MATH 1271 Fall 2011, Midterm #1
Handout date: Thursday 6 October 2011

PRINT YOUR NAME: SOLUTIONS

PRINT YOUR TA'S NAME:

WHAT SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Assume that $\lim_{x\to 100}(f(x)) = 4$ and $\lim_{x\to 200}(g(x)) = 5$. At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

(a) $\lim_{x\to 300}[(f(x)) + (g(x))] = 9$

(b) $\lim_{x\to 4}(f(x)) = 100$

(c) $\lim_{x\to 2}\dfrac{f(x)}{g(x)} = 4/5$

(d) $\lim_{x\to 300}[(f(x)) + (g(x))]$ does not exist

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $\lim_{x\to -\infty}\left[\dfrac{2x^2 - x}{\underbrace{4x^2 + x}_{2}}\right]$. Circle one of the following answers:

$$\frac{2x^2}{4x^2} = \frac{1}{2} \longrightarrow \frac{1}{2}$$

(a) ∞

(b) $-\infty$

(c) 1/2

(d) $-1/2$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Compute $\lim_{x\to -\infty}\left[\dfrac{\sqrt{4x^4 - x}}{\underbrace{8x^2 + x}_{2}}\right]$. Circle one of the following answers:

$$\frac{\sqrt{4x^4}}{8x^2} = \frac{2x^2}{8x^2} = \frac{1}{4} \longrightarrow \frac{1}{4}$$

(a) 1/4

(b) $-1/4$

(c) 1/2

(d) $-1/2$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Compute $\ln(e^{-(5^2)})$. Circle one of the following answers:

$= -25$

(a) 25

(b) -10

(c) -25

(d) DOES NOT EXIST

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Compute $\lim_{x\to 0} \frac{2x^3+5x^2}{\underbrace{7x(\sin x)}_{\approx}}$. Circle one of the following answers:

(a) 2/7

(b) 5/7

$\frac{5x^2}{7x \cdot x} = \frac{5}{7}$

(c) ∞

(d) 0

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute the largest $\delta > 0$ such that: $0 < |x-1| < \delta$ implies $|(2x+4) - 6| < 0.1$. Circle one of the following answers:

(a) 0.2

(b) 0.1

$\delta = \frac{0.1}{2} = 0.05$

(c) 0.025

(d) 0.01

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) If $\lim_{x \to a} f(x) = \infty$, then $\lim_{x \to a^-} f(x) = \infty$.

True

b. (5 pts) There is a function with three horizontal asymptotes.

False

c. (5 pts) If f and g are continuous at 3, then $f + g$ MUST be continuous at 3 as well.

True

d. (5 pts) Every polynomial is continuous.

True

e. (5 pts) The function $f(x) = |x|$ is differentiable at 0.

False

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2a,b

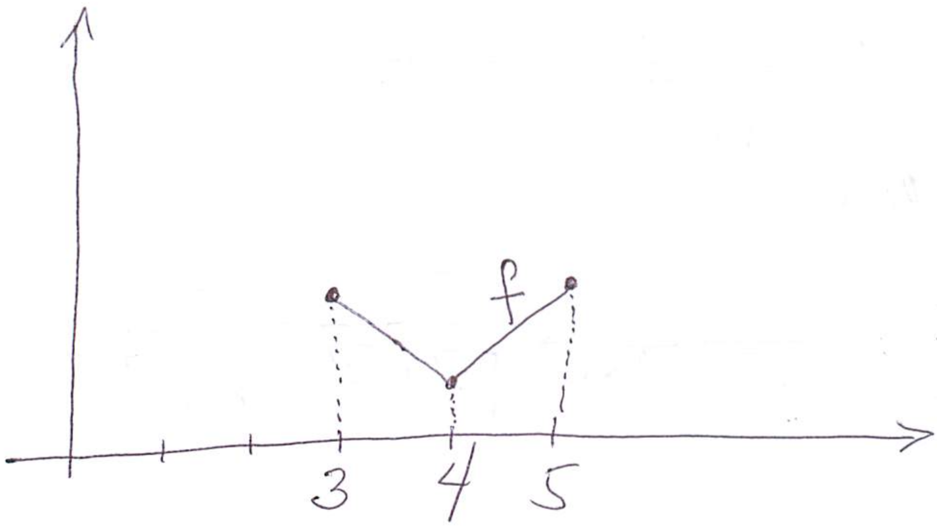
III. 3

III. 4a,b

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Draw a single graph showing a function $f : [3,5] \to \mathbb{R}$ with *all* of the following properties:

