Financial Mathematics

Constructions of measure spaces

1700-1.

Find the Lebesgue measure (in $\mathbb{R}^2$) of

$$([1,3]\times[5,9])\cup([3,6]\times[7,17]).$$

1700-2.

Find the Lebesgue measure (in $\mathbb{R}^2$) of

$$\bigcup_{n=1}^{\infty}\,[n,n+7^{-n}]\times[n,n+5^{-n}].$$

1700-3. Define $f : [2, 8] \to \mathbb{R}$ by

$$f(x) = \begin{cases} -1, & \text{if } 2 \le x \le 4 \\ 0, & \text{if } 4 < x < 6 \\ 8, & \text{if } 6 \le x \le 8. \end{cases}$$

Let λ_1 be the restriction, to $[2, 8]$, of Lebesgue measure on $\mathbb{R}$.

Write $f_*(\lambda_1/6)$ as a linear combination of delta masses.

1700-4.

Let $X := \{H, T\}$, with the discrete σ-algebra.

Let $Y := X \times X \times X \times \cdots$.
with the product σ-algebra,

Let μ be the measure on X defined by
$\mu(\{H\}) = 0.7$ and $\mu(\{T\}) = 0.3$.

Let $\nu := \mu \times \mu \times \mu \times \cdots$, a msr def'd on Y.

$\forall$integers $j \geq 1$, let $p_j : Y \to X$
be the jth coordinate projection,
defined by $p_j(x_1, x_2, x_3, \ldots) = x_j$.

(cont'd on next slide).

1700-4 (cont'd).

Let S be the set of all $y \in Y$ such that

$$p_5(y) = p_{10}(y) = p_{15}(y).$$

Let S_0 be the set of all $y \in Y$ such that

$$p_5(y) = p_{10}(y) = p_{15}(y) = p_{20}(y) = \cdots.$$

Compute $\nu(S)$ and $\nu(S_0)$.

Note: The measure space $(Y, \mathcal{B}, \mu)$ models an infinite sequence of flips of a biased coin that comes up heads 70% of the time. You are being asked to compute the probability that flips 5, 10 and 15 come up the same, as well as the probability that flips 5,10,15,20,... come up the same.