Financial Mathematics

Measures on the reals

2700-1. Define $f, g : \mathbb{R} \to \mathbb{R}$ by $f(x) = e^x$,

$$g(x) = \begin{cases} x^5 + 1, & \text{if } x < 6 \\ x^5 + 4, & \text{if } x \geq 6. \end{cases}$$

Compute $\displaystyle\int_5^9 f\, dg$.

2700-2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = e^{-x}$.

Let λ denote Lebesgue measure on $\mathbb{R}$.

Find the CDF of $f\lambda$.

2700-3. Let $I := [0, 1]$.

Let $f : I \to \mathbb{R}$ be defined by $f(x) = e^{-x}$.

Let λ_1 denote Leb. measure on I.

a. Find the CDF of $f_*(\lambda_1)$.

b. Find a PDF for $f_*(\lambda_1)$.

$\forall a \in \mathbb{R}$, $\delta_a :=$ the point mass on $\mathbb{R}$ concentrated at a.

as $n \to \infty$.

2700-4. Find the limit of the point masses $\delta_{4+(1/n)}$,

2700-5. For all integers $n \geq 1$,

$$\text{let } \mu_n := \frac{\delta_{1/n} + \delta_{2/n} + \cdots + \delta_{2n/n}}{2n}.$$

Find $\lim\limits_{n\to\infty} \mu_n$.

2700-6. Let λ be Lebesgue measure on $\mathbb{R}$.

Let $\Omega := [0,1]$. Let $\lambda_1 := \mathbf{1}_{[0,1]}^{\mathbb{R}} \lambda$,

let $\lambda_2 := \mathbf{1}_{[0,2]}^{\mathbb{R}} \lambda$ and let $\lambda_3 := \mathbf{1}_{[3,4]}^{\mathbb{R}} \lambda$.

Find a random variable $X : \Omega \to \mathbb{R}$ s.t.

$$\delta_X = (1/2)\lambda_1 + (1/6)\lambda_2 + (1/6)\lambda_3.$$

2700-7.

Let $f_n(x) := \begin{cases} 0, & \text{if } x \le -1/n \\ (n + n^2x)/2, & \text{if } -1/n \le x \le 0 \\ (n - n^2x)/2, & \text{if } 0 \le x \le 1/n \\ 0, & \text{if } 1/n \le x \end{cases}$

Let $\lambda :=$ Lebesgue measure on $\mathbb{R}$.

Find the limit of $f_n\lambda$, as $n \to \infty$.