MATH 1271 Spring 2014, Midterm #2
Handout date: Thursday 17 April 2014
Instructor: Scot Adams

PRINT YOUR NAME:
SOLUTIONS
Version C

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I. Multiple choice

A. (5 pts) (no partial credit) Let $y = x^2 + x$. Compute dy, evaluated at $x = 10$, $dx = 0.1$. Circle one of the following answers:

$$\| \quad (2x+1)\,dx$$

(a) 1.22

(b) 1.2

(c) 2.11

(d) 2.1

(e) NONE OF THE ABOVE

$$(2\cdot 10+1)(0.1) = (21)(0.1) = 2.1$$

B. (5 pts) (no partial credit) Find the derivative of $(2+x^4)^{\sin x}$ w.r.t. x. Circle one of the following answers:

(a) $[(2+x^4)^{\sin x}][(\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))]$

(b) $[(2+x^4)^{\sin x}][(\cos x)(\ln(2+x^4)) + (\sin x)(4x^3/(2+x^4))]$

(c) $[(2+x^4)^{\sin x}][(\sin x)(\ln(2+x^4))]$

(d) $[(2+x^4)^{\sin x}][(\cos x)(4x^3/(2+x^4))]$

(e) NONE OF THE ABOVE

LD: $\left[(2+x^4)^{\sin x}\right]\left[\frac{d}{dx}\left[(\sin x)(\ln(2+x^4))\right]\right]$

C. (5 pts) (no partial credit) Suppose $f''(x) = (x-7)^3(x-8)^4$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is concave down on $(-\infty, 7]$ and up on $[7, \infty)]$.

(b) f is concave up on $(-\infty, 7]$ and down on $[7, \infty)]$.

(c) f is concave up on $(-\infty, 7]$, down on $[7, 8]$ and up on $[8, \infty)$.

(d) f is concave down on $(-\infty, 7]$, up on $[7, 8]$ and down on $[8, \infty)$.

(e) NONE OF THE ABOVE

f'' neg 0^3 pos 0^4 pos

7 8

$= 2e^{-1} + 2e^{5}$

D. (5 pts) (no partial credit) Let $f(x) = e^{3x-4}$. Recall that $L_2 S_1^5 f$ denotes the left endpoint Riemann sum, from 1 to 5, of f, with two subintervals. Which of these is equal to $L_2 S_1^5 f$? Circle one of the following answers:

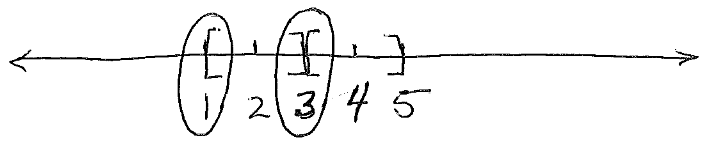
(a) $e^5 + e^{11}$

(b) $e^2 + e^8$

(c) $2(e^5 + e^{11})$

(d) $2(e^2 + e^8)$

(e) NONE OF THE ABOVE



$f(1) = e^{3-4} = e^{-1}$

$f(3) = e^{9-4} = e^{5}$

E. (5 pts) (no partial credit) Let $f(x) = e^{2x} + 3x$. What is the iterative formula of Newton's method used to solve $f(x) = 0$? Circle one of the following answers:

(a) $x_{n+1} = x_n - \dfrac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3}$

(b) $x_{n+1} = x_n - \dfrac{2e^{2x_n} + 3}{e^{2x_n} + 3x_n}$

(c) $x_{n+1} = x_n - \dfrac{e^{2x_n} + 3x_n}{e^{2x_n} + 3}$

(d) $x_{n+1} = x_n - \dfrac{e^{2x_n} + 3}{e^{2x_n} + 3x_n}$

(e) NONE OF THE ABOVE

$$x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3}$$

F. (5 pts) (no partial credit) Let $f(x) = \tan^2(5x^4 + 1)$. Compute $\int_{-1}^{-1} f(x)\,dx$. Circle one of the following answers:

(a) -1

(b) $-\sqrt{2}/2$

(c) 0

(d) 20

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

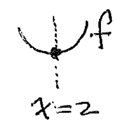
a. (5 pts) Let $f, g : \mathbb{R} \to \mathbb{R}$ be any two differentiable functions such that, for all $x \in \mathbb{R}$, $f'(x) = g'(x)$. Then $f = g$.

$f(x) = 1, \quad g(x) = 2$

False

b. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be any function such that $f'(2) = 0$ and $f''(2) > 0$. Assume that f'' is defined on $\mathbb{R}$. Then f has a local minimum at 2.

