-\infty}^{\infty} x^{15} e^{-x^2/2}\, dx$$

a. $(15!) \cdot \sqrt{2\pi}$

b. $1 \cdot 3 \cdot 5 \cdot \cdots \cdot 15 \cdot \sqrt{2\pi}$

$\boxed{\text{c. } 0}$

d. $15!\sqrt{2\pi}$



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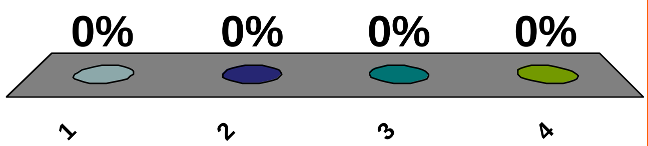
$$\int_{-\infty}^{\infty} x^{16} e^{-x^2/2}\, dx$$

a. $(15!) \cdot \sqrt{2\pi}$

b. $1 \cdot 3 \cdot 5 \cdot \cdots \cdot 15 \cdot \sqrt{2\pi}$

c. 0

d. $2 \cdot 4 \cdot 6 \cdot \cdots \cdot 16 \cdot \sqrt{2\pi}$



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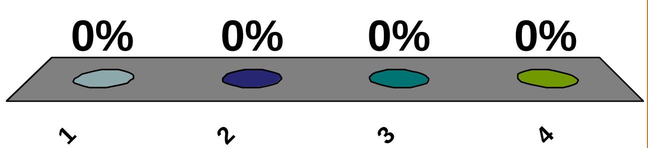
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x^{25}e^{-x^2/2}\,dx$$

a. $25!$

b. $1 \cdot 3 \cdot 5 \cdot \cdots \cdot 25$

c. $\boxed{0}$

d. $2 \cdot 4 \cdot 6 \cdot \cdots \cdot 26$



0 of 5

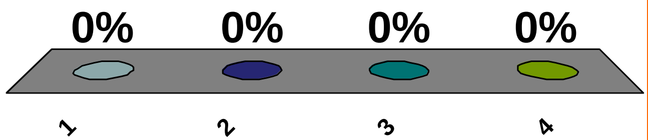
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^{26} e^{-x^2/2} \, dx$$

a. $25!$

b. $1 \cdot 3 \cdot 5 \cdot \cdots \cdot 25$

c. 0

d. $2 \cdot 4 \cdot 6 \cdot \cdots \cdot 26$



0 of 5

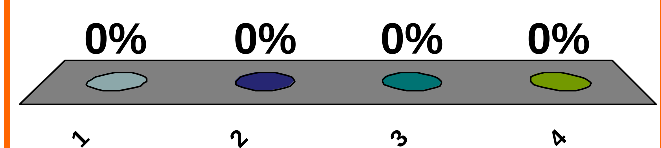
Assume $\Pr[X = 3] = 0.4$ and $\Pr[X = 8] = 0.6$. Find $\mathrm{E}[X]$.

a. $(0.4)(0.6)(8-3)$

b. $(0.4)(0.6)(8-3)^2$

c. $\sqrt{(0.4)(0.6)}(8-3)$

d. $\boxed{(0.4)(3)+(0.6)(8)}$



0 of 5

Assume $\Pr[X = 2] = 0.1$, $\Pr[X = -2] = 0.9$.

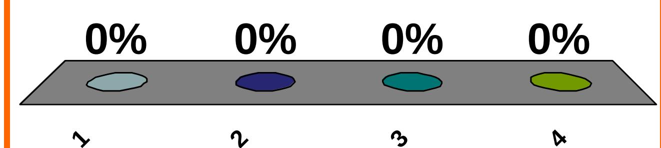
Find $\mathrm{E}[X]$.

a. $(0.1)(0.9)(2-(-2))$

b. $(0.1)(0.9)(2-(-2))^2$

c. $\sqrt{(0.1)(0.9)}(2-(-2))$

d. $(0.1)(2) + (0.9)(-2)$



0 of 5

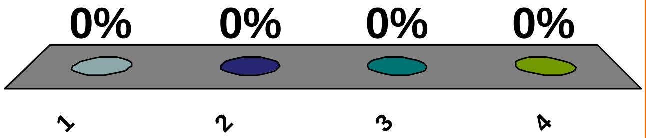
Assume $\Pr[X = 3] = 0.4$ and $\Pr[X = 8] = 0.6$.
Find $\mathrm{Var}[X]$.

a. $(0.4)(0.6)(8-3)$

b. $(0.4)(0.6)(8-3)^2$

c. $\sqrt{(0.4)(0.6)}(8-3)$

d. $(0.4)(3) + (0.6)(8)$



0 of 5

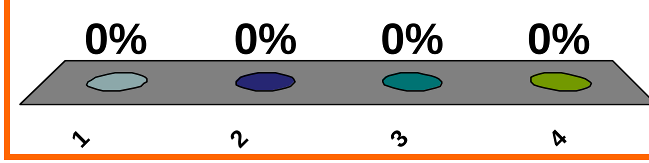
Assume $\Pr[X = 2] = 0.1$, $\Pr[X = -2] = 0.9$.
Find $\mathrm{Var}[X]$.

