Financial Mathematics

One variable differential calculus review

$$f(x) = x^2$$

$$\frac{d}{dx}[f(x)] = 2x = f'(x)$$

$$f(t) = t^2$$

$$\frac{d}{dt}[f(t)] = 2t = f'(t)$$

$$f = (\bullet)^2$$

$$f^{\text{I}} = 2(\bullet)$$

$$f'(t) = 2tc$$

ALLOWABLE NOTATION:

$$\frac{d}{dx}[f(x)]$$

$$\frac{d}{dt}[f(t)]$$

f'

NOT ALLOWED:







I should ask your permission...
USUALLY AVOIDED:

$$[f(x)]'$$
 $[f(u)]'$ $[f(t)]^{\bullet} := \frac{d}{dt}[f(t)]$

ALLOWABLE NOTATION:

$$u := \sin x$$

$$u := \sin x$$
 $\sin' = \cos$

$$\frac{du}{dx} = \frac{d}{dx}u = \frac{d}{dx}\sin x = \cos x$$

$$v := t^5$$

$$\frac{dv}{dt} = \frac{d}{dt}v = \frac{d}{dt}t^5 = 5t^4$$

USUALLY AVOIDED:

$$u := \sin x$$

$$u := \sin x$$
 directly $v := t^5$

$$u' = (\sin x)' = \cos x$$

$$\dot{v} = (t^5)^{\bullet} = 5t^4$$

$$f(x) = \cos(x) \qquad \frac{d}{dx}[f(x)] = -\sin(x) = f'(x)$$

$$f(t) = \cos(t) \qquad \frac{d}{dt}[f(t)] = -\sin(t) = f'(t)$$

 $f=\cos \qquad \qquad f'$ plug in x and t

,

$$f(x) = \ln(x)$$

$$\frac{d}{dx}[f(x)] = \frac{1}{x} = f'(x)$$

$$f(t) = \ln(t)$$

$$\frac{d}{dt}[f(t)] = \frac{1}{t} = f'(t)$$

$$f = In$$



$$\frac{d}{dx}[f(x)] = f'(x)$$

start with a function make it an expression of x differentiate w.r.t. x

differentiate the function make it an expression of \boldsymbol{x}

$$\frac{d}{dt}[f(t)] = f'(t)$$

start with a function make it an expression of t differentiate w.r.t. t

differentiate the function make it an expression of t

$x : \rightarrow t$

Product Rule

("Differentiation by parts")

1st 2nd
part part $\frac{d}{dx}[f(x)][g(x)] =$

$$f(x) = \left[\frac{d}{dx} (g(x)) \right] + \left[\frac{d}{dx} (f(x)) \right]$$

the the the derivative of the part of the 2nd part 1st part

the

2nd

part

$$(fg)' = fg' + f'g$$

Product Rule ("Differentiation by parts")

$$\frac{d}{dt} \left[[f(t)] \left[g(t) \right] \right] =$$

part

Product Rule ("Differentiation by parts")

$$\frac{d}{du} \begin{bmatrix} u^2 & e^u \end{bmatrix} = \begin{bmatrix} u^2 \end{bmatrix} \begin{bmatrix} e^u \end{bmatrix} + \begin{bmatrix} 2u \end{bmatrix} \begin{bmatrix} e^u \end{bmatrix}$$
the 1st part of the 2nd part 1st part of the 2nd part 1st part

Product Rule ("Differentiation by parts")

Many equivalent alternatives, e.g., . .

$$\frac{d}{du} \begin{bmatrix} u^2 & e^u \end{bmatrix} = 2u e^u + u^2 e^u$$

$$= \left[\begin{array}{c} u^2 \end{array} \right] \qquad \left[\begin{array}{c} e^u \end{array} \right] \qquad + \left[\begin{array}{c} 2u \end{array} \right] \qquad \left[\begin{array}{c} e^u \end{array} \right]$$

part

11

the the the derivative of the of the the 1st part of the of the 2nd part 1st part

Quotient Rule

$$\lim_{\text{low}} \left[\frac{f}{g} \right]' = \frac{g f'}{g^2}$$

and underneath, low squared'll go.

