Financial Mathematics

The tower law
and taking out what you know

3100-1. Define a PCRV X by

$$X(\omega) := \begin{cases} 5, & \text{if } \omega \in [0, 0.25] \\ 7, & \text{if } \omega \in (0.25, 1]. \end{cases}$$

Let $\mathcal{F}$ be the σ-subalgebra generated by $\{[0, 0.4], (0.4, 1]\}$.

Let $\mathcal{G}$ be the σ-subalgebra generated by $\{[0, 0.25], (0.25, 0.4], (0.4, 0.80], (0.80, 1]\}$.

a. Compute $\mathsf{E}[X|\mathcal{G}]$.

b. Compute $\mathsf{E}[\mathsf{E}[X|\mathcal{G}]|\mathcal{F}]]$.

c. Compute $\mathsf{E}[\mathsf{E}[\mathsf{E}[X|\mathcal{G}]|\mathcal{F}]]]$.

d. Compute $\mathsf{E}[X]$.

3100-2. Let $I := [0,1]$.

Let $\Omega := I^3 = I \times I \times I$.

Define $U : \Omega \to \mathbb{R}$ by $U(s,t,u) := s^4 + t^5 + e^u$.

Define $X, Y : \Omega \to \mathbb{R}^2$ by

$$X(s,t,u) = (t,u), \quad Y(s,t,u) = (s,u).$$

Let $\mathcal{F} := \mathcal{S}_X$.

Let $\mathcal{G} := \mathcal{S}_Y$.

Let $\mathcal{H} := \langle \mathcal{F} \cup \mathcal{G} \rangle_\sigma$.

a. Compute $\mathsf{E}[U|\mathcal{F}]$.

b. Compute $\mathsf{E}[\mathsf{E}[U|\mathcal{F}]|\mathcal{G}]]$.

c. Compute $\mathsf{E}[U|\mathcal{H}]$.

d. Compute $\mathsf{E}[\mathsf{E}[\mathsf{E}[U|\mathcal{F}]|\mathcal{G}]]]$.

e. Compute $\mathsf{E}[U]$.