a. $(0.1)(0.9)(2 - (-2))$

b. $(0.1)(0.9)(2 - (-2))^2$

c. $\sqrt{(0.1)(0.9)}(2 - (-2))$

d. $(0.1)(2) + (0.9)(-2)$



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$$\mathrm{E}\left[\sum^n \mathcal{B}^{0.2,2}_{0.8,-3}\right]$$

(a) $-(0.2)(2) - (0.8)(-3)$

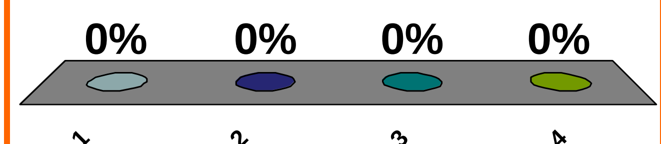
(b) $(0.2)(2) - (0.8)(-3)$

(c) $(0.2)(2) + (0.8)(-3)$

(d) none of the above

Correct answer:

$$n[(0.2)(2) + (0.8)(-3)]$$



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

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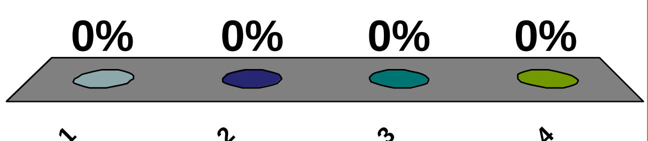
$$\mathrm{SD}\left[\sum^n \mathcal{B}^{0.2,2}_{0.8,-3}\right]$$

(a) $n(0.2)(0.8)[2+3]^2$

(b) $n\sqrt{(0.2)(0.8)}[2+3]$

(c) $\sqrt{n}\sqrt{(0.2)(0.8)}[2+3]$

(d) none of the above



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

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$$Z_n := (C_1 + \cdots + C_n)/\sqrt{n}$$

$$\lim_{n\to\infty} \mathsf{E}[e^{3Z_n - 5}] = ??$$

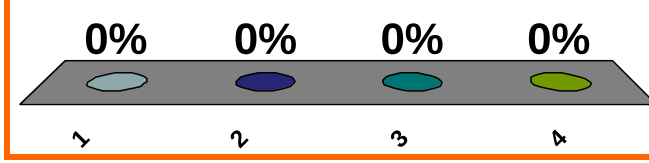
a. $\int_{-\infty}^{\infty} e^{3x-5}\, dx$

b. $\int_{-\infty}^{\infty} \left[e^{3x-5}\right] \left[e^{-x^2/2}\right] dx$

c. 0

d. none of the above

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[e^{3x-5}\right] \left[e^{-x^2/2}\right] dx$$



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Assume $X = \begin{cases} 1, & \text{on } [0, 0.3] \\ 2, & \text{on } (0.3, 1] \end{cases}$

and $Y = \begin{cases} 10^7, & \text{on } [0, 0.3) \\ 0, & \text{on } [0.3, 1] \end{cases}$

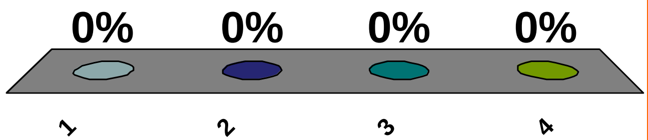
Find $\mathrm{Corr}[X, Y]$.

a. $(0.3)(0.7)$

b. 0

c. 1

d. -1



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$$\text{Assume} \quad X = \begin{cases} 10^{10^{100}}, & \text{on } [0, 0.7] \\ -2, & \text{on } (0.7, 1] \end{cases}$$

$$\text{and} \quad Y = \begin{cases} 2, & \text{on } [0, 0.7) \\ 0, & \text{on } [0.7, 1] \end{cases}$$

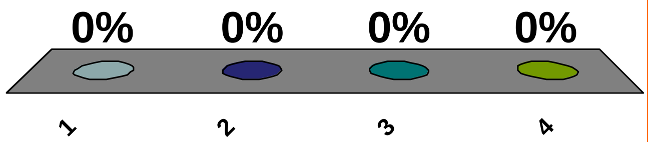
Find $\mathrm{Corr}[X, Y]$.

a. $(0.3)(0.7)$

b. 0

c. $\boxed{1}$

d. -1



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Assume $X = \begin{cases} -10^{10^{100}}, & \text{on } [0, 0.7] \\ -2, & \text{on } (0.7, .1] \end{cases}$

and $Y = \begin{cases} 2, & \text{on } [0, 0.7) \\ 5, & \text{on } [0.7, .1] \end{cases}$

