MATH 1271 Spring 2013, Midterm #1
Handout date: Thursday 21 February 2013

PRINT YOUR NAME:

SOLUTIONS

VERSION C

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Compute $\lim_{x\to 0}\left[\underbrace{\frac{(3x^2-8x^4)(\cos x)}{4x(\sin x)}}_{?x\to 0}\right]$. Circle one of the following answers:

(a) 3/4

(b) -2

(c) 0

(d) This limit does not exist.

(e) NONE OF THE ABOVE

$$\frac{(3x^2)(1)}{4x(x)} \underset{x\neq 0}{=} \frac{3}{4} \xrightarrow{x\to 0} \frac{3}{4}$$

B. (5 pts) (no partial credit) Compute $\lim_{x\to -\infty}\left[\underbrace{\frac{x^3+2x^2-4x}{2x^4-7x^2}}_{?x\to -\infty}\right]$ Circle one of the following answers:

(a) 4/7

(b) $-4/7$

(c) 0

(d) ∞

(e) NONE OF THE ABOVE

$$\frac{x^3}{2x^4} = \frac{1}{2x} \xrightarrow{x\to -\infty} 0$$

C. (5 pts) (no partial credit) (no partial credit) A line passes through $(5, 40)$ and $(3, 80)$. Find its slope. Circle one of the following answers:

(a) 40

(b) 10

(c) 0

(d) -10

(e) NONE OF THE ABOVE

$$\frac{80-40}{3-5} = \frac{40}{-2} = -20$$

D. (5 pts) (no partial credit) What is the largest number x such that $|x-3| \leq 0.005$?

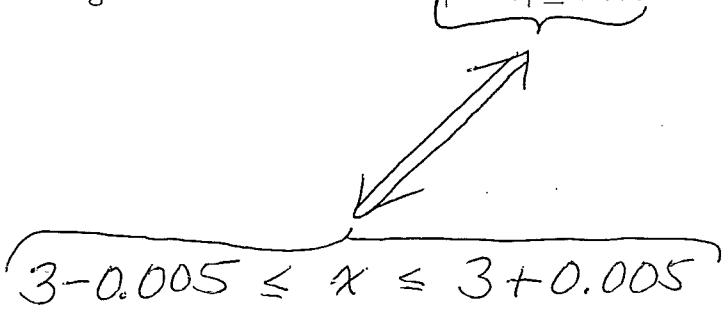
(a) 2.995

(b) 3.005

(c) 3

(d) -2.995

(e) NONE OF THE ABOVE



E. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x \to 4}(h(x)) = 7$? Circle one of the following answers:

(a) If x is close to 4, but not equal to 4, then $h(x)$ is close to 7, but not equal to 7.

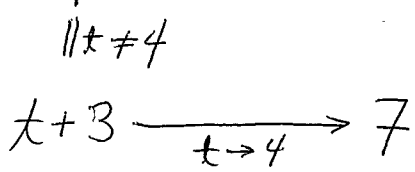
(b) If $h(x)$ is close to 4, then x is close to 7.

(c) If x is close to 4, but not equal to 4, then $h(x)$ is close to 7.

(d) If $h(x)$ is close to 7, but not equal to 7, then x is close to 4, but less than 4.

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute $\lim_{t \to 4}\left[\frac{t^2-t-12}{t-4}\right]$. Circle one of the following answers:



(a) 7

(b) 8

(c) 9

(d) 10

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) Let f be any algebraic function. If $\lim_{x\to\infty} f(x) = 1/3$, then $\lim_{x\to-\infty} f(x) = 1/3$.

False

b. (5 pts) Let f be any function. If $\lim_{x\to 3} f(x)$ exists, then f is continuous at 3.

False

c. (5 pts) Let $f(x) = |x|$. Then $f(x)$ is differentiable at $x = 0$.

False

d. (5 pts) Let f be the restriction of sin to $[-\pi/2, 0]$. Then f is a one-to-one function.

True

e. (5 pts) $\lim_{x\to 0} \dfrac{\sin^2 x}{x^2} = 1$.

True

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find all horizontal asymptotes to

$$y = \frac{\sqrt[4]{x^4 + 2x + 5}}{7x - 3} =: f(x)$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines, **NOT** numbers.)

$$f(x) \underset{x \to \pm\infty}{\sim} \frac{\sqrt[4]{x^4}}{7x} = \frac{|x|}{7x} = \frac{\pm x}{7x} \overset{x \neq 0}{=\!=} \pm \frac{1}{7}$$

$x \downarrow \pm\infty$

$$\pm \frac{1}{7}$$

$y = -\frac{1}{7}$ and $y = \frac{1}{7}$ are the horizontal asymptotes.

2. (15 pts) Compute $\lim_{n\to\infty}\left(1+\frac{63}{n}\right)^n$.

$x = \frac{n}{63}$

$$\lim_{n\to\infty}\left(1+\frac{63}{n}\right)^n$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{63x}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{63}$$

$$\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{63}$$

$$e^{63}$$

3. (10 pts) Compute $\lim_{x\to-\infty}\left[\underbrace{\frac{2x^2+\cos^2 x}{5x^2+2}}_{\therefore\ f(x)}\right]$.

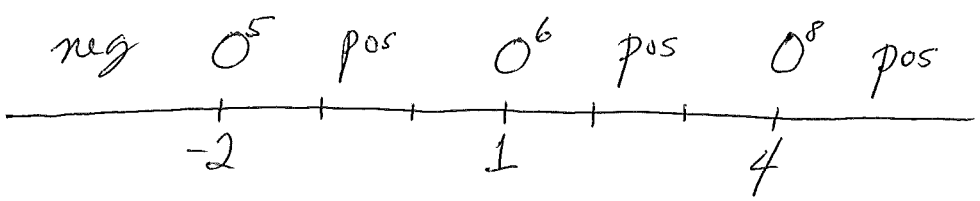
$$\left.\begin{array}{c}1\\ \vee\\ \cos^2 x\\ \vee\\ 0\end{array}\right\}\Longrightarrow\left\{\begin{array}{l}\dfrac{2x^2+1}{5x^2+2}\ \underset{x\to-\infty}{\sim}\ \dfrac{2x^2}{5x^2}\ \underset{x\neq 0}{=}\ \dfrac{2}{5}\ \xrightarrow[x\to-\infty]{}\ \dfrac{2}{5}\\ \vee\\ f(x)\\ \vee\\ \dfrac{2x^2+0}{5x^2+2}\ \underset{x\to-\infty}{\sim}\ \dfrac{2x^2}{5x^2}\ \underset{x\neq 0}{=}\ \dfrac{2}{5}\ \xrightarrow[x\to-\infty]{}\ \dfrac{2}{5}\end{array}\right.$$

By the Squeeze Thm,

$$\lim_{x\to-\infty} f(x) = \frac{2}{5}$$

4. (10 pts) Let $f(x) = (x+2)^5(x-1)^6(x-4)^8$. Find all of the maximum intervals of positivity and negativity for f.

f



f is negative on $(-\infty, -2)$,

positive on $(-2, 1)$,

positive on $(1, 4)$,

and positive on $(4, \infty)$.