

Financial Mathematics

Signed measures

2800-1. Define $v : \mathbb{R} \rightarrow \mathbb{R}$ by $v(x) = x^2 - x$.

a. Find two nondecreasing functions

$$f, g : \mathbb{R} \rightarrow \mathbb{R} \text{ s.t. } v = f - g.$$

b. Define $h : \mathbb{R} \rightarrow \mathbb{R}$ by $h(x) = x^4$.

Compute the Stieltjes integral $\int_{-2}^7 h \, dv$.

c. Find two differentiable functions

$$f_0, g_0 : \mathbb{R} \rightarrow \mathbb{R} \text{ s.t. } v = f_0 - g_0,$$

$$\text{s.t. } f_0' > 0 \text{ on } \mathbb{R},$$

$$\text{and s.t. } g_0' > 0 \text{ on } \mathbb{R}.$$

2800-2. Let $g := \mathbf{1}_{[5,6]}^{[4,6]} : [4, 6] \rightarrow \{0, 1\}$

be the characteristic function of $[5, 6]$ on $[4, 6]$,

which is defined by $g(x) = \begin{cases} 1, & \text{if } 5 \leq x \leq 6, \\ 0, & \text{if } 4 \leq x < 5, \end{cases}$

Define $h : [4, 6] \rightarrow \mathbb{R}$ by $h(x) = e^x [g(x)]$.

Define $f : [4, 6] \rightarrow \mathbb{R}$ by $f(x) = x^3$.

Compute the Stieltjes integral $\int_4^6 f \, dh$.