Exercise: Write out the formulas for

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right]$$
 and $\frac{d}{dt} \left[\frac{f(t)}{g(t)} \right]$

Quotient Rule Low dee high $\frac{1}{5}$ high dee low, $\frac{[\cos t] [\cos t] - [\sin t][-\sin t]}{\cos^2 t}$ and underneath,

low squared'll go.
$$\frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t} = \sec^2 t$$

 $tan' = sec^2$

e.g.:

 $\lceil \sin t \rceil$

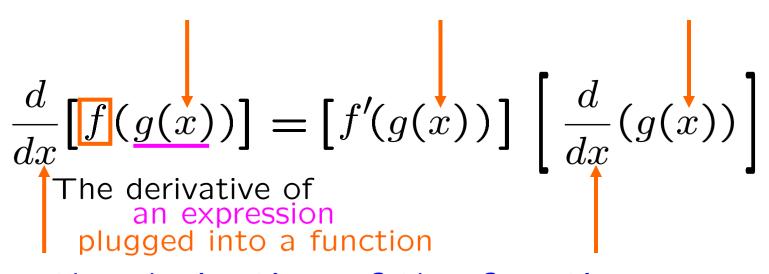
 $L\cos t$

[tan t]

13

$x:\to t$

Chain Rule



Take the derivative of the function.

Plug in the expression.

Chain Rule

$$\frac{d}{dt} [f(g(t))] = [f'(g(t))] \left[\frac{d}{dt} (g(t)) \right]$$
The derivative of an expression plugged into a function

Take the derivative of the function.

Plug in the expression.

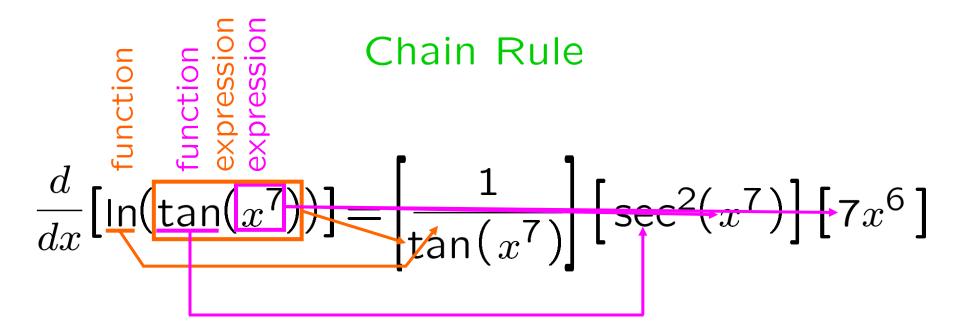
Chain Rule
$$\frac{d}{dx} \left[\ln(\sin(x)) \right] = \left[\frac{1}{\sin(x)} \right] \left[\frac{d}{dx} \cos(x) \right]$$

$$= \cot(x)$$

Take the derivative of the function.

Plug in the expression.

This expression is itself an expression inside a function.



Take the derivative of the function.

Plug in the expression.

Practice Problem

$$\frac{d}{dx} \frac{e^x \sin x}{\cos^7(x^3)}$$

$$\left[\cos^7(x^3)\right]\left[-\left[e^x \sin x\right]\right]$$

$$\cos^{14}(x^3)$$

18

Practice Problem

$$\frac{d}{dx} \frac{e^x \sin x}{\cos^7(x^3)}$$

$$[\cos^{7}(x^{3})][e^{x}\sin x + e^{x}\cos x]$$

$$-[e^{x}\sin x][$$

$$\cos^{14}(x^{3})$$

Practice Problem

$$\frac{d}{dx} \frac{e^x \sin x}{\cos^7(x^3)} =$$

$$\left[\frac{\cos(x^3)}{\right]^7$$

$$7[\cos(x^3)]^{6}$$

$$\left[\cos^{7}(x^{3})\right]\left[e^{x}\sin x + e^{x}\cos x\right]$$

$$-[e^x \sin x][7 \cos^6(x^3)][-\sin(x^3)][3x^2]$$

 $\cos^{14}(x^3)$

Logarithmic Derivative

$$\frac{d}{dx}\left[\ln(f(x))\right] = \left[\frac{1}{f(x)}\right] \left[\frac{d}{dx}[f(x)]\right] = \frac{f'(x)}{f(x)}$$

