MATH 1271 Fall 2013, Midterm #2
Handout date: Thursday 14 November 2013
Instructor: Scot Adams

PRINT YOUR NAME:

SOLUTIONS
Version A

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I. Multiple choice

A. (5 pts) (no partial credit) Let f be a function such that $f'(x) = 4(\cos(4x))$. Suppose, also, that $f(0) = 1$. Which of the following is an equation of the tangent line to the graph of f at $(0, 1)$. Circle one of the following answers:

(a) $y = 4(x - 1)$

(b) $y - 1 = 4(\cos(4x))x$

[✓ (c)] $y - 1 = 4x$

(d) $y = -4(\sin(x))(x - 1)$

(e) NONE OF THE ABOVE

slope $= f'(0) = 4$

$y-1 = 4(x-0)$

$y-1 = 4x$

B. (5 pts) (no partial credit) Find $\underbrace{\text{the logarithmic derivative of } x^2+3x-8 \text{ w.r.t. } x}_{\begin{gathered}\|\\ \dfrac{2x+3}{x^2+3x-8}\end{gathered}}$. Circle one of the following answers:

(a) $-\left[\dfrac{2x+3}{x^2+3x-8}\right]$

(b) $\dfrac{x^2+3x-8}{2x+3}$

(c) $(\ln(x^2)) + 3(\ln x) - (\ln 8)$

(d) $\ln(2x + 3)$

[✓ (e)] NONE OF THE ABOVE

C. (5 pts) (no partial credit) Suppose

- $f'(1) = 1$, $f''(1) = 3$, 
- $f'(2) = 0$, $f''(2) = -2$, 
- $f'(3) = 0$, $f''(3) = 6$. 

At which of the numbers 1, 2 and 3 does f have a local minimum? Circle one of the following answers:

(a) 1, but not 2, and not 3

(b) 2, but not 1, and not 3

[✓ (c)] 3, but not 1, and not 2

(d) both 2 and 3, but not 1

(e) NONE OF THE ABOVE

$-(x^2-4x+3) = -(x-1)(x-3)$

$\|$

D. (5 pts) (no partial credit) Suppose $f''(x) = -x^2 + 4x - 3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.

[✓ (b)] f is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.

(c) f is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.

(d) f is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.

(e) NONE OF THE ABOVE

f'' neg 0 pos 0 neg

1 3

E. (5 pts) (no partial credit) Let $f(x) = x^3 + 4x + 3$. What is the iterative formula of Newton's method used to solve $f(x) = 0$? Circle one of the following answers:

(a) $x_{n+1} = x_n + \dfrac{3x_n^2 + 4}{x_n^3 + 4x_n + 3}$

(b) $x_{n+1} = x_n - \dfrac{3x_n^2 + 4}{x_n^3 + 4x_n + 3}$

(c) $x_{n+1} = x_n + \dfrac{x_n^3 + 4x_n + 3}{3x_n^2 + 4}$

[✓ (d)] $x_{n+1} = x_n - \dfrac{x_n^3 + 4x_n + 3}{3x_n^2 + 4}$

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Let $y = x^2 + x$. Compute Δy, evaluated at $x = 10$, $\Delta x = 0.1$. (Hint: $\Delta(x^2) = 2x(\Delta x) + (\Delta x)^2$.)

(a) 1.22

[✓ (b)] 2.11

(c) 2.1

(d) 1.2

(e) NONE OF THE ABOVE

$\|$

$[\Delta(x^2)] + [\Delta(x)]$

$\|$

$[2x(\Delta x) + (\Delta x)^2] + [\Delta x]$

$[2\cancel{(10)}\cancel{(0.1)} + (0.1)^2] + [0.1]$

$\|$

$2 + 0.01 + 0.1 = 2.11$

II. True or false (no partial credit):

a. (5 pts) Let f be any function such that $f(3) = 5$ and such that $f''(3) = 0$. Then $(3, 5)$ is a point of inflection for f.

$f(x) = (x-3)^4 + 5$ False

b. (5 pts) If f has a global maximum at c, then c is a critical number for f.

Fermat's Theorem True

c. (5 pts) Let f be any function such that $f(x)$ is increasing on $1 < x < 3$. Then $f'(2) > 0$.

$f(x) = (x-2)^3$ False

d. (5 pts) Let $f, g : \mathbb{R} \to \mathbb{R}$ be any two differentiable functions such that, for all $x \in \mathbb{R}$, $f'(x) = g'(x)$. Then $f - g$ is a constant.

