# VARIATIONS ON PRACTICE TEST 4

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52-1. Consider the following system of linear equations over the real numbers, where $x$, $y$ and $z$ are variables and $b$ is a real constant.

$$\begin{array}{rcrcrcr} x & + & 2y & + & z & = & 0 \\ 2x & + & 4y & + & 3z & = & 0 \\ x & + & 3y & + & bz & = & 0 \end{array}$$

Which of the following statements are true?

I. There exists a value of $b$ for which the system has no solution.
II. There exists a value of $b$ for which the system has exactly one solution.
III. There exists a value of $b$ for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

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52-2. Consider the following system of linear equations over the real numbers, where $x$, $y$ and $z$ are variables and $b$ is a real constant.

$$\begin{array}{rcrcrcl} x & + & 2y & + & z & = & 0 \\ 2x & + & 4y & + & 3z & = & 0 \\ 3x & + & 6y & + & bz & = & 0 \end{array}$$

Which of the following statements are true?

I. There exists a value of $b$ for which the system has no solution.
II. There exists a value of $b$ for which the system has exactly one solution.
III. There exists a value of $b$ for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) III only

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53-1. In the complex plane, let $C$ be the circle $|z+2| = 2$ with negative (clockwise) orientation. Compute $\displaystyle\int_C \frac{dz}{(z-1)(z+3)^2}$.

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53-2. In the complex plane, let $C$ be the circle $|z| = 4$ with negative (clockwise) orientation. Compute $\displaystyle\int_C \frac{dz}{(z-1)(z+3)^2}$.

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54-1. Assume that, in a certain two-dimensional world, the wind velocity at any point $(x, y)$ is $(-11x + 10y, -10x + 14y)$. A small particle is simply pushed by the wind. Its position at any time $t$ is given by $(f(t), g(t))$. Assume that its velocity at time $t$ is

$$(\quad -11[f(t)] + 10[g(t)] \quad , \quad -10[f(t)] + 14[g(t)] \quad ).$$

Because its velocity at time $t$ is also given by $(f'(t), g'(t))$, its motion will satisfy the equations:

$$f'(t) = -11[f(t)] + 10[g(t)], \qquad g'(t) = -10[f(t)] + 14[g(t)].$$

Assume that the initial position of the particle is $(f(0), g(0)) = (0, 1)$. We stand at the origin and watch the particle. Along what slope line will we look, asymptotically, as $t \to \infty$? That is, compute $\lim_{t\to\infty} \frac{g(t)}{f(t)}$.

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54-2. Assume that, in a certain two-dimensional world, the wind velocity at any point $(x, y)$ is $(-y, x)$. A small particle is simply pushed by the wind. Its position at any time $t$ is given by $(f(t), g(t))$. Assume that its velocity at time $t$ is $(-[g(t)], f(t))$. Because its velocity at time $t$ is also given by $(f'(t), g'(t))$, its motion will satisfy the equations:

$$f'(t) = -[g(t)], \qquad g'(t) = f(t).$$

Assume that the initial position of the particle is $(f(0), g(0)) = (2, 0)$. Find its position $(f(t), g(t))$ at any time $t$.

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55-1. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. True or False: If $f'(0) = 0$, then $f(x)$ has a local extremum at $x = 0$.

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55-2. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. True or False: If $f(x)$ has a local extremum at $x = 0$, then $f'(0) = 0$.

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55-3. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. True or False: If $f'(x)$ has a local extremum at $x = 0$, then $f(x)$ has a point of inflection at $x = 0$.

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55-4. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. True or False: If $f(x)$ has a point of inflection at $x = 0$, then $f'(x)$ has a local extremum at $x = 0$.

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55-5. Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable. True or False: If $f''(0) = 0$, then $f(x)$ has a point of inflection at $x = 0$.

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55-6. Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable. True or False: If $f(x)$ has a point of inflection at $x = 0$, then $f''(0) = 0$.

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56-1. True or false: For any metric $d$ on $\mathbb{R}$, there is a norm $\| \bullet \|$ on $\mathbb{R}$ such that, for all $x, y \in \mathbb{R}$, $d(x, y) = \|x - y\|$.

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56-2. True or false: For every norm $\| \bullet \|$ on $\mathbb{R}$, there is an inner product $\langle \bullet, \bullet \rangle$ on $\mathbb{R}$ such that, for all $x \in \mathbb{R}$, we have $\|x\|^2 = \langle x, x \rangle$.

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56-3. True or false: For every norm $\| \bullet \|$ on $\mathbb{R}^2$, there is an inner product $\langle \bullet, \bullet \rangle$ on $\mathbb{R}^2$ such that, for all $v \in \mathbb{R}^2$, we have $\|v\|^2 = \langle v, v \rangle$.

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57-1. Let $\mathbb{R}$ be the field of real numbers and $\mathbb{R}[x]$ the ring of polynomials in $x$ with coefficients in $\mathbb{R}$. Which of the following subsets of $\mathbb{R}[x]$ is a subring of $\mathbb{R}[x]$?

I. All polynomials in $\mathbb{R}[x]$ whose coefficient of $x^2$ is zero

II. All polynomials in $\mathbb{R}[x]$ all of whose terms have even degree, including the zero polynomial.

III. All polynomials in $\mathbb{R}[x]$ whose coefficients are nonnegative real numbers.

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58-1. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and injective. Let $U$ be an open subset of $\mathbb{R}$. True or false: $f(U)$ is necessarily an open subset of $\mathbb{R}$.

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58-2. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Let $U$ be an open subset of $\mathbb{R}$. True or false: $f(U)$ is necessarily an open subset of $\mathbb{R}$.

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58-3. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Let $U$ be an open subset of $\mathbb{R}$. True or false: $f^{-1}(U)$ is necessarily an open subset of $\mathbb{R}$.

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58-4. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Let $B$ be a bounded subset of $\mathbb{R}$. True or false: $f(B)$ is necessarily a bounded subset of $\mathbb{R}$.

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58-5. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Let $B$ be a bounded subset of $\mathbb{R}$. True or false: $f^{-1}(B)$ is necessarily a bounded subset of $\mathbb{R}$.

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58-6. Let $f : (0,1) \to \mathbb{R}$ be continuous. Let $B$ be a bounded subset of $\mathbb{R}$. Assume that $B \subseteq (0,1)$. True or false: $f(B)$ is necessarily a bounded subset of $\mathbb{R}$.

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