MATH 1271 Spring 2013, Midterm #2
Handout date: Thursday 4 April 2013

PRINT YOUR NAME:

SOLUTIONS
Version D

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Compute $\lim_{x\to-\infty}(2x^3+4x-3)e^{-x}$. Circle one of the following answers:

(a) -3

(b) 0

(c) 2

(d) ∞

(e) NONE OF THE ABOVE

"$(2(-\infty)^3)(e^{-(-\infty)})$

$= (-\infty)(\infty) = -\infty$ "

B. (5 pts) (no partial credit) Compute $\frac{d}{dx}[\ln|(2x+1)(3x-4)|]$. Circle one of the following answers:

(a) $\left|\frac{2}{2x+1}+\frac{3}{3x-4}\right|$

(b) $\frac{6}{(2x+1)(3x+4)}$

(c) $\left|\frac{6}{(2x+1)(3x+4)}\right|$

(d) $\frac{2}{2x+1}+\frac{3}{3x-4}$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

(a) $[\cos^2(xy)][y+xy']$

(b) $2[\sin(xy)][\cos(y+xy')]$

(c) $2[\sin(xy)][\cos(xy)][y+xy']$

(d) $2[\sin(xy)][\cos(xy)]$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Let f be a function such that $f'(x) = 3e^{4x}$. Suppose, also, that $f(0) = 1$. Which of the following is an equation of the tangent line to the graph of f at $(0, 1)$. Circle one of the following answers:

(a) $y = 3e^{4x}(x - 1)$

(b) $y - 1 = 3e^{4x}x$

(c) $y = 1 + 3x$

(d) $y = 3(x - 1)$

(e) NONE OF THE ABOVE

slope $= f'(0) = 3e^{4\cdot 0} = 3$

$y - 1 = 3(x - 0)$

$y = 1 + 3x$

E. (5 pts) (no partial credit) The Quotient Rule says that $(f/g)'$ is equal to what? Circle one of the following answers:

(a) $(fg' - gf')/g^2$

(b) $(gf' - fg')/g^2$

(c) g'/f'

(d) f'/g'

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Suppose $f'(x) = (x - 1)^5(x - 2)^6(x - 3)^8$. Which of the following is a maximal interval of decrease for f? Circle one of the following answers:

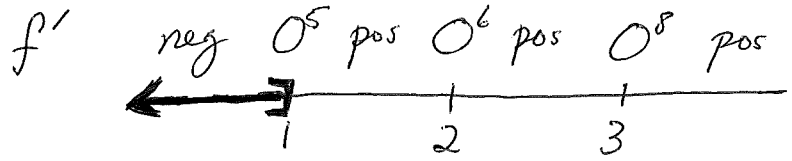
(a) $(-\infty, 1]$

(b) $[1, \infty)$

(c) $(-2, \infty)$

(d) $[2, \infty)$

(e) NONE OF THE ABOVE



II. True or false (no partial credit):

a. (5 pts) Let f and g be any two functions such that $\lim_{x\to 5} f(x) = 1$ and $\lim_{x\to 5} g(x) = 0$. Then $\lim_{x\to 5} \frac{f(x)}{g(x)} = \infty$.

False "$\frac{1}{0}$" is (slightly) indeterminate

b. (5 pts) Let g be any function such that $\lim_{x\to\infty} [g(x)] = \infty$. Then $\lim_{x\to\infty} [(1/x)^{g(x)}] = 0$.

True "$1/\infty = 0^+$" and "$(0^+)^\infty = 0$"

c. (5 pts) If $f' > 0$ on an interval I, then f is increasing on I.

True

d. (5 pts) Let f and g be any two functions such that $\lim_{x\to a} [f(x)] = \infty$ and $\lim_{x\to a} [g(x)] = \infty$. Then $\lim_{x\to a} [(f(x)) - (g(x))] = 0$.

False "$\infty - \infty$" is indeterminate

e. (5 pts) Let u be any expression of x. Then $(d/dx)(e^{2u}) = e^{2u}(du/dx)$.

False $(e^{2u})(2\,(du/dx))$

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5. a,b,c

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx}\left[\frac{e^{-x^4}}{4-\cot(x^2)}\right]$. (Here e^{-x^4} means $e^{(-x^4)}$.)

$$=\frac{[4-\cot(x^2)][e^{-x^4}][-4x^3]-[e^{-x^4}][\csc^2(x^2)][2x]}{[4-\cot(x^2)]^2}$$

2. (5 pts) Compute $\frac{d}{dx}\left[(3+2\sin x)^{5-x}\right]$.

$$=\left[(3+2\sin x)^{5-x}\right]\left[\frac{d}{dx}\left[(5-x)(\ln(3+2\sin x))\right]\right]$$

$$=\left[(3+2\sin x)^{5-x}\right]\left[(-1)(\ln(3+2\sin x))+(5-x)\left(\frac{2\cos x}{3+2\sin x}\right)\right]$$

3. (10 pts) Find an equation for the tangent line to $7x^{-3} - 5xy + y^2 = 4x - y$ at $(1, 3)$.

$m :=$ slope of this tangent line

$$-21x^{-4} - 5y - 5xy' + 2yy' = 4 - y'$$

1 3 1 m 3 m m

$$-21 - 15 - 5m + 6m = 4 - m$$

$$-36 + m = 4 - m$$

$$2m = 40$$

$$m = 20$$

$$\boxed{y - 3 = 20(x - 1)}$$

4. (10 pts) Compute $\lim_{x\to 0} (e^x - 3\sin x)^{7/x}$.

$$=$$

$$e^{\lim_{x\to 0} (7/x)(\ln(e^x - 3\sin x))}$$

$$=$$

$$e^{\lim_{x\to 0} \frac{7(\ln(e^x - 3\sin x))}{x}}$$

$$= \text{LH}\ \tfrac{0}{0}$$

$$e^{\lim_{x\to 0} \frac{7\left(\frac{e^x - 3\cos x}{e^x - 3\sin x}\right)}{1}}$$

$$=$$

$$e^{\frac{7\left(\frac{1-3}{1-0}\right)}{1}} = e^{-14}$$

5. Let $y = x^4$. Then $\triangle y = px^3(\triangle x) + qx^2(\triangle x)^2 + rx(\triangle x)^3 + s(\triangle x)^4$, for some real numbers p, q, r, s.

a. (5 pts) Compute p, q, r and s.

$$\Delta y = (x+\Delta x)^4 - x^4$$

$$= \cancel{x^4} + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - \cancel{x^4}$$

$$p = 4 \qquad q = 6 \qquad r = 4 \qquad s = 1$$

b. (5 pts) Assuming $\triangle x \neq 0$, compute $\frac{\triangle y}{\triangle x}$.

$$\| \Delta x \neq 0$$

$$4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3$$

c. (5 pts) Compute $\lim_{\triangle x \to 0} \frac{\triangle y}{\triangle x}$.

$$\|$$

$$4x^3$$