MATH 1271 Spring 2014, Midterm #1
Handout date: Thursday 27 February 2014
Instructor: Scot Adams

PRINT YOUR NAME:

SOLUTIONS
Version C

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I. Multiple choice

A. (5 pts) (no partial credit) Compute $\underbrace{[d/dx][2e^3 + 5\sin x]}_{\substack{\| \\ 0 + 5\cos x}}$. Circle one of the following answers:

(a) $-5\cos x$

(b) $6e^2 + 5\cos x$

(c) $6e^3 + 5\cos x$

(d) $2e^3 + 5\cos x$

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $\Delta(x^3 - x^2)$. Circle one of the following answers:

(a) $3x^2 + 3x(\Delta x) + (\Delta x)^2 - 2x - (\Delta x)$

(b) $(3x^2 - 2x)(\Delta x)$

(c) $3x^2 - 2x$

(d) $3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2x(\Delta x) - (\Delta x)^2$

(e) NONE OF THE ABOVE

1

1 1

1 2 1

1 3 3 1

$\Delta(x^3) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$

$\Delta(x^2) = 2x(\Delta x) + (\Delta x)^2$

C. (5 pts) (no partial credit) Compute $\left[\frac{d}{dx}\right]\left[\frac{e^x}{x^4 - 8x}\right]$. Circle one of the following answers:

(a) $\dfrac{(e^x)(4x^3 - 8) - (x^4 - 8x)(e^x)}{(x^4 - 8x)^2}$

(b) $\dfrac{(x^4 - 8x)(e^x) - (e^x)(4x^3 - 8)}{(x^4 - 8x)^2}$

(c) $\dfrac{xe^{x-1}}{4x^3 - 8}$

(d) $\dfrac{e^x}{4x^3 - 8}$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Let $g(x) = [8 - 3x]\left[\frac{x-5}{x-5}\right]$. What is the largest $\delta > 0$ such that $0 < |x - 5| < \delta \quad \Rightarrow \quad |(g(x)) + 7| < 0.6$? Circle one of the following answers:

(a) 0.2

$\frac{\pm 0.6}{-3} = \mp 0.2$

(b) -0.2

(c) 1.8

$\delta = 0.2$

(d) -0.3

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x \to \infty} (f(x)) = -\infty$? Circle one of the following answers:

(a) If $f(x)$ is very negative, then x is very positive.

(b) If $f(x)$ is very positive, then x is very negative.

(c) If x is very positive, then $f(x)$ is very negative.

(d) If x is very negative, then $f(x)$ is very positive.

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Let $f(t) = \tan^2 t$. Compute $f'(\pi/4)$.
(Hint: $f(t) = (\tan t)(\tan t)$.) Circle one of the following answers:

(a) 4

$f'(t) = (\sec^2 t)(\tan t) + (\tan t)(\sec^2 t)$

$= 2(\tan t)(\sec^2 t)$

(b) -1

(c) $-\sqrt{2}/2$

$f'(\pi/4) = 2(1)\left(\frac{1}{1/2}\right) = 4$

(d) 1

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) If f and g are both differentiable at 3, then $f^3g + 2f$ is also differentiable at 3.

True

b. (5 pts) Let f and g be any two functions such that $f'(4) = 10$ and $g'(4) = 20$. Then $(f+g)'(4) = 30$.

True

c. (5 pts) If P is any polynomial of degree 4 and Q is any polynomial of degree 3, then $\lim_{x\to\infty} \left[\frac{P(x)}{Q(x)}\right] = \infty$.

$$\frac{-x^4}{x^3} = -x \xrightarrow{x\to\infty} -\infty$$

False

d. (5 pts) $\frac{d}{dx}\left[\frac{\sin x}{x^2}\right] = \frac{\cos x}{2x}$.

False

e. (5 pts) $\lim_{x\to 0} \frac{1-\sin x}{x} = 0$.

"$\frac{1}{0^{\pm}} = \pm\infty$"

False

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3ab

III. 4abc

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute $\dfrac{d}{dx}\left[\dfrac{(3x^2+4)(\cot x)}{2+e^x}\right]$.

$$= \frac{[2+e^x][(6x)(\cot x)+(3x^2+4)(-\csc^2 x)]-[(3x^2+4)(\cot x)][e^x]}{[2+e^x]^2}$$

2. (10 pts) Compute $\lim_{x \to 0} \left[\frac{(\sin(3x))(e^{4x})(x^5 - x^4 + 5x^2)}{(\sin(5x))(\tan^2 x)} \right]$.

$$\wr\ x \to 0$$

$$\frac{(3x)(1)(5x^2)}{(5x)(x^2)}$$

$$\|$$

$$\frac{15x^3}{5x^3}$$

$$\|\ x \neq 0$$

$$3$$

$$\Big\downarrow\ x \to 0$$

$$3$$

3. Let $f(x) = x^6 - 6x^4 - e^{-3}$.

a. (5 pts) Find all $a \in \mathbb{R}$ such that the graph of f has a horizontal tangent line at $(a, f(a))$.

$$f'(x) = 6x^5 - 24x^3 - 0$$
$$= 6x^3(x^2 - 4)$$
$$= 6x^3(x+2)(x-2)$$

$(a = 0)$ OR $(a = -2)$ OR $(a = 2)$

b. (5 pts) Find all the maximal intervals on which f' is negative.

f'	neg	0	pos	0^3	neg	0	pos
		-2		0		2	

f' is negative on $(-\infty, -2)$
and on $(0, 2)$.

4. Let $y = -2x^3 + 2x$. Then $\Delta y = ax^2(\Delta x) + bx(\Delta x)^2 + c(\Delta x)^3 + k(\Delta x)$, for some real numbers a, b, c, k.

a. (5 pts) Compute a, b, c and k.

1
1 1
1 2 1
1 3 3 1

$$\Delta(x^3) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$$

$$\Delta y = -6x^2(\Delta x) - 6x(\Delta x)^2 - 2(\Delta x)^3 + 2(\Delta x)$$

$$a = -6 \qquad b = -6 \qquad c = -2 \qquad k = 2$$

b. (5 pts) Assuming $\Delta x \neq 0$, compute $\dfrac{\Delta y}{\Delta x}$.

$\| \; \Delta x \neq 0$

$$-6x^2 - 6x(\Delta x) - 2(\Delta x)^2 + 2$$

c. (5 pts) Compute $\displaystyle\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$.

$\|$

$$-6x^2 - 0 - 0 + 2 = -6x^2 + 2$$