Financial Mathematics
Basics of stochastic differential equations

3700-1.

Define $V : \mathbb{R}^2 \to \mathbb{R}^2$ by $V(x, y) = (2, -4)$.
Find the maximal flowline for V
footed at the point $(-2, 0)$.

3700-2.

Define $V : \mathbb{R}^2 \to \mathbb{R}^2$ by $V(x, y) = (3x, -4y)$.
Find the maximal flowline for V
footed at the point $(2, 5)$.

3700-3. Let $M := \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$.

Find a 2×2 matrix C s.t. $C^{-1}MC$ is a diagonal matrix.

3700-4. Define $V : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$V(x,y) = (-x + 2y, -x - 4y).$$

Find the maximal flowline for V footed at $(1, -1)$.

3700-5. Let W_t be a Brownian motion. Let X_t be the solution to

$$dX_t = 4\,dW_t - 2\,dt, \qquad X_0 = 1.$$

Compute $\mathsf{E}[e^{X_5 - 3X_7}]$.