Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.
I. Multiple choice

A. (5 pts) (no partial credit) Suppose \( f''(x) = -x^2 + 4x - 3 \). At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

(a) \( f \) is concave up on \((-\infty, -3]\), down on \([-3, -1]\) and up on \([-1, \infty)\).

(b) \( f \) is concave up on \((-\infty, 1]\), down on \([1, 3]\) and up on \([3, \infty)\).

(c) \( f \) is concave down on \((-\infty, 1]\), up on \([1, 3]\) and down on \([3, \infty)\).

(d) \( f \) is concave down on \((-\infty, -3]\), up on \([-3, -1]\) and down on \([-1, \infty)\).

(e) NONE OF THE ABOVE

\[ f'' \quad \text{neg} \quad 0 \quad \text{pos} \quad 0 \quad \text{neg} \]

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B. (5 pts) (no partial credit) Find the logarithmic derivative of \((2 + x^4)^{\cos x}\) w.r.t. \( x \).

(a) \((-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))\)

(b) \((\cos x)(\ln(2 + x^4))\)

(c) \((-\sin x)(4x^3/(2 + x^4))\)

(d) \((\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))\)

(e) NONE OF THE ABOVE

\[ \frac{d}{dx} \left[ \left(\cos x\right) \left(\ln \left(2+x^4\right)\right) \right] \]

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C. (5 pts) (no partial credit) Find the derivative of \((2 + x^4)^{\cos x}\) w.r.t. \( x \).

(a) \([2 + x^4]^{\cos x} \left[ (-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4)) \right]\)

(b) \([2 + x^4]^{\cos x} \left[ (\cos x)(\ln(2 + x^4)) \right]\)

(c) \([2 + x^4]^{\cos x} \left[ (-\sin x)(4x^3/(2 + x^4)) \right]\)

(d) \([2 + x^4]^{\cos x} \left[ (\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4)) \right]\)

(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Find the logarithmic derivative of \( x^2 + 7x - 8 \) w.r.t. \( x \).

- (a) \( \frac{x^2 + 7x - 8}{2x + 7} \)
- (b) \( \frac{2x + 7}{x^2 + 7x - 8} \)
- (c) \( \ln(2x + 7) \)
- (d) \( (\ln(x^2)) + 7(\ln x) - (\ln 8) \)
- (e) NONE OF THE ABOVE

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E. (5 pts) (no partial credit) Find an equation of the tangent line to \( 4x^2y - 2y^3 = 2 \) at the point \( (1, 1) \).

- (a) \( y - 1 = 4(x - 1) \)
- (b) \( y - 1 = 3(x - 1) \)
- (c) \( y - 1 = 2(x - 1) \)
- (d) \( y - 1 = x - 1 \)
- (e) NONE OF THE ABOVE

\[
8xy + 4x^2y' - 6y^2y' = 0
\]

\[
y' = \frac{-8xy}{4x^2 - 6y^2}
\]

\[
\text{slope} = \frac{-8}{4 - 6} = -8 = 4
\]

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F. (5 pts) (no partial credit) Compute \( [d/dx][\sin(\cos(e^x + 3))] \).

- (a) \( \cos(\cos(e^x + 3)) \)
- (b) 0
- (c) \( [\cos(\cos(e^x + 3))][\cos(e^x + 3)][e^x + 3] \)
- (d) \( [\cos(\cos(e^x + 3))][-\sin(e^x + 3)][e^x + 3] \)
- (e) NONE OF THE ABOVE

\[
\left[ \cos \left( \cos \left( e^x + 3 \right) \right) \right] \left[ -\sin \left( e^x + 3 \right) \right] \left[ e^x \right]
\]
II. True or false (no partial credit):

a. (5 pts) If \( f \) is increasing on an interval \( I \), then \( f' > 0 \) on \( I \).

\[ \text{F} \]

b. (5 pts) Assume that \( \lim_{x \to a} [f(x)] = 0 = \lim_{x \to a} [g(x)] \). Assume also that \( \lim_{x \to a} \frac{f'(x)}{g'(x)} = -\infty \).

Then \( \lim_{x \to a} \frac{f(x)}{g(x)} = -\infty \).

\[ \text{T} \]

c. (5 pts) Every global minimum of a function \( f : \mathbb{R} \to \mathbb{R} \) occurs at a critical number for \( f \).

\[ \text{T} \]

d. (5 pts) If \( f'(7) = 0 \) and \( f''(7) > 0 \), then \( f \) has a local maximum at 7.

\[ \text{U} \]

\[ \text{F} \]

e. (5 pts) If two functions have the same derivative, then they are equal.

\[ \text{F} \]

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1ab.

III. 2.

III. 3,4.

III. 5.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute \( \frac{d}{dx} \left[ \frac{2x^3 - 8}{7 + \arctan(2x)} \right] \).

\[
\left[ 7 + \arctan(2x) \right] [6x^2] - [2x^3 - 8] \left[ \frac{1}{1 + (2x)^2} \right]^2 \left[ 7 + \arctan(2x) \right]^2
\]

b. (5 pts) Compute \( \frac{d}{dx} [(4 - \sin x)^x] \).

\[
\left[ (4 - \sin x)^x \right] \left[ \frac{d}{dx} \left[ x (\ln (4 - \sin x)) \right] \right]
\]

\[
\left[ (4 - \sin x)^x \right] \left[ (\ln (4 - \sin x)) + x \left( \frac{-\cos x}{4 - \sin x} \right) \right]
\]
2. (10 pts) Using implicit differentiation, find \( y' = dy/dx \), assuming that \((x - y^2)^5 = x\).

\[
[5(x-y^2)^4][1-2yy'] = 1
\]

\[
[5(x-y^2)^4] - [10y(x-y^2)^3]y' = 1
\]

\[
y' = \frac{1 - 5(x-y^2)^4}{-10y(x-y^2)^4}
\]
3. (5 pts) Let \( f(x) = 7x + x^5 \). Then \( f \) is a one-to-one function. Let \( g := f^{-1} \). Then \( f(1) = 8 \), so \( g(8) = 1 \). Compute \( g'(8) \).

\[
g'(8) = \frac{1}{f'(1)} = \frac{1}{\left[7 + 5x^4\right]_{x=1}} = \frac{1}{12}
\]

4. (10 pts) Find the maximal intervals of concavity for \( f(x) = -3x^5 + 20x^4 + 12x - 7 \). For each interval, state clearly whether \( f \) is concave up or concave down on that interval.

\[
f'(x) = -15x^4 + 80x^3 + 12
\]

\[
f''(x) = -60x^3 + 240x^2
= -60x^2(x - 4)
\]

\[
f'' \quad \text{pos} \quad 0 \quad \text{neg} \quad \text{pos} \quad \text{neg}
\]

\[
\begin{array}{c|c|c|c|c}
& 0 & 4 & 0 & \infty \\
\hline
f & \text{pos} & 0 & \text{neg} & \text{pos}
\end{array}
\]

\( f \) is concave up on \((-\infty, 4]\)

\( f \) is concave down on \([4, \infty)\)
5. (10 pts) Compute \[ \lim_{x \to 1} \left[ \frac{\ln x}{\cos(\pi x/2)} \right]. \]