Financial Mathematics
Lebesgue measure

1500-1.
Find the Lebesgue measure (in $\mathbb{R}$) of $[1,3] \cup \mathbb{Q}$.

1500-2.
Find the Lebesgue measure (in $\mathbb{R}$) of
$$[2/3, 1] \setminus (7/9, 10/9).$$

1500-3. Let $\mathcal{S}$ be the σ-algebra
on $[0, 200)$ generated by
$$[0, 60), \ [40, 115), \ [115, 200).$$

a. List all of the "minimal sets" in $\mathcal{S}$,
i.e., list all of those $S \in \mathcal{S}$ s.t. $\forall A \subseteq S$,
$$A \in \mathcal{S} \ \Rightarrow \ A = \emptyset \text{ or } A = S.$$

b. How many sets are there in $\mathcal{S}$?

1500-4. Let $\mathcal{S}$ be the σ-algebra on $\mathbb{R}$ generated by all intervals of the form (a, b), with $a, b \in \mathbb{Q}$.

   a. Show that $\{e\} \in \mathcal{S}$.

   b. Show that $\{e, \pi\} \in \mathcal{S}$.

   c. Show that $(e, \pi) \in \mathcal{S}$.

   d. Show that $(e, \pi] \in \mathcal{S}$.