MVT True

e. (5 pts) Let f and g be any two functions such that $\lim_{x \to 7} f(x) = \infty$ and $\lim_{x \to 7} g(x) = \infty$. Then $\lim_{x \to 7} [1/(f(x))]^{g(x)} = 0$.

“$[1/\infty]^\infty = [0^+]^\infty = 0$” True

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1.

III. 2.

III. 3.

III. 4.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute $d\left[\dfrac{e^{-x^4}}{4+(\tan(x^2))}\right]$. (Here e^{-x^4} means $e^{\left(-x^4\right)}$.)

WARNING: You are asked to find the differential, d, and *NOT* the derivative, d/dx.

$$\frac{[4+(\tan(x^2))][(e^{-x^4})(-4x^3)]-[e^{-x^4}][(\sec^2(x^2))(2x)]}{[4+(\tan(x^2))]^2}\,dx$$

2. (10 pts) Say $f'(x) = e^{x^2}$. (Here e^{x^2} means $e^{(x^2)}$.) Let $g(x) = f(2x+1)$. Find $g'(1)$.

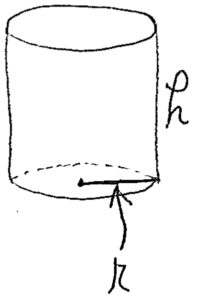
$$g'(x) \overset{CR}{=} (f'(2x+1))(2)$$

$$g'(1) = (f'(3))(2)$$

$$= (e^{3^2})(2)$$

$$= 2e^9$$

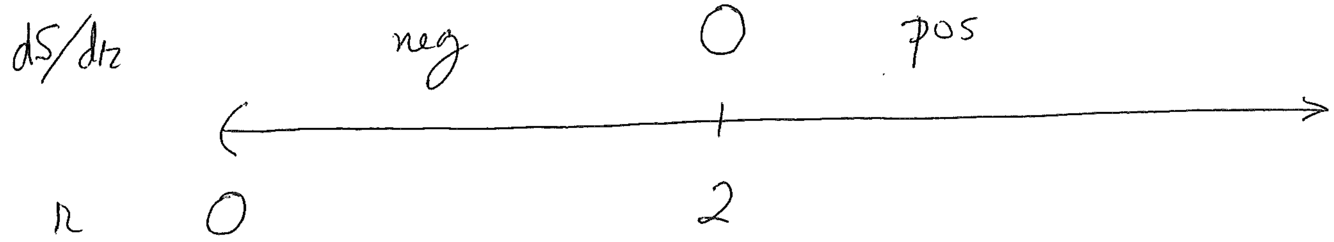
3. (15 pts) We must design a cylindrical can with an open top that contains 8π cubic feet of volume inside, and which minimizes surface area. Find the height, h, and the radius, r, of such a can. (Remember: The surface area includes the side of the can, and the bottom of the can, but the can has no top).



$$\cancel{\pi} r^2 h = 8\cancel{\pi}$$
$$h = 8r^{-2}$$

minimize $S := \pi r^2 + 2\pi r h = \pi r^2 + (2\pi r)(8r^{-2}) = \pi r^2 + 16\pi r^{-1}$

$$\frac{dS}{dr} = 2\pi r - 16\pi r^{-2} = 2\pi\left(r - \frac{8}{r^2}\right) = 2\pi\left(\frac{r^3-8}{r^2}\right)$$



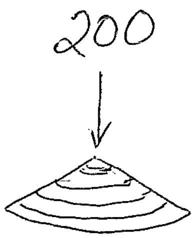
On $r>0$, S attains its global minimum only

at $r = 2$ ft

$h = (8)(2^{-2}) = 2$ ft

4. (10 pts) Sand is accumulating in a conical pile whose base radius is, at all times, twice its height. Sand is being added at 200 cubic feet per hour. At what rate is the height of the pile increasing, at the moment when the height is 5 feet?

$t = \text{time}$ $\quad \bullet = \frac{d}{dt}$ $\quad t_0$ $\quad * = [t :\longrightarrow t_0]$



$r := \text{base radius}$
$h := \text{height}$
$V := \text{volume}$

$r = 2h$

$\dot{V} = 200$

$h_* = 5$

$\dot{h}_* = ?$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2h)^2 h = \frac{4}{3}\pi h^3$$

$$200 = \dot{V} = \frac{4}{3}\pi(3h^2\dot{h}) = 4\pi h^2 \dot{h}$$

$$200 = 4\pi h_*^2 \dot{h}_*$$

$$= 4\pi(5^2)(?)$$

$$? = \frac{200}{4\pi(5^2)} = \frac{200}{100\pi} = \frac{2}{\pi} \text{ ft/hr}$$