Financial Mathematics

Conditional probability and expectation

3000-1. Let Y, Z be indep. std normal RVs.

Compute $\Pr[(Y+2Z>3)|(-2Y+Z>7)]$.

Hint: Let λ^2 be Lebesgue measure on $\mathbb{R}^2$.

Let $f(x) := e^{-x^2/2}/\sqrt{2\pi}$.

Let $g(s,t) := [f(s)][f(t)]$, so $\delta_{Y,Z} = g\lambda^2$.

Also, $g(s,t) = e^{-(s^2+t^2)/2}/(2\pi)$

so g is invariant under rotations,

so $\delta_{Y,Z}$ is invariant under rotations.

Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be a carefully chosen rot'n.

Let $(U,V) := R(Y,Z)$.

Choose R s.t. U is a multiple of $Y+2Z$ and

and s.t. V is a multiple of $-2Y+Z$.

Then $\delta_{U,V} = R_*(\delta_{Y,Z}) = \delta_{Y,Z}$,

so U and V are indep. std normal RVs.

3000-2. Define a PCRV X by

$$X(\omega) := \begin{cases} 7, & \text{if } \omega \in [0, 0.35] \\ -3, & \text{if } \omega \in (0.35, 1]. \end{cases}$$

Let $\mathcal{F}$ be the σ-subalgebra generated by $\{[0, 0.6], (0.6, 1]\}$.

Let $\mathcal{G}$ be the σ-subalgebra generated by $\{[0, 0.15], (0.15, 0.6], (0.6, 0.80], (0.80, 1]\}$.

a. Compute $\mathrm{E}[X|\mathcal{G}]$.

b. Compute $\mathrm{E}[X|\mathcal{F}]$.

c. Compute $\mathrm{E}[X]$.

3000-3. Let $I := [0,1]$.

Let $\Omega := I^3 = I \times I \times I$, with Leb. measure.

Define $V : \Omega \to \mathbb{R}$ by $V(s,t,u) := s^3 + t^4 + e^u$.

Define $X, Y : \Omega \to \mathbb{R}$ by

$$X(s,t,u) = s, \qquad Y(s,t,u) = t.$$

Let $\mathcal{F} := \mathcal{S}_X$.

Let $\mathcal{G} := \mathcal{S}_Y$.

Let $\mathcal{H} := \langle \mathcal{F} \cup \mathcal{G} \rangle_\sigma$.

a. Compute $\mathrm{E}[V|\mathcal{F}]$.

b. Compute $\mathrm{E}[V|\mathcal{H}]$.

c. Compute $\mathrm{E}[\quad \mathrm{E}[V|\mathcal{F}] \quad | \quad \mathcal{G}]$.