

MATH 1271 Fall 2012, Midterm #2
Handout date: Thursday 8 November 2012

PRINT YOUR NAME:

SOLUTIONS
Version D

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Suppose $f'(x) = -(x-1)^2(x-2)(x-3)^2$. Which of the following is a maximal interval of increase for f ? Circle one of the following answers:

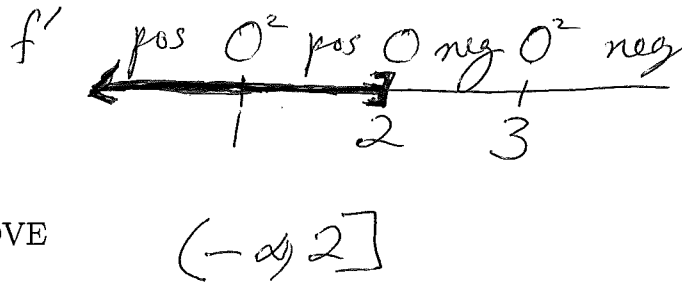
(a) $(-\infty, 1]$

(b) $[1, \infty)$

(c) $(2, \infty)$

(d) $[2, \infty)$

(e) NONE OF THE ABOVE



B. (5 pts) (no partial credit) Suppose $f''(x) = x^2 + 4x + 3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

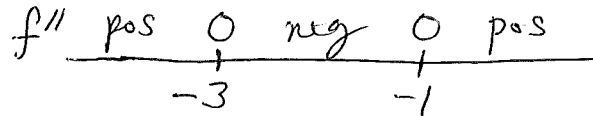
(a) f is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.

(b) f is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.

(c) f is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.

(d) f is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.

(e) NONE OF THE ABOVE



C. (5 pts) (no partial credit) Compute the derivative of $\ln(x^{\arctan x})$, with respect to x , on the interval $x > 0$. Circle one of the following answers:

(a) $\frac{1}{x^{\arctan x}}$

(b) $\frac{\ln x}{1+x^2} + \frac{\arctan x}{x}$

(c) $\frac{1}{x^{\sec^2 x}}$

(d) $x^{\sec^2 x}$

(e) NONE OF THE ABOVE

$$\frac{d}{dx} \left[(\arctan x)(\ln x) \right]$$

D. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

- (a) $2[\sin(xy)][\cos(y + xy')]$
 - (b) $2[\sin(xy)][y + xy']$
 - (c) $[\cos^2(xy)][y + xy']$
 - (d) $2[\sin(xy)][\cos(xy)][y + xy']$
 - (e) NONE OF THE ABOVE
-

E. (5 pts) (no partial credit) Find the logarithmic derivative of $(2 + \sin(2x))^{\cos x}$ w.r.t. x . Circle one of the following answers:

- (a) $(-\sin x) \left(\frac{2 \cos(2x)}{2 + \sin(2x)} \right)$
 - (b) $(\cos x)(\ln(2 + \sin(2x))) + (-\sin x) \left(\frac{2 \cos(2x)}{2 + \sin(2x)} \right)$
 - (c) $(\cos x)(\ln(2 + \sin(2x)))$
 - (d) $(-\sin x)(\ln(2 + \sin(2x))) + (\cos x) \left(\frac{2 \cos(2x)}{2 + \sin(2x)} \right)$
 - (e) NONE OF THE ABOVE
-

F. (5 pts) (no partial credit) Find the derivative of $(2 + \sin(2x))^{\cos x}$ w.r.t. x . Circle one of the following answers:

- (a) $[(2 + \sin(2x))^{\cos x}] \left[(-\sin x) \left(\frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]$
- (b) $[(2 + \sin(2x))^{\cos x}] \left[(\cos x)(\ln(2 + \sin(2x))) + (-\sin x) \left(\frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]$
- (c) $[(2 + \sin(2x))^{\cos x}] [(\cos x)(\ln(2 + \sin(2x)))]$
- (d) $[(2 + \sin(2x))^{\cos x}] \left[(-\sin x)(\ln(2 + \sin(2x))) + (\cos x) \left(\frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]$
- (e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) If $f' > 0$ on an interval I , then f is increasing on I .

