MATH 1271 Spring 2014, Midterm #1
Handout date: Thursday 27 February 2014
Instructor: Scot Adams

PRINT YOUR NAME:
SOLUTIONS
Version B

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I. Multiple choice

A. (5 pts) (no partial credit) Let $f(t) = \cot^2 t$. Compute $f'(\pi/4)$.
(Hint: $f(t) = (\cot\ t)(\cot\ t)$.) Circle one of the following answers:

(a) 1

(b) $-\sqrt{2}/2$

(c) -4

(d) -1

(e) NONE OF THE ABOVE

$$f'(t) = (-\csc^2 t)(\cot t) + (\cot t)(-\csc^2 t)$$
$$= -2(\cot t)(\csc^2 t)$$
$$f'(\pi/4) = -2(1)\left(\frac{1}{1/2}\right) = -4$$

B. (5 pts) (no partial credit) Compute $\underbrace{[d/dx][2e^3 + 5\sin x]}_{\parallel}$. Circle one of the following answers:

$0 + 5\cos x$

(a) $6e^2 + 5\cos x$

(b) $6e^3 + 5\cos x$

(c) $-5\cos x$

(d) $5\cos x$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x\to-\infty}(f(x)) = \infty$? Circle one of the following answers:

(a) If x is very negative, then $f(x)$ is very positive.

(b) If x is very positive, then $f(x)$ is very negative.

(c) If $f(x)$ is very positive, then x is very negative.

(d) If $f(x)$ is very negative, then x is very positive.

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Compute $\left[\frac{d}{dx}\right]\left[\frac{e^x}{x^4 - 8x}\right]$. Circle one of the following answers:

$$= \frac{(x^4-8x)(e^x)-(e^x)(4x^3-8)}{(x^4-8x)^2}$$

(a) $\dfrac{(x^4 - 8x)(e^x) - (e^x)(4x^3 - 8)}{\sqrt{x^4 - 8x}}$

(b) $\dfrac{(x^4 - 8x)(e^x) - (e^x)(4x^3 - 8)}{x^4 - 8x}$

(c) $\dfrac{e^x}{4x^3 - 8}$

(d) $\dfrac{xe^{x-1}}{4x^3 - 8}$

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Compute $\triangle(x^3 - x^2)$. Circle one of the following answers:

(a) $3x^2(\triangle x) + 3x(\triangle x)^2 + (\triangle x)^3 - 2x(\triangle x) - (\triangle x)^2$

(b) $3x^2 - 2x$

(c) $(3x^2 - 2x)(\triangle x)$

(d) $3x^2 + 3x(\triangle x) + (\triangle x)^2 - 2x - (\triangle x)$

(e) NONE OF THE ABOVE

1
1 1
1 2 1
1 3 3 1

$\Delta(x^3) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$

$\Delta(x^2) = 2x(\Delta x) + (\Delta x)^2$

F. (5 pts) (no partial credit) Let $g(x) = [8 - 3x]\left[\frac{x-5}{x-5}\right]$. What is the largest $\delta > 0$ such that $0 < |x - 5| < \delta \quad \Rightarrow \quad |(g(x)) + 7| < 0.6$? Circle one of the following answers:

(a) 0.3

(b) -0.3

(c) 0.2

(d) 1.8

(e) NONE OF THE ABOVE

$\frac{\pm 0.6}{-3} = \mp 0.2$

$\delta = 0.2$

II. True or false (no partial credit):

a. (5 pts) If P is any polynomial of degree 3 and Q is any polynomial of degree 2, then $\lim_{x\to-\infty}\left[\frac{P(x)}{Q(x)}\right]=-\infty.$

$\frac{-x^3}{x^2} = -x \xrightarrow[x\to-\infty]{} \infty$

False

b. (5 pts) $\lim_{x\to 0}\frac{1-\cos x}{x}=0.$

$\frac{1-(1-x^2/2!+\cdots)}{x} \underset{x\to 0}{\sim} \frac{x^2/2}{x} \overset{x\neq 0}{=} \frac{x}{2} \xrightarrow{x\to 0} 0$

True

c. (5 pts) If f and g are both differentiable at 3, then f^2g-f is also differentiable at 3.

True

d. (5 pts) Let f and g be any two functions such that $f'(4)=10$ and $g'(4)=20$. Then $(f+g)'(4)=30$.

True

e. (5 pts) $\frac{d}{dx}\left[\frac{\sin x}{e}\right]=\frac{\cos x}{e}.$

True

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3ab

III. 4abc

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute $\dfrac{d}{dx}\left[\dfrac{(2x^2+8x)(\csc x)}{5+e^x}\right]$.

$$\frac{[5+e^x]\left[(4x+8)(\csc x)+(2x^2+8x)(-(\csc x)(\cot x))\right]-\left[(2x^2+8x)(\csc x)\right][e^x]}{[5+e^x]^2}$$

2. (10 pts) Compute $\lim\limits_{x\to 0}\left[\dfrac{(\sin(3x))(\cos(2x))(3x^5-4x^4-2x^2)}{x(\sec(-x))(\tan^2 x)}\right]$.

$$\wr \; x \to 0$$

$$\frac{(3x)\,(1)\,(-2x^2)}{x\left(\frac{1}{1}\right)\left(\frac{x}{1}\right)^2}$$

$$\|$$

$$\frac{-6x^3}{x^3}$$

$$\| \; x \neq 0$$

$$-6$$

$$\downarrow \; x \to 0$$

$$-6$$

3. Let $f(x) = -3x^5 + 5x^3 + 2e^7$.

a. (5 pts) Find all $a \in \mathbb{R}$ such that the graph of f has a horizontal tangent line at $(a, f(a))$.

$$f'(x) = -15x^4 + 15x^2 + 0$$
$$= -15x^2(x^2 - 1)$$
$$= -15x^2(x+1)(x-1)$$

$(a = 0)$ OR $(a = -1)$ OR $(a = 1)$

b. (5 pts) Find all the maximal intervals on which f' is negative.

f' neg 0 pos 0^2 pos 0 neg

-1 $\quad$ 0 $\quad$ 1

f' is negative on $(-\infty, -1)$ and on $(1, \infty)$.

4. Let $y = 2x^3 - x$. Then $\Delta y = ax^2(\Delta x) + bx(\Delta x)^2 + c(\Delta x)^3 + k(\Delta x)$, for some real numbers a, b, c, k.

a. (5 pts) Compute a, b, c and k.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$\Delta(x^3) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$$

$$\Delta y = 6x^2(\Delta x) + 6x(\Delta x)^2 + 2(\Delta x)^3 - \Delta x$$

$$a = 6 \qquad b = 6 \qquad c = 2 \qquad d = -1$$

b. (5 pts) Assuming $\Delta x \neq 0$, compute $\underbrace{\dfrac{\Delta y}{\Delta x}}_{\|\,\Delta x \neq 0}$.

$$6x^2 + 6x(\Delta x) + 2(\Delta x)^2 - 1$$

c. (5 pts) Compute $\underbrace{\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}}_{\|}$.

$$6x^2 + 0 + 0 - 1 = 6x^2 - 1$$