MATH 1271 Spring 2013, Midterm #2
Handout date: Thursday 4 April 2013

PRINT YOUR NAME: SOLUTIONS
Version C

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Compute \[d/dx[\sin^2(xy)]\]. Circle one of the following answers:

(a) \[\cos(xy)(y + xy')\]
(b) \[2\sin(xy)(y + xy')\]
(c) \[2\sin(xy)[\cos(y + xy')]\]
(d) \[2\sin(xy)[\cos(xy)]\]
(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) The Quotient Rule says that \((f/g)’\) is equal to what? Circle one of the following answers:

(a) \((gf’ - fg’)/g^2\)
(b) \((fg’ - gf’)/g^2\)
(c) \(g'/f’\)
(d) \(f'/g’\)
(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Let \(f\) be a function such that \(f'(x) = 6e^{4x}\). Suppose, also, that \(f(0) = 1\). Which of the following is an equation of the tangent line to the graph of \(f\) at \((0,1)\). Circle one of the following answers:

(a) \(y = 1 + 6x\)
(b) \(y = 6(x - 1)\)
(c) \(y - 1 = 6e^{4x}x\)
(d) \(y = 6e^{4x}(x - 1)\)
(e) NONE OF THE ABOVE

\[
\text{slope } = f'(0) = 6e^{4\cdot0} = 6
\]
\[
y - 1 = 6(x - 0)
\]
\[
y = 1 + 6x
\]
D. (5 pts) (no partial credit) Compute \( \frac{d}{dx} \ln|(2x + 1)(3x - 4)| \). Circle one of the following answers:

(a) \( \frac{2}{2x+1} + \frac{3}{3x-4} \)

(b) \( \frac{6}{(2x+1)(3x+4)} \)

(c) \( \frac{2}{2x+1} + \frac{3}{3x-4} \)

(d) \( \frac{6}{(2x+1)(3x+4)} \)

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Suppose \( f'(x) = -2(x - 1)^5(x - 2)^6(x - 3)^8 \). Which of the following is a maximal interval of increase for \( f \)? Circle one of the following answers:

(a) \((-\infty, 1]\)

(b) \((-\infty, 2)\)

(c) \([1, \infty)\)

(d) \([2, \infty)\)

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute \( \lim_{x \to \infty} \frac{(2x^2 + 4x)e^{-x} - 3}{(2x^2 + 4x - 3e^x)e^{-3}} \). Circle one of the following answers:

(a) \(\infty\)

(b) 2

(c) 0

(d) -3

(e) NONE OF THE ABOVE

\[ \lim_{x \to \infty} \frac{4}{e^x} - 3 \]

\[ e^x = \infty \quad \text{and} \quad \frac{4}{\infty} = 0 \]
II. True or false (no partial credit):

a. (5 pts) Let \( f \) and \( g \) be any two functions such that \( \lim_{x \to 5} f(x) = 1 \) and \( \lim_{x \to 5} g(x) = 0 \). Then \( \lim_{x \to 5} \frac{f(x)}{g(x)} = \infty \).

\[ \text{False} \quad \frac{1}{0} \text{ is (slightly) indeterminate} \]

b. (5 pts) Let \( u \) be any expression of \( x \). Then \( (d/dx)(e^u) = e^u \).

\[ \text{False} \quad e^u \left(\frac{du}{dx}\right) \]

c. (5 pts) If \( f' > 0 \) on an interval \( I \), then \( f \) is increasing on \( I \).

\[ \text{True} \]

d. (5 pts) Let \( g \) be any function such that \( \lim_{x \to \infty} g(x) = \infty \). Then \( \lim_{x \to \infty} \left(\frac{1}{g(x)}\right) = 0 \).

\[ \text{True} \quad \frac{1}{\infty} = 0^+ \text{ and } (0)^\infty = 0 \]

e. (5 pts) Let \( f \) and \( g \) be any two functions such that \( \lim_{x \to a} f(x) = \infty \) and \( \lim_{x \to a} g(x) = \infty \). Then \( \lim_{x \to a} [(f(x)) - (g(x))] = 0 \).

\[ \text{False} \quad \infty - \infty \text{ is indeterminate} \]

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5. a,b,c
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute \( \frac{d}{dx} \left[ \frac{e^{-x^4}}{4 \sin(x^2)} \right] \). (Here \( e^{-x^4} \) means \( e^{-x^4} \).

\[
\left[ 4 - \sin(x^2) \right] \left[ e^{-x^4} \right] \left[ -4x^2 \right] - \left[ e^{-x^4} \right] \left[ -\cos(x^2) \right] \left[ 2x \right] \\
\left[ 4 - \sin(x^2) \right]^2
\]

2. (5 pts) Compute \( \frac{d}{dx} \left[ (2 + 3x^2) \tan x \right] \).

\[
\left[ (2 + 3x^2) \tan x \right] \left[ \frac{d}{dx} \left[ \left( \tan x \right) \left( \ln (2 + 3x^2) \right) \right] \right] \\
\left[ (2 + 3x^2) \tan x \right] \left[ \left( \sec^2 x \right) \left( \ln (2 + 3x^2) \right) + \left( \tan x \right) \left( \frac{6x}{2 + 3x^2} \right) \right]
\]
3. (10 pts) Find an equation for the tangent line to $-6e^x - 5xy + y^2 = 4x - y$ at $(0, 2)$.

$m := \text{slope of this tangent line}$

$-6e^x - 5y - 5xy' + 2yy' = 4 - y'$

$0 \quad 2 \quad 0 \quad m \quad 2m \quad m$

$-6 - 10 - 0 + 4m = 4 - m$

$-16 + 4m = 4 - m$

$5m = 20$

$m = 4$

$y - 2 = 4x$
4. (10 pts) Compute \( \lim_{x \to 0} (2e^x - \cos x)^{3/x} \).

\[
\lim_{x \to 0} \left( \frac{3}{x} \right) \left( \ln (2e^x - \cos x) \right)
\]

\[
\lim_{x \to 0} \frac{3 \left( \ln (2e^x - \cos x) \right)}{x}
\]

\[
\lim_{x \to 0} \frac{3 \left( \frac{2e^x + \sin x}{2e^x - \cos x} \right)}{1}
\]

\[
\lim_{x \to 0} \frac{3 \left( \frac{2 + 0}{2 - 1} \right)}{1} = e^6
\]
5. Let \( y = x^3 \). Then \( \Delta y = ax^2(\Delta x) + bx(\Delta x)^2 + c(\Delta x)^3 \), for some real numbers \( a, b, c \).

a. (5 pts) Compute \( a, b \) and \( c \).

\[
\Delta y = (x + \Delta x)^3 - x^3 \\
= x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3
\]

\[
\begin{align*}
\text{a} & \quad \| \\
\text{b} & \quad \| \\
3 & \quad 3 & \quad 1
\end{align*}
\]

b. (5 pts) Assuming \( \Delta x \neq 0 \), compute \( \frac{\Delta y}{\Delta x} \).

\[
\| \Delta x \neq 0
\]

\[
3x^2 + 3x(\Delta x) + (\Delta x)^2
\]

c. (5 pts) Compute \( \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \).

\[
\| \\
3x^2
\]