Financial Mathematics

Estimating variance

3400-1. Let λ be Lebesgue msr on $\mathbb{R}$.

Let $\Omega := (0,1)$. Let $\lambda_1 := \lambda|\Omega$.

Let $Z := \Phi^{-1} : \Omega \to \mathbb{R}$.

Define $X : \Omega^2 \to \mathbb{R}^2$ by $X(s,t) = (Z(s), Z(t))$.

Let $\mu := X_*(\lambda_1 \times \lambda_1)$. joint distribution of two indep std normal RVs

Let $\nu := \lambda \times \lambda$.

a. Compute $\dfrac{d\mu}{d\nu}$. joint PDF of joint distribution of two indep std normal RVs

Define $Y : \Omega^2 \to \mathbb{R}$ by $Y(s,t) = 5(Z(s)) - 2(Z(t))$.

Let $\tau := Y_*(\lambda_1 \times \lambda_1)$. distribution of a lin. comb. of two indep std normal RVs

b. Compute $\dfrac{d\tau}{d\lambda}$. PDF of distribution of sum of two indep std normal RVs

Hint: Use rotational invariance of μ.

NOTE: The numbers below are made up.
I'd be interested to know the correct standard deviation of heart rate (in the US population).

3400-2. Suppose we have 75 measurements of heart rates with a sample standard deviation of 8.3 beats per minute.

Using a χ^2-table, find a 99% confidence interval for the standard deviation of the population.