VERSION A

MATH 1271 Fall 2011, Midterm #2
Handout date: Thursday 10 November 2011

PRINT YOUR NAME: SOLUTIONS

PRINT YOUR TA'S NAME:

WHAT SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Suppose $f'(x) = -x^2 + 3x - 2$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is increasing on $(-\infty, 1]$, decreasing on $[1, 2]$ and increasing on $[2, \infty)$.

[✓ (b)] f is decreasing on $(-\infty, 1]$, increasing on $[1, 2]$ and decreasing on $[2, \infty)$.

(c) f is increasing on $(-\infty, -2]$, decreasing on $[-2, -1]$ and increasing on $[-1, \infty)$.

(d) f is decreasing on $(-\infty, -2]$, increasing on $[-2, -1]$ and decreasing on $[-1, \infty)$.

(e) NONE OF THE ABOVE

$$f'(x) = -(x^2 - 3x + 2)$$
$$= -(x-1)(x-2)$$

f' neg 0 pos 0 neg
1 2

B. (5 pts) (no partial credit) Find the logarithmic derivative of $x^2 + 3x - 8$ w.r.t. x.

[✓ (a)] $\dfrac{2x+3}{x^2+3x-8}$

(b) $\dfrac{x^2+3x-8}{2x+3}$

(c) $(\ln(x^2)) + 3(\ln x) - (\ln 8)$

(d) $\ln(2x+3)$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Find the slope of the tangent line to $y = (x^3 + 4)e^{2x}$ at the point $(0, 4)$.

(a) 2

(b) 4

(c) 6

[✓ (d)] 8

(e) NONE OF THE ABOVE

$$\left[3x^2 \cdot e^{2x} + (x^3+4)\cdot e^{2x} \cdot 2\right]_{x \to 0}$$
$$\|$$
$$0 + 4 \cdot e^0 \cdot 2 = 8$$

D. (5 pts) (no partial credit) Find the logarithmic derivative of $(2+\sin x)^x$ w.r.t. x.

(a) $[(2+\sin x)^x]\left[(\ln(2+\sin x))+\left(\dfrac{x\cos x}{2+\sin x}\right)\right]$

[✓ (b)] $(\ln(2+\sin x))+\left(\dfrac{x\cos x}{2+\sin x}\right)$

(c) $\ln(\cos x)$

(d) $\cos x$

(e) NONE OF THE ABOVE

$$\frac{d}{dx}\left[x\left(\ln\left(2+\sin x\right)\right)\right]$$

E. (5 pts) (no partial credit) Find the derivative of $(2+\sin x)^x$ w.r.t. x.

[✓ (a)] $[(2+\sin x)^x]\left[(\ln(2+\sin x))+\left(\dfrac{x\cos x}{2+\sin x}\right)\right]$

(b) $(\ln(2+\sin x))+\left(\dfrac{x\cos x}{2+\sin x}\right)$

(c) $\ln(\cos x)$

(d) $\cos x$

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute $\lim_{x\to 0}\underbrace{\left[\dfrac{\sin^2 x}{4x^3+2x^2}\right]}_{\wr}$.

(a) 2

(b) 1

[✓ (c)] 1/2

(d) 1/4

(e) NONE OF THE ABOVE

$$\frac{x^2}{2x^2}=\frac{1}{2}\longrightarrow\frac{1}{2}$$

II. True or false (no partial credit):

a. (5 pts) If $f'(3) = 0$ and $f''(3) < 0$, then f has a local maximum at 3.

T

b. (5 pts) Every local extremum occurs at a critical number.

T

c. (5 pts) Every global extremum occurs at a critical number.

T

d. (5 pts) If f is increasing on an interval I, then $f' > 0$ on I.

F

e. (5 pts) If f and g are differentiable, then $\frac{d}{dx}[(f(x))(g(x))] = [f'(x)][g'(x)]$.

F

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1.