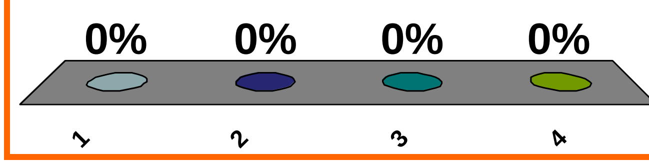
Find $\mathrm{Corr}[X, Y]$.

a. $(0.3)(0.7)$

b. 0

c. $\boxed{1}$

d. -1



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$$A = \begin{bmatrix} s & t \\ 0 & u \end{bmatrix},\ A^t = \begin{bmatrix} s & 0 \\ t & u \end{bmatrix},\ M = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix}.$$

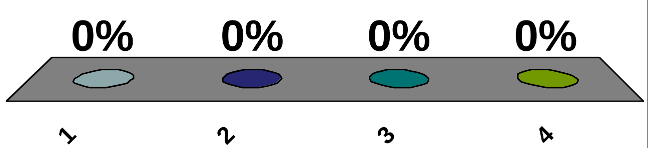
Find (upper triangular) A s.t. $AA^t = M$.

a. $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$



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$$B = \begin{bmatrix} s & 0 \\ t & u \end{bmatrix}, \quad B^t = \begin{bmatrix} s & t \\ 0 & u \end{bmatrix}, \quad M = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix}.$$

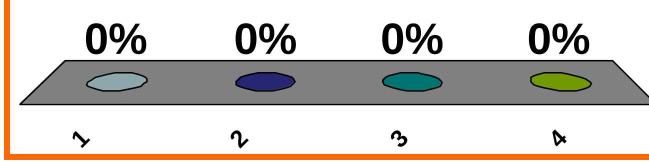
Find (lower triangular) B s.t. $B^t B = M$.

a. $\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$



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$$A = \begin{bmatrix} s & t \\ 0 & u \end{bmatrix}, \quad \begin{aligned} X_1 &= sZ_1 + tZ_2, \\ X_2 &= uZ_2, \end{aligned} \quad M = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix},$$

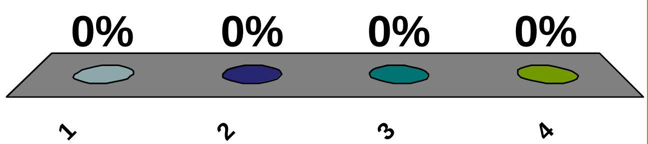
Find (lower triangular) A s.t. var/covar matrix of X_1, X_2 is M.

a. $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$



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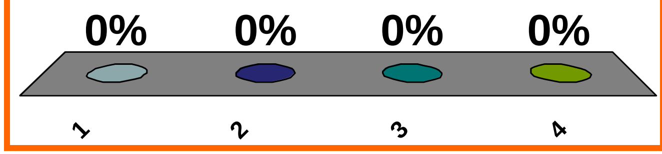
Assume $\Pr[X = 2] = 0.1$, $\Pr[X = -2] = 0.9$. Find the Fourier transform of the dist. of X.

(a) $2e^{(0.1)it} - 2e^{(0.9)it}$

(b) $2e^{-(0.1)it} - 2e^{-(0.9)it}$

(c) $(0.1)e^{2it} + (0.9)e^{-2it}$

(d) $(0.1)e^{-2it} + (0.9)e^{2it}$



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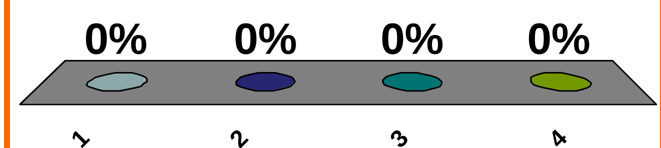
Assume $\Pr[X = 4] = 0.3$, $\Pr[X = 3] = 0.7$.
Find the Fourier transform of the dist. of X.

(a) $4e^{(0.3)it} + 3e^{(0.7)it}$

(b) $(0.3)e^{-4it} + (0.7)e^{-3it}$

(c) $(0.3)e^{4it} + (0.7)e^{3it}$

(d) $4e^{-(0.3)it} + 3e^{-(0.7)it}$



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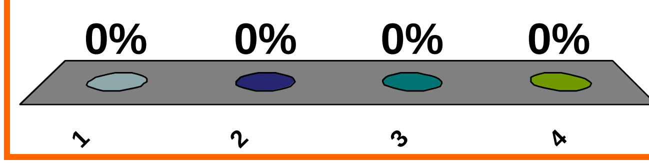
Assume $\Pr[X = 7] = 0.6$, $\Pr[X = 5] = 0.4$.
Find the Fourier transform of the dist. of X.

(a) $(0.6)e^{7it} + (0.4)e^{5it}$

(b) $7e^{(0.6)it} + 5e^{(0.4)it}$

(c) $7e^{-(0.6)it} + 5e^{-(0.4)it}$

(d) $(0.6)e^{-7it} + (0.4)e^{-5it}$



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