e.g.:
$$\frac{d}{dx}[\ln(x^2 + 3x + 7)] = \frac{2x + 3}{x^2 + 3x + 7}$$

$$[f(x)] \times$$

$$[f(x)] \left[\frac{d}{dx} [\ln(f(x))] \right] = \left[\frac{1}{f(x)} \right] \left[\frac{d}{dx} [f(x)] \right]$$

Principle of Logarithmic Differentiation

$$\frac{d}{dx}[f(x)] = [f(x)] \left[\frac{d}{dx} [\ln(f(x))] \right]$$

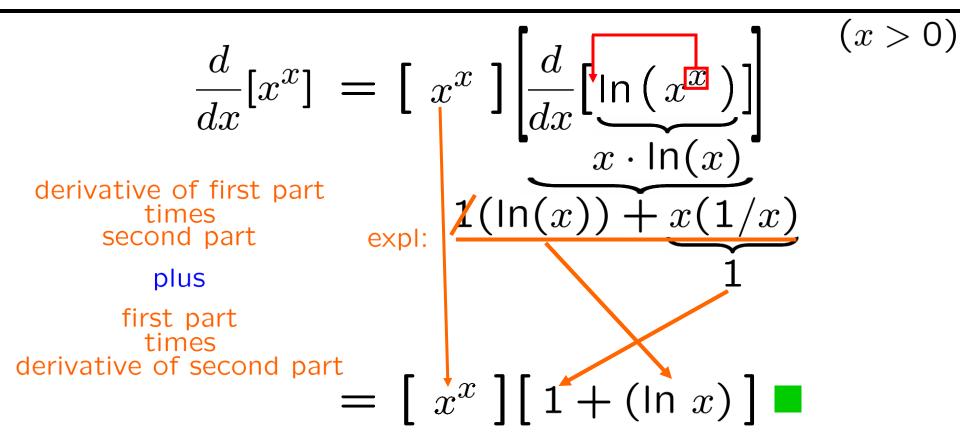
Principle of Logarithmic Differentiation: To compute the derivative of an expression, multiply the expression by its logarithmic derivative.

e.g.:
$$\frac{d}{dx}[x^x] = \left[x^x \right] \left[\frac{d}{dx} \left[\ln(x^x) \right] \right]$$
 (x > 0)

Principle of Logarithmic Differentiation

$$\frac{d}{dx}[f(x)] = [f(x)] \underbrace{\frac{d}{dx}[\ln(f(x))]}_{\text{logarithmic derivative of } f(x)}$$

Principle of Logarithmic Differentiation: To compute the derivative of an expression, multiply the expression by its logarithmic derivative.



Common sol'n:

$$\frac{1}{y}\frac{dy}{dx} = \frac{4}{5} \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2 + 8} - 5 \cdot \frac{3}{3x + 7}$$

$$\frac{dy}{dx} = y\left(\frac{4}{5} \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2 + 8} - 5 \cdot \frac{3}{3x + 7}\right)$$

$$\frac{dy}{dx} = \frac{x^{4/5}\sqrt{x^2 + 8}}{(3x+7)^5} \left(\frac{4}{5}\sqrt{\frac{1}{x}} + \frac{1}{2} \cdot \frac{2x}{x^2 + 8} - 5 \cdot \frac{3}{3x+7}\right)$$

My sol'n:

$$\frac{dy}{dx} = \frac{x^{4/5}\sqrt{x^2 + 8}}{(3x + 7)^5} \left(\frac{4}{5} \cdot \frac{\Box}{x} + \frac{1}{2} \cdot \frac{\Box}{x^2 + 8} - 5 \cdot \frac{\Box}{3x + 7}\right)$$



STOP