III. 2.

III. 3,4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute $\frac{d}{dx}\left[\frac{2x^3-8}{\arctan x}+xe^{\sin x}\right]$

$$\left[\frac{(\arctan x)(6x^2)-(2x^3-8)\left(\frac{1}{1+x^2}\right)}{(\arctan x)^2}\right]+$$

$$\left[(e^{\sin x})+x(e^{\sin x})(\cos x)\right]$$

2. (10 pts) Using implicit differentiation (and logarithmic differentiation), find $y' = dy/dx$, assuming that $(2+y^2)^{xy} = 9$.

$$\cancel{\left[(2+y^2)^{xy}\right]}\left[\frac{d}{dx}\left[xy\left(\ln\left(2+y^2\right)\right)\right]\right]=0$$

$$y(\ln(2+y^2)) + xy'(\ln(2+y^2)) + xy\left(\frac{2yy'}{2+y^2}\right)=0$$

$$y' = \frac{-y(\ln(2+y^2))}{x(\ln(2+y^2)) + \frac{2xy^2}{2+y^2}}$$

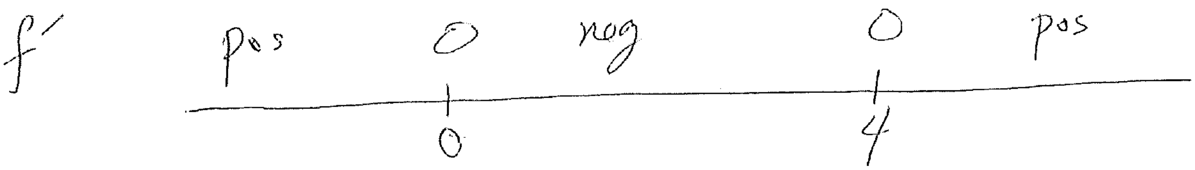
3. (5 pts) Suppose f is 1-1 and $g = f^{-1}$ is the inverse of f. Suppose $f(3) = 4$ and $f'(3) = 27$. Compute $g(4)$ and $g'(4)$.

$$g(4) = 3$$

$$g'(4) = \frac{1}{27}$$

4. (10 pts) Find the maximal intervals of increase and decrease for $f(x) = x^3 - 6x^2 + 5$.

$$f'(x) = 3x^2 - 12x$$
$$= 3x(x-4)$$



f is increasing on $(-\infty, 0]$,

decreasing on $[0, 4]$,

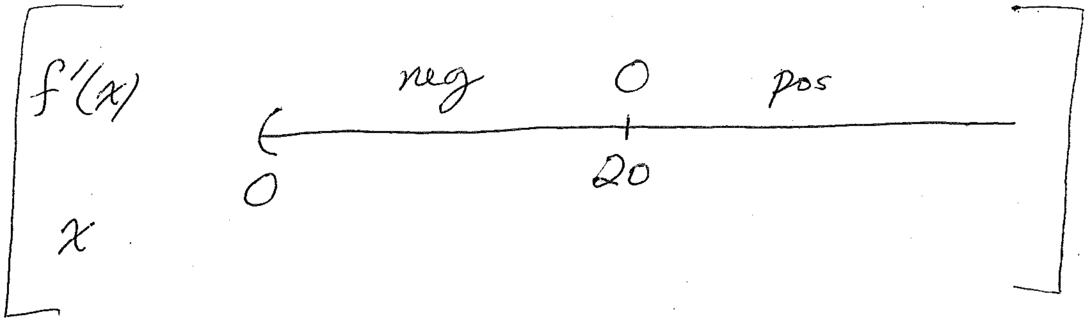
increasing on $[4, \infty)$

5. (10 pts) Among all pairs of positive numbers x and y such that $xy = 100$, find the global maximum value of $x + 4y$, provided it exists. Then find the global minimum value, provided it exists. (NOTE: If the global maximum value does not exist, you need to state that clearly to receive full credit. If it does exist, for full credit, you'll need to compute $x + 4y$; computing x and/or y alone is insufficient. These same comments apply to the global minimum value.)

$$y = \frac{100}{x}, \quad x > 0, \quad y > 0$$

Let $f(x) = x + 4y = x + \frac{400}{x} = x + 400x^{-1}$

$$f'(x) = 1 - 400x^{-2} = 1 - \frac{400}{x^2} = \frac{x^2 - 400}{x^2} = \frac{(x+20)(x-20)}{x^2}$$



On $x > 0$,

$f(x)$ has <u>no</u> global maximum

and has one global minimum

at $x = 20$, $y = \frac{100}{20} = 5$

with global minimum value

$$f(20) = 20 + \frac{400}{20} = 40.$$