True

b. (5 pts) If f is increasing on an interval I , then $f' > 0$ on I .

False

c. (5 pts) If f and g are differentiable at a number a , then $fg + f + g$ is differentiable at a .

True

d. (5 pts) Assume that $\lim_{x \rightarrow 0} [f(x)] = 0 = \lim_{x \rightarrow 0} [g(x)]$. Assume also that $\lim_{x \rightarrow 0} \left[\frac{f'(x)}{g'(x)} \right]$ does not exist. Then $\lim_{x \rightarrow 0} \left[\frac{f(x)}{g(x)} \right]$ does not exist.

False

e. (5 pts) Assume that $\lim_{x \rightarrow 3} [f(x)] = 0 = \lim_{x \rightarrow 3} [g(x)]$. Assume also that $\lim_{x \rightarrow 3} \frac{f'(x)}{g'(x)} = 7$. Then $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)} = 7$.

True

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx} \left[\frac{e^{x^4} - 8}{5 + \sec(x^2)} \right]$. (Here e^{x^4} means $e^{(x^4)}$.)
 //

$$\frac{[5 + \sec(x^2)][4x^3 e^{x^4}] - [e^{x^4} - 8][\sec(x^2)][\tan(x^2)][2x]}{[5 + \sec(x^2)]^2}$$

2. (5 pts) Compute $\frac{d}{dx} [(5 - \sin x)^7 \arcsin x]$.

//

$$[(5 - \sin x)^7 \arcsin x] \left[\frac{d}{dx} [(7 \arcsin x)(\ln(5 - \sin x))] \right]$$

//

$$[(5 - \sin x)^7 \arcsin x] \left[\left(\frac{7}{\sqrt{1-x^2}} \right) (\ln(5 - \sin x)) + (7 \arcsin x) \left(\frac{-\cos x}{5 - \sin x} \right) \right]$$

3. (10 pts) Find an equation for the tangent line to $x^3 + xy + y^3 = 11$ at $(1, 2)$.

$$3x^2 + y + xy' + 3y^2y' = 0$$

$$y' = \frac{-3x^2 - y}{x + 3y^2}$$

$$\text{slope} = \frac{-3 \cdot 1 - 2}{1 + 3 \cdot 4} = \frac{-5}{13}$$

$$\text{eq'n: } y - 2 = -\frac{5}{13}(x - 1)$$

4. (15 pts) Compute $\lim_{x \rightarrow 0} ((\cos x) - (\sin x))^{9/x}$.

$$\begin{aligned} & \parallel \\ e \lim_{x \rightarrow 0} (9/x) (\ln((\cos x) - (\sin x))) \end{aligned}$$

$$\begin{aligned} & \parallel \\ e \lim_{x \rightarrow 0} \frac{9(\ln(\cos x) - (\sin x)))}{x} \end{aligned}$$

\parallel L'H $\frac{0}{0}$

$$e \lim_{x \rightarrow 0} \frac{9 \left(\frac{-(\sin x) - (\cos x)}{(\cos x) - (\sin x)} \right)}{1}$$

$$\begin{aligned} & \parallel \\ e \frac{9 \left(\frac{-0 - 1}{1 - 0} \right)}{1} \end{aligned}$$

$$\begin{aligned} & \parallel \\ e^{-9} \end{aligned}$$

5. (10 pts) Find the global maximum and minimum value of $f(x) = -x^3 + 3x^2 - 3x - 9$ on the interval $0 \leq x \leq 2$.

$$\begin{aligned}f'(x) &= -3x^2 + 6x - 3 \\ &= -3(x^2 - 2x + 1) \\ &= -3(x-1)^2\end{aligned}$$

$$f' \quad \begin{array}{ccc} \text{neg} & 0^2 & \text{neg} \\ \hline & 1 & \end{array}$$

f is decreasing on $\mathbb{R} = (-\infty, \infty)$.

On $0 \leq x \leq 2$,

f attains global min at $x=2$,

$$\begin{aligned}\text{with global min value } f(2) &= -8 + 12 - 6 - 9 \\ &= 4 - 15 = -11, \text{ and}\end{aligned}$$

f attains global max at $x=0$

$$\text{with global max value } f(0) = -9.$$