MATH 1271 Spring 2012, Midterm #1
Handout date: Thursday 16 February 2012

PRINT YOUR NAME:

SOLUTIONS
Version B

PRINT YOUR TA'S NAME:

WHAT SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) A line passes through $(1, 40)$ and $(3, 80)$. Find its slope. Circle one of the following answers:

(a) 10

(b) 20

(c) 30

(d) 40

(e) NONE OF THE ABOVE

$$\frac{80-40}{3-1} = \frac{40}{2} = 20$$

B. (5 pts) (no partial credit) A particle travels along a number line. Its position at time 1 is 40 and its position at time 3 is 80. Find its average velocity between time 1 and time 3. Circle one of the following answers:

(a) 10

(b) 20

(c) 30

(d) 40

(e) NONE OF THE ABOVE

$$\frac{80-40}{3-1} = \frac{40}{2} = 20$$

C. (5 pts) (no partial credit) Compute the largest $\delta > 0$ such that: $0 < |x - 1| < \delta$ implies $|(2x + 7) - 9| < 0.05$. Circle one of the following answers:

(a) 0.2

(b) 0.1

(c) 0.025

(d) 0.01

(e) NONE OF THE ABOVE

$$\frac{0.05}{2} = 0.025$$

D. (5 pts) (no partial credit) Compute $\lim_{x\to 0} \frac{2x^3 - 5x^2}{\underbrace{7x(\sin x)}_{\approx x\to 0}}$. Circle one of the following answers:

$$\frac{-5\cancel{x^2}}{7\cancel{x}\cdot x} \xrightarrow{x\to 0} -\frac{5}{7}$$

(a) 0

(b) ∞

(c) $-5/7$

(d) $2/7$

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Let $y = x^2 - x$. Find Δy. Circle one of the following answers:

(a) $(x + \Delta x)^2 - (x + \Delta x)$

(b) $[(x + \Delta x)^2 - (x + \Delta x)] + [x^2 - x]$

(c) $[x^2 - x] - [(x + \Delta x)^2 - (x + \Delta x)]$

(d) $[(x + \Delta x)^2 - (x + \Delta x)] - [x^2 - x]$

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Assume that $\lim_{x\to 200}(f(x)) = 4$ and $\lim_{x\to 200}(g(x)) = 5$. At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

(a) $\lim_{x\to 400}[(f(x)) + (g(x))] = 9$

(b) $\lim_{x\to 200}[(f(x)) + (g(x))] = 20$

(c) $\lim_{x\to 1}\left[\frac{f(x)}{g(x)}\right] = \frac{4}{5}$

(d) $\lim_{x\to 300}[(f(x)) + (g(x))]$ does not exist

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) The function $f(x) = |x|$ is continuous at every real number.

True

b. (5 pts) There is a function with five vertical asymptotes.

True

c. (5 pts) A tangent line to the graph of a function cannot intersect the graph of the function more than once.

False

d. (5 pts) For every real number x, $\ln(e^x) = 1$.

False

e. (5 pts) If a function is differentiable at 2, then it is continuous at 0.

False

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1a,b

III. 2

III. 3

III. 4a,b

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute $\underbrace{\lim_{h\to 0} \frac{\sqrt{5+2h}-\sqrt{5-h}}{h} \cdot \frac{\sqrt{5+2h}+\sqrt{5-h}}{\sqrt{5+2h}+\sqrt{5-h}}}$

$$= \frac{(\cancel{5}+2h)-(\cancel{5}-h)}{h} \frac{1}{\sqrt{5+2h}+\sqrt{5-h}}$$

$$= \frac{3\cancel{h}}{\cancel{h}} \frac{1}{\sqrt{5+2h}+\sqrt{5-h}}$$

$$\Big\downarrow h\to 0$$

$$\frac{3}{\sqrt{5}+\sqrt{5}} = \frac{3}{2\sqrt{5}}$$

b. (5 pts) Compute $\underbrace{\lim_{h\to 0} \frac{\frac{1}{5+2h}-\frac{1}{5-h}}{h}}$.

$$= \frac{(5-h)-(5+2h)}{h(5+2h)(5-h)} = \frac{-3\cancel{h}}{\cancel{h}(5+2h)(5-h)}$$

$$\Big\downarrow h\to 0$$

$$\frac{-3}{5\cdot 5} = -\frac{3}{25}$$

2. (10 pts) Find all the horizontal asymptotes to $y = \underbrace{\frac{\sqrt{9x^2+5}}{x+1}}_{f(x)}$.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{\sqrt{9x^2}}{x}$$

$$= \lim_{x \to \pm\infty} \frac{(3)(\pm \cancel{x})}{\cancel{x}} = \pm 3$$

$y = -3$ and $y = 3$ are the horizontal asymptotes.

3. (10 pts) Compute $\lim_{x \to 0} \left(\frac{7x^3 + 4x^2}{8x \sin x} \right)$.

$$\parallel$$

$$\lim_{x \to 0} \frac{4\cancel{x^2}}{(8\cancel{x})(\cancel{x})} = \frac{4}{8} = \frac{1}{2}$$

4. On the planet of Gallifrey, in an alternate universe, a dropped object travels $t^3 + t^2$ feet during its first t seconds of free fall.

a. (10 pts) For $h \neq 0$, the average velocity between time $t = 2$ seconds and time $t = 2 + h$ seconds is given by a quadratic polynomial in h of the form $ah^2 + bh + c$. Find the coefficients a, b and c.

$$\frac{\left[t^3+t^2\right]_{t:\to 2}^{t:\to 2+h}}{\left[t\right]_{t:\to 2}^{t:\to 2+h}} = \frac{\left[(2+h)^3+(2+h)^2\right]-\left[2^3+2^2\right]}{h}$$

$$= \frac{\cancel{2^3}+3\cdot 2^2\cdot h+3\cdot 2\cdot h^2+h^3+\cancel{2^2}+2\cdot 2\cdot h+h^2-\cancel{2^3}-\cancel{2^2}}{h}$$

$$= \frac{12h + 6h^2 + h^3 + 4h + h^2}{h}$$

$$= \frac{h^3+7h^2+16h}{h} \overset{h\neq 0}{=} h^2+7h+16 \quad ft/sec$$

$a=1 \quad b=7 \quad c=16$

b. (5 pts) Find the instantaneous velocity at time $t = 2$ seconds.

$$\lim_{h\to 0} \frac{\left[t^3+t^2\right]_{t:\to 2}^{t:\to 2+h}}{\left[t\right]_{t:\to 2}^{t:\to 2+h}} = \lim_{h\to 0}\left(h^2+7h+16\right)$$

$$= 16 \quad ft/sec$$