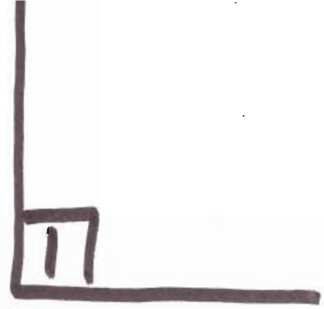
$1 \quad GP = 10^6$ $\qquad G = 10^{100}$ 

$2 \quad GPP = 10^{GP}$

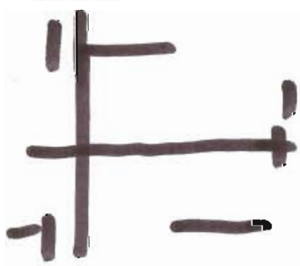
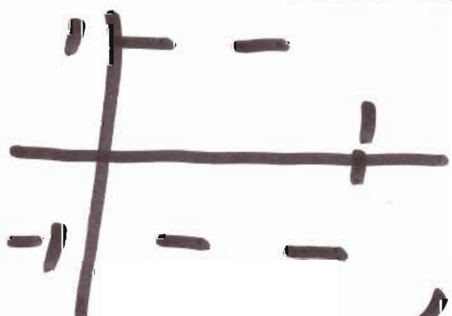
$\vdots \qquad \vdots$

$GP \qquad BP$

$$dt = 1/BP$$

$$CF_0, CF_{dt}, CF_{2dt}, \ldots : \underset{[0,1]}{\underset{\parallel}{\Omega}} \longrightarrow \mathbb{R}$$

SRV_s

 ,  , ...

$$CF_t^2 = 1$$

$$W_0, W_{dt}, W_{2dt}, \ldots : \Omega \longrightarrow R$$

$$W_t = (CF_0 + \cdots + CF_{t-dt})\sqrt{dt}$$

$$dW_t = W_{t+dt} - W_t = CF_t \sqrt{dt}$$

$$dW_t^2 = dt$$

$$\frac{dB_t}{B_t} = r_t \, dt$$

$$\left.\begin{array}{l} r_t = r \\ B_0 = 1 \end{array}\right\} \Longrightarrow B_t \doteq e^{rt}$$

$$\mathcal{F}_t := \mathcal{F}(W_0, \ldots, W_{t-dt})$$

$\forall F = f(A)$,

$$\bar{F} = f(S_t) = F\big|_{A \to S_t}$$

$\forall F_t = f(t, A)$,

$$\bar{F}_t = f(t, S_t) = F_t\big|_{A \to S_t}$$

$$\frac{dS_t}{S_t} = \bar{\sigma}_t \, dW_t + \bar{\mu}_t \, dt$$

Can solve, if σ_t, μ_t consts.

expr in t const

Given r_t, B_0

Given σ_t, μ_t, S_0

expr in t, s const

Given T, K consts,

find ϕ_t, ψ_t $\ni$:

if $V_t := \overline{\phi}_t S_t + \overline{\psi}_t B_t$

then $dV_t = \overline{\phi}_t \, dS_t + \overline{\psi}_t \, dB_t$

and $V_T = (S_T - K)^+$

$\dot{H}_t = \frac{\partial}{\partial t} H_t$; $H_t' := \frac{\partial}{\partial s} H_t$

Thm. Say F_t satisfies

$$\dot{F}_t + \frac{1}{2}\sigma_t^2 s^2 F_t'' = r_t (F_t - s F_t')$$

and $\phi_t = F_t'$

and $\psi_t = (F_t - s F_t')/B_t$

Then $\overline{F}_t = \overline{\phi}_t S_t + \overline{\psi}_t B_t$

and $d\overline{F}_t = \overline{\phi}_t \, dS_t + \overline{\psi}_t \, dB_t$

Solve PDE w/ $F_T = (s-K)^+$

Let ϕ_t, ψ_t be as above

Let $V_t = \overline{\phi}_t S_t + \overline{\psi}_t B_t$. Etc.

