

Financial Mathematics

Basics of integration

Let λ denote Lebesgue measure on \mathbb{R} .

2330-1. Let $I := [1, 5]$.

Compute $\int_I (3s^2 + 7s - 3) d\lambda(s)$, if it exists.

2330-2. Let $\mathbb{R}_+ := (0, \infty)$.

Let $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be defined by $f(x) = x^7$.

Let $a, b \in \mathbb{R}_+$ and assume that $a < b$.

Let $I := [a, b]$ be the closed interval
from a to b .

Let λ_+ be Lebesgue measure on \mathbb{R}_+ .

a. Compute $(f\lambda_+)([a, b])$.

b. Compute $(f_*\lambda_+)([a, b])$.

c. Define $g : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ by

$$g(y) = \frac{1}{f'(f^{-1}(y))} = \frac{1}{7(\sqrt[7]{y})^6}.$$

Compute $(g\lambda_+)([a, b])$.