(2nd derivative test)



True

c. (5 pts) Assume that $\lim_{x\to a}[f(x)] = 1 = \lim_{x\to a}[g(x)]$. Assume also that $\lim_{x\to a} \dfrac{f'(x)}{g'(x)} = 3$. Then $\underbrace{\lim_{x\to a} \dfrac{f(x)}{g(x)}}_{= \frac{1}{1} = 1} = 3$.

False

d. (5 pts) If f is continuous on $[a, b]$, then $\displaystyle\int_a^b (f(x))\,dx = \lim_{n\to\infty}[L_n S_a^b f]$.

Definition of $\int_a^b (f(x))\,dx$

True

e. (5 pts) $\underbrace{\dfrac{d}{dx}\left[\displaystyle\int_1^x \sin(e^t)\,dt\right]}_{= \sin(e^x)} = \cos(e^x)$.

FTC

False

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1.

III. 2.

III. 3.

III. 4.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find an antiderivative w.r.t. x of $\cos^2(x-3)$. (Hint: $\cos(2\theta) = -1 + 2(\cos^2\theta)$.)

$$1 + [\cos(2\theta)] = 2(\cos^2\theta)$$

$$\frac{1}{2} + \frac{1}{2}[\cos(2\theta)] = \cos^2\theta$$

$$\theta :\longrightarrow x-3$$

$$\frac{1}{2} + \frac{1}{2}[\cos(2x-6)] = \cos^2(x-3)$$

Antiderivative:

$$\frac{1}{2}x + \frac{1}{2}\left[\frac{\sin(2x-6)}{2}\right]$$

2. (10 pts) Let $f(x) = \int_x^{x^4} \underbrace{\sqrt{2^t + 8t^2 + 6}}_{G'(t)}\, dt$. Compute $f'(1)$.

$$f(x) \overset{FTC}{=\!=} \left(G(x^4)\right) - \left(G(x)\right)$$

$$f'(x) \overset{CR}{=\!=} \left(G'(x^4)\right)\left(4x^3\right) - \left(G'(x)\right)$$

$$f'(1) = \left(G'(1)\right)(4) - \left(G'(1)\right)$$

$$= \left(G'(1)\right)(4-1) = \left(G'(1)\right)(3)$$

$$= \left(\sqrt{2+8+6}\right)(3) = \left(\sqrt{16}\right)(3)$$

$$= 12$$

3. (15 pts) We are asked to design a large cup in the shape of an inverted (*i.e.*, upside down) cone. The cup is to have an open top, and must contain $\pi/3$ cubic feet of volume inside. Let r be the radius of the top of the cup. On the interval $r > 0$, find the choice of r (in feet) that minimizes the surface area, A, of the cup. (HINT: Our local precalculus expert shows us the formula that relates A to r. It is $A = (\pi r)\sqrt{r^2 + r^{-4}}$.)

$$w := r^2 + r^{-4}$$

$$\frac{dA}{dr} = \pi\sqrt{w} + (\pi r)\left(\frac{2r - 4r^{-5}}{2\sqrt{w}}\right)$$

$$= \pi\left(\frac{w}{\sqrt{w}}\right) + (\pi r)\left(\frac{r - 2r^{-5}}{\sqrt{w}}\right)$$

$$= \pi\left(\frac{r^2 + r^{-4}}{\sqrt{w}}\right) + \pi\left(\frac{r^2 - 2r^{-4}}{\sqrt{w}}\right)$$

$$= \pi\left(\frac{2r^2 - r^{-4}}{\sqrt{w}}\right)\frac{r^4}{r^4} = \pi\left(\frac{2r^6 - 1}{r^4\sqrt{w}}\right)$$

dA/dr	(	neg	0	pos
r	0		$\sqrt[6]{\frac{1}{2}}$	

On $r > 0$, A attains a global minimum only at $r = \sqrt[6]{\frac{1}{2}}$ ft.

4. (10 pts) A square-based pyramid is growing. Its height is always equal to the length, s, of the sides of its base. Assume that its volume is always growing at a rate of 10 cubic feet per minute. Find the rate of growth in s (in feet per minute) at the moment when the volume is 9 cubic feet. (HINT: According to our local precalculus expert, its volume, V, is given by $V = s^3/3$.)

t_0

$* := [t :\to t_0]$

$? := \dot{s}_*$

$$9 = V_* = s_*^3/3$$

$$27 = s_*^3$$

$$3 = s_*$$

$$10 = \dot{V} = \cancel{3} s^2 \dot{s} / \cancel{3} = s^2 \dot{s}$$

$$10 = s_*^2 \dot{s}_* = (9)(?)$$

$$? = \frac{10}{9} \text{ ft/min}$$