Pf: $F_t = \phi_t A + \psi_t B_t$

$$\bar{F}_t = \bar{\phi}_t S_t + \bar{\psi}_t B_t$$

Want: $d\bar{F}_t = \bar{\phi}_t \, dS_t + \bar{\psi}_t \, dB_t$

Itô: $d\bar{F}_t = \dot{\bar{F}}_t \, dt + \bar{F}_t' \, dS_t$

$$+ \frac{1}{2} \bar{F}_t'' \, dS_t^2$$

Want: $\dot{\bar{F}}_t \, dt + \frac{1}{2} \bar{F}_t'' \, dS_t^2 = \bar{\psi}_t \, dB_t$

$$dS_t = \bar{\sigma}_t S_t \, dW_t + \bar{\mu}_t S_t \, dt$$

$$dS_t^2 = \bar{\sigma}_t^2 S_t^2 \, dt$$

$$dB_t = r_t B_t \, dt$$

<u>Want</u>: $\overline{\dot{F}_t} + \frac{1}{2}\overline{F_t''}\sigma_t^2 S_t^2 = \overline{\psi}_t r_t B_t$

<u>Want</u>: $\dot{F}_t + \frac{1}{2}F_t''\sigma_t^2 \Delta^2 = \psi_t r_t B_t$

$$\dot{F}_t + \frac{1}{2}F_t''\sigma_t^2\Delta^2 = r_t\left(F_t - \Delta F_t'\right)$$

$$= r_t\left(\psi_t B_t\right)$$

QED

$$\forall F = f(x)$$

$$\bar{F} = f(X_t)$$

$$F' = f'(x) = \frac{d}{dx}(f(x))$$

Thm. Say $dX_t = \bar{\sigma}_t dW_t + \bar{\mu}_t dt$

Say p_t satisfies: $\forall f \in C_c^\infty(\mathbb{R})$

$$\mathbb{E}[\bar{f}] = \int_{-\infty}^{\infty} f p_t \, dx.$$

Then

$$\dot{p}_t = -(\mu_t p_t)' + \frac{1}{2}(\sigma_t^2 p_t)''$$

$$\frac{d}{dt}\mathbb{E}[\bar{f}] = \int_{-\infty}^{\infty} f \, \dot{p}_t \, dx$$

$$d\bar{f} = \overline{f'}\, dX_t + \frac{1}{2}\overline{f''}\, dX_t^2 =$$

$$f'\big(\bar{\sigma}_t \underbrace{dW_t}_{\big\downarrow\, \mathbb{E}[\,\cdot\,|\,\mathcal{F}_t] \;\; 0} + \bar{\mu}_t\, dt\big) + \frac{1}{2}\overline{f''}\big(\bar{\sigma}_t^2\, dt\big)$$

Tower Law Trick:

$$\mathbb{E}[d\bar{f}] = \mathbb{E}\left[\overline{f'}\bar{\mu}_t + \frac{1}{2}\overline{f''}\bar{\sigma}_t^2\right] dt$$

$$\mathbb{E}\left[\overline{f'\mu_t + \tfrac{1}{2} f'' \sigma_t^2}\right] = \int_{-\infty}^{\infty} f \, \dot{p}_t \, dx$$

$$\|$$

$$\int_{-\infty}^{\infty} \left(f'\mu_t + \tfrac{1}{2} f'' \sigma_t^2\right) p_t \, dx$$

$$\|$$

$$-\int_{-\infty}^{\infty} f \left(\mu_t p_t\right)' dx + \int_{-\infty}^{\infty} \tfrac{1}{2} f \left(\sigma_t^2 p_t\right)'' dx$$

$$\therefore \dot{p}_t = -\left(\mu_t p_t\right)' + \tfrac{1}{2}\left(\sigma_t^2 p_t\right)''$$

QED