(•) Its domain is the interval $[3,5]$.
(•) It is continuous on $[3,5]$.
(•) It is differentiable on $(3,4)$ and on $(4,5)$.
(•) It is not differentiable at 4.



2. a. (10 pts) Compute $\lim_{h\to 0} \frac{\sqrt{7+h}-\sqrt{7-h}}{h}$.

$$\underbrace{\frac{\sqrt{7+h}-\sqrt{7-h}}{h}}$$

$$\| $$

$$\frac{\sqrt{7+h}-\sqrt{7-h}}{h} \cdot \frac{\sqrt{7+h}+\sqrt{7-h}}{\sqrt{7+h}+\sqrt{7-h}}$$

$$\|$$

$$\frac{\cancel{7}+h-\cancel{7}+h}{h} \cdot \frac{1}{\sqrt{7+h}+\sqrt{7-h}}$$

$$\|_{h\neq 0}$$

$$\frac{2}{\sqrt{7+h}+\sqrt{7-h}} \xrightarrow[h\to 0]{} \frac{2}{\sqrt{7}+\sqrt{7}} = \frac{2}{2\sqrt{7}} = \frac{1}{\sqrt{7}}$$

b. (5 pts) Compute $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x-h}}{h}$.

replace $7 \longmapsto x$ above

Answer: $\frac{1}{\sqrt{x}}$

3. (10 pts) Compute $\lim_{x\to-\infty}\left(\sqrt{x^2+4x}-\sqrt{x^2-5x}\right)$.

$$\frac{\sqrt{x^2+4x}-\sqrt{x^2-5x}}{1}\quad\frac{\sqrt{x^2+4x}+\sqrt{x^2-5x}}{\sqrt{x^2+4x}+\sqrt{x^2-5x}}$$

$$=$$

$$\frac{\cancel{x^2}+4x-\cancel{x^2}+5x}{\sqrt{x^2+4x}+\sqrt{x^2-5x}}$$

$$=\ x<0$$

$$\frac{9x}{\sqrt{x^2+4x}+\sqrt{x^2-5x}}\cdot\frac{-1/x}{+1/\sqrt{x^2}}$$

$$=$$

$$\frac{-9}{\sqrt{1+\frac{4}{x}}+\sqrt{1-\frac{5}{x}}}\xrightarrow[x\to-\infty]{}\frac{-9}{\sqrt{1}+\sqrt{1}}$$

$$=$$

$$-\frac{9}{2}$$

4. On the planet of Gallifrey, in an alternate universe, a dropped object travels t^3 feet during its first t seconds of free fall.

a. (5 pts) For $h \neq 0$, the average velocity between time $t = 2$ seconds and time $t = 2 + h$ seconds is given by a quadratic polynomial in h of the form ah^2+bh+c. Find the coefficients a, b and c.

$$\frac{(2+h)^3-2^3}{h} = \frac{\cancel{2^3}+3\cdot 2^2 h+3\cdot 2\cdot h^2+h^3-\cancel{2^3}}{h}$$

$$= \frac{12h+6h^2+h^3}{h} \overset{h\neq 0}{=} 12+6h+h^2$$

$$\parallel$$

$$h^2+6h+12 \ \text{ft/sec}$$

$a=1 \qquad b=6 \qquad c=12$

b. (5 pts) Find the instantaneous velocity at time $t = 2$ seconds.

$$\lim_{h\to 0}(h^2+6h+12) = 12 \ \text{feet/sec}$$