MATH 1271 Spring 2013, Midterm #1
Handout date: Thursday 21 February 2013

PRINT YOUR NAME:

SOLUTIONS
VERSION A

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x\to 4^-}(h(x)) = 7$? Circle one of the following answers:

(a) If x is close to 4, but not equal to 4, then $h(x)$ is close to 7, but not equal to 7.

(b) If x is close to 4, but less than 4, then $h(x)$ is close to 7.

(c) If $h(x)$ is close to 7, but not equal to 7, then x is close to 4, but less than 4.

(d) If $h(x)$ is close to 4, then x is close to 7.

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $\lim_{x\to 0}\left[\frac{(3x^5 - 8x^4)(\cos x)}{4x^3(\sin x)}\right]$. Circle one of the following answers:

(a) 3/4

(b) -2

(c) 0

(d) This limit does not exist.

(e) NONE OF THE ABOVE

$\lim_{x\to 0}$

$\frac{(-8x^4)(1)}{4x^3(x)} \underset{x\neq 0}{=} \frac{-8}{4} = -2$

$x \to 0$

-2

C. (5 pts) (no partial credit) Compute $\lim_{t\to 3}\left[\frac{t^2 + t - 12}{t - 3}\right]$. Circle one of the following answers:

(a) 5

(b) 6

(c) 7

(d) 8

(e) NONE OF THE ABOVE

$\lim_{t\to 3}$

$\| t \neq 3$

$t + 4 \xrightarrow[t\to 3]{} 7$

D. (5 pts) (no partial credit) (no partial credit) A line passes through $(1, 40)$ and $(3, 80)$. Find its slope. Circle one of the following answers:

(a) 5

(b) 10

(c) 15

(d) 20

(e) NONE OF THE ABOVE

$$\frac{80-40}{3-1} = \frac{40}{2} = 20$$

E. (5 pts) (no partial credit) What is the largest number x such that $|x-3| \leq 0.005$?

(a) 2.995

(b) 3

(c) 3.005

(d) -2.995

(e) NONE OF THE ABOVE

$$3-0.005 \leq x \leq 3+0.005$$

F. (5 pts) (no partial credit) Compute $\lim_{x\to-\infty}\left[\frac{x^3+2x^2-4x}{2x^4-7x^2}\right]$ Circle one of the following answers:

(a) 4/7

(b) $-4/7$

(c) 1/2

(d) $-1/2$

(e) NONE OF THE ABOVE

$$\frac{x^3}{2x^4} = \frac{1}{2x} \xrightarrow{x\to-\infty} 0$$

II. True or false (no partial credit):

a. (5 pts) Let $f(x) = |x|$. Then $f(x)$ is differentiable at $x = 1$.

True

b. (5 pts) Let f be the restriction of sin to $[0, \pi]$. Then f is a one-to-one function.

False

c. (5 pts) $\lim_{x \to 0} \dfrac{1 - \cos x}{x} = 1$.

False

d. (5 pts) Let f be any function. If $\lim_{x \to 3} f(x)$ exists, then 3 is in the domain of f.

False

e. (5 pts) Let f be any rational function. If $\lim_{x \to \infty} f(x) = 2$, then $\lim_{x \to -\infty} f(x) = 2$.

True

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find all horizontal asymptotes to

$$y = \frac{\sqrt[3]{8x^3 + 2x + 5}}{7x - 3} =: f(x)$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines, **NOT** numbers.)

$$f(x) \underset{x\to\pm\infty}{\sim} \frac{\sqrt[3]{8x^3}}{7x} = \frac{2x}{7x} \underset{x\neq 0}{=} \frac{2}{7} \xrightarrow[x\to\pm\infty]{} \frac{2}{7}$$

$y = \frac{2}{7}$ is the only horizontal asymptote

2. (15 pts) Compute $\lim_{n\to\infty}\left(1+\frac{45}{n}\right)^n$.

$x = \frac{n}{45}$

$$= \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{45x}$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{45}$$

$$= \left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{45}$$

$$= e^{45}$$

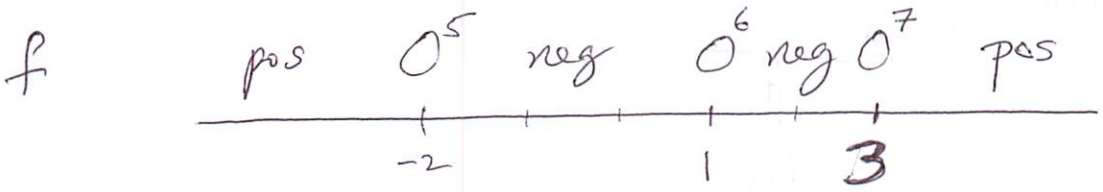
3. (10 pts) Compute $\lim_{x\to\infty}\left[\underbrace{\frac{2x^2+\cos^2 x}{4x^2+2}}_{\| \\ f(x)}\right]$.

$$\left.\begin{array}{l}1\\ \vee\\ \cos^2 x\\ \vee\\ 0\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\dfrac{2x^2+1}{4x^2+2}\underset{x\to\infty}{\sim}\dfrac{2x^2}{4x^2}\underset{x\neq 0}{=}\dfrac{2}{4}\xrightarrow[x\to\infty]{}\dfrac{2}{4}\\ \vee\\ f(x)\\ \vee\\ \dfrac{2x^2+0}{4x^2+2}\underset{x\to\infty}{\sim}\dfrac{2x^2}{4x^2}\underset{x\neq 0}{=}\dfrac{2}{4}\xrightarrow[x\to\infty]{}\dfrac{2}{4}\end{array}\right.$$

By the Squeeze Thm,

$$\lim_{x\to\infty} f(x) = \frac{2}{4} = \frac{1}{2}$$

4. (10 pts) Let $f(x) = (x+2)^5(x-1)^6(x-3)^7$. Find all of the maximum intervals of positivity and negativity for f.



f is positive on $(-\infty, -2)$,

negative on $(-2, 1)$,

negative on $(1, 3)$,

and positive on $(3, \infty)$.