Financial Mathematics

Definition of integration

Let λ denote Lebesgue measure on $\mathbb{R}$.

2300-1. Compute $\int_{\mathbb{R}} x^5 \, d\lambda(x)$, if it exists.

2300-2. Compute $\int_{\mathbb{R}} x^6 \, d\lambda(x)$, if it exists.

2300-3. Compute $\int_{\mathbb{R}} e^{-x^2/4} \, d\lambda(x)$, if it exists.

2300-4. Compute $\int_{\mathbb{R}} e^{x^2/4} \, d\lambda(x)$, if it exists.

2300-5. Let μ be the probability measure on $\mathbb{R}$ defined by $\mu(\{3\}) = 0.25$
$\mu(\{5\}) = 0.65$
$\mu(\{8\}) = 0.10$.

Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \int_{\mathbb{R}} (x - y)^2 \, d\mu(y).$$

Find the value of x that minimizes $f(x)$.

Hint:

Note that $\mu = (0.25)\delta_3 + (0.65)\delta_5 + (0.10)\delta_9$.

Use the fact that, $\forall$Borel spaces $(M, \mathcal{B})$,

$\forall p \in M$, $\forall$Borel $g : M \to \mathbb{R}$,

$$\int_M g(y) \, d\delta_p(y) = g(p).$$