

MATH 1271 Spring 2012, Midterm #1  
Handout date: Thursday 16 February 2012

PRINT YOUR NAME:

SOLUTIONS  
Version A

PRINT YOUR TA'S NAME:

WHAT SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Assume that  $\lim_{x \rightarrow 200} (f(x)) = 4$  and  $\lim_{x \rightarrow 200} (g(x)) = 5$ . At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer c.

(a)  $\lim_{x \rightarrow 200} [(f(x)) + (g(x))] = 9$

(b)  $\lim_{x \rightarrow 100} [(f(x)) + (g(x))] = 9$

(c)  $\lim_{x \rightarrow 1} \left[ \frac{f(x)}{g(x)} \right] = \frac{4}{5}$

(d)  $\lim_{x \rightarrow 300} [(f(x)) + (g(x))]$  does not exist

(e) NONE OF THE ABOVE

---

B. (5 pts) (no partial credit) A particle travels along a number line. Its position at time 1 is 40 and its position at time 3 is 80. Find its average velocity between time 1 and time 3. Circle one of the following answers:

(a) 10

(b) 20

(c) 30

(d) 40

(e) NONE OF THE ABOVE

---

$$\frac{80 - 40}{3 - 1} = \frac{40}{2} = 20$$

C. (5 pts) (no partial credit) A line passes through (1, 40) and (3, 80). Find its slope. Circle one of the following answers:

(a) 10

(b) 20

(c) 30

(d) 40

(e) NONE OF THE ABOVE

$$\frac{80 - 40}{3 - 1} = \frac{40}{2} = 20$$

D. (5 pts) (no partial credit) Let  $y = x^2 - x$ . Find  $\Delta y$ . Circle one of the following answers:

- (a)  $(x + \Delta x)^2 - (x + \Delta x)$
  - (b)  $[(x + \Delta x)^2 - (x + \Delta x)] + [x^2 - x]$
  - (c)  $[(x + \Delta x)^2 - (x + \Delta x)] - [x^2 - x]$
  - (d)  $[x^2 - x] - [(x + \Delta x)^2 - (x + \Delta x)]$
  - (e) NONE OF THE ABOVE
- 

E. (5 pts) (no partial credit) Compute  $\lim_{x \rightarrow 0} \frac{2x^3 - 5x^2}{7x(\sin x)}$ . Circle one of the following answers:

- (a) 0
- (b)  $\infty$
- (c)  $5/7$
- (d)  $2/7$
- (e) NONE OF THE ABOVE

$$\frac{-5x^2}{7x \cdot x} \xrightarrow{x \rightarrow 0} -\frac{5}{7}$$

F. (5 pts) (no partial credit) Compute the largest  $\delta > 0$  such that:  $0 < |x - 1| < \delta$  implies  $|(2x + 7) - 9| < 0.05$ . Circle one of the following answers:

- (a) 0.2
  - (b) 0.1
  - (c) 0.025
  - (d) 0.01
  - (e) NONE OF THE ABOVE
- 

$$\frac{0.05}{|2|} = 0.025$$

II. True or false (no partial credit):

a. (5 pts) For every real number  $x$ ,  $\ln(e^x) = x$ .

*True*

b. (5 pts) There is a function with three vertical asymptotes.

*True*

c. (5 pts) A tangent line to the graph of a function cannot intersect the graph of the function more than once.

*False*

d. (5 pts) The function  $f(x) = |x|$  is differentiable at every real number.

*False*

e. (5 pts) If a function is differentiable at 0, then it is continuous at 0.

*True*

---

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES  
PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1a,b

III. 2

III. 3

III. 4a,b

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute  $\lim_{h \rightarrow 0} \frac{\sqrt{5+2h} - \sqrt{5-h}}{h} \cdot \frac{\sqrt{5+2h} + \sqrt{5-h}}{\sqrt{5+2h} + \sqrt{5-h}}$

$$\frac{(\sqrt{5+2h}) - (\sqrt{5-h})}{h} \cdot \frac{1}{\sqrt{5+2h} + \sqrt{5-h}}$$

||

$$\frac{3h}{h} \cdot \frac{1}{\sqrt{5+2h} + \sqrt{5-h}}$$

$\downarrow h \rightarrow 0$

$$\frac{3}{\sqrt{5} + \sqrt{5}} = \frac{3}{2\sqrt{5}}$$

b. (5 pts) Compute  $\lim_{h \rightarrow 0} \frac{\frac{1}{5+2h} - \frac{1}{5-h}}{h}$

$$\frac{(5-h) - (5+2h)}{h(5+2h)(5-h)} = \frac{-3h}{h(5+2h)(5-h)}$$

$\downarrow h \rightarrow 0$

$$\frac{-3}{5 \cdot 5} = -\frac{3}{25}$$

2. (10 pts) Find all the horizontal asymptotes to  $y = \frac{\sqrt{9x^2 + 5}}{x+1}$ .

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} f(x) &= \lim_{x \rightarrow \pm\infty} \frac{\sqrt{9x^2}}{x} \\ &= \lim_{x \rightarrow \pm\infty} \frac{(3)(\pm\cancel{x})}{\cancel{x}} = \pm 3 \end{aligned}$$

$y = -3$  and  $y = 3$  are the horizontal asymptotes.

3. (10 pts) Compute  $\lim_{x \rightarrow 0} \left( \frac{7x^3 + 4x^2}{8x \sin x} \right)$ .

||

$$\lim_{x \rightarrow 0} \frac{4x^2}{(8x)(x)} = \frac{4}{8} = \frac{1}{2}$$

4. On the planet of Gallifrey, in an alternate universe, a dropped object travels  $t^3 + t^2$  feet during its first  $t$  seconds of free fall.

a. (10 pts) For  $h \neq 0$ , the average velocity between time  $t = 2$  seconds and time  $t = 2 + h$  seconds is given by a quadratic polynomial in  $h$  of the form  $ah^2 + bh + c$ . Find the coefficients  $a$ ,  $b$  and  $c$ .

$$\begin{aligned} \frac{[t^3 + t^2]_{t=2+h} - [t^3 + t^2]_{t=2}}{h} &= \frac{[(2+h)^3 + (2+h)^2] - [2^3 + 2^2]}{h} \\ &= \frac{2^3 + 3 \cdot 2^2 \cdot h + 3 \cdot 2 \cdot h^2 + h^3 + 2^2 + 2 \cdot 2 \cdot h + h^2 - 2^3 - 2^2}{h} \\ &= \frac{12h + 6h^2 + h^3 + 4h + h^2}{h} \\ &= \frac{h^3 + 7h^2 + 16h}{h} \stackrel{h \neq 0}{=} h^2 + 7h + 16 \quad \text{ft/sec} \\ &\quad a=1 \quad b=7 \quad c=16 \end{aligned}$$

b. (5 pts) Find the instantaneous velocity at time  $t = 2$  seconds.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{[t^3 + t^2]_{t=2+h} - [t^3 + t^2]_{t=2}}{h} &= \lim_{h \rightarrow 0} (h^2 + 7h + 16) \\ &= 16 \quad \text{ft/sec} \end{aligned}$$