**WARNING:** This file has not been double checked. If you notice any typographical errors, please let me know. Thanks. - Scot

# **BEGIN TOPIC:** precalculus

e. (S11 Midterm 01, Topic precalculus) For any real number x,  $x^2 \ge 0$ .

ANSWER: True

**TOPIC:** precalculus

c. (S11 Midterm 01, Topic precalculus) For any  $\alpha, \beta \in \mathbb{R}$ ,  $\sin(\alpha+\beta) = (\sin(\alpha)) + (\sin(\beta))$ .

ANSWER: False

**TOPIC:** precalculus

6. (S11 Midterm 01, Topic precalculus) (no partial credit) Solve the equation  $3x + 4 = \frac{2x + 7}{5}$  for x. Circle one of the following answers:

- (a) x = 0
- (b) x = 1
- (c) x = 2
- (d) x = 3
- (e) NONE OF THE ABOVE

**ANSWER:** (e), x = -1

**TOPIC:** precalculus

1. (S11 Midterm 01, Topic precalculus) (no partial credit) Compute  $\frac{x}{y} + \frac{2}{z^3}$ . Circle one of the following answers:

1

- (a)  $\frac{xz^3}{2+y}$
- (b)  $\frac{x+2}{y+z^3}$
- (c)  $\frac{x+2}{yz^3}$
- $(d) \frac{xz^3 + 2y}{yz^3}$
- (e) NONE OF THE ABOVE

# **END TOPIC:** precalculus

# **BEGIN TOPIC: 0010**

a. (S11 Midterm 01, Topic 0010) The interval  $[-1, \infty)$  is closed.

ANSWER: True

**TOPIC:** 0010 (5-20)

- 3. (S11 Midterm 01, Topic 0010) (no partial credit) Which of the following intervals does NOT contain an integer? Only one answer is correct; circle it.
  - (a) [1, 100]
  - (b) (1,100)
  - (c) [1,2)
  - (d) (1,2)
  - (e) (1,3)

ANSWER: (d)

**TOPIC:** 0010

## END TOPIC: 0010

**BEGIN TOPIC: 0020** 

END TOPIC: 0020

#### **BEGIN TOPIC: 0030**

9. (S11 Midterm 01, Topic 0030) (no partial credit) Let  $P(x) = (x^3 + 2x^2 + x + 1)(2x^4 + 5x^2 + 2x - 3)(x^6 + x + 8)$ . What is the degree of the polynomial P? (NOTE: You needn't write out P(x); all we need is its degree.)

**ANSWER:** 3 + 4 + 6 = 13

**TOPIC:** 0030

b. (S11 Midterm 01, Topic 0030) The difference of two polynomials is always a polynomial.

ANSWER: True

**TOPIC:** 0030

# END TOPIC: 0030

# **BEGIN TOPIC: 0040**

4. (S13 Midterm 01A, Topic 0040(2-3)) Let  $f(x) = (x+2)^5(x-1)^6(x-3)^7$ . Find all of the maximum intervals of positivity and negativity for f.

**ANSWER:** f is positive on  $(-\infty, -2)$ , negative on (-2, 1), negative on (1, 3), positive on  $(3, \infty)$  **TOPIC:** 0040(2-3)

4. (F12 Midterm 01A, Topic 0040(2-4)) Let  $f(x) = (x+1)^3(x-2)^4(x-5)$ . Find all of the maximum intervals of positivity and negativity for f.

**ANSWER:** positive on  $(-\infty, -1)$ , negative on (-1, 2), negative on (2, 5), positive on  $(5, \infty)$  **TOPIC:** 0040 (2-4)

4. (S11 Midterm 01, Topic 0040) (no partial credit) Find all real solutions of  $x^2+3x+2=0$ . Circle one of the following answers:

(a) 
$$\frac{-3 \pm \sqrt{12 - 4}}{2}$$

- (b) 1 and 2
- (c) -1 and -2
- (d) There are no real solutions.
- (e) NONE OF THE ABOVE

ANSWER: (c)

**TOPIC:** 0040

2. (S11 Midterm 01, Topic 0040) (no partial credit) Let P(x) = (x-1)(x-2). Which of the following is true? (Only one answer is correct; circle it.)

- (a) P is positive on the interval (1, 2).
- (b) P is negative on the interval (1, 2).
- (c)  $P(10^6)$  is negative.
- (d) P is equal to zero on the interval  $(2, \infty)$ .
- (e) P is negative on the interval  $(-\infty, 1)$ .

ANSWER: (b)

**TOPIC:** 0040(2)

### END TOPIC: 0040

#### **BEGIN TOPIC: 0050**

B. (F13 Midterm 01B, Topic 0050(7)) (no partial credit) What is the smallest number x such that  $|x+3| \le 0.002$ ? Circle one of the following answers:

- (a) -2.998
- (b) 3
- (c) -3.002
- (d) 2.998
- (e) NONE OF THE ABOVE

**ANSWER:** (c)

**TOPIC:** 0050(7)

E. (F13 Midterm 01A, Topic 0050(7)) (no partial credit) What is the largest number x such that  $|x + 3| \le 0.002$ ? Circle one of the following answers:

- (a) 3
- (b) -2.998
- (c) 3.002
- (d) 2.998
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0050(7)

E. (S13 Midterm 01A, Topic 0050(7)) (no partial credit) What is the largest number x such that  $|x-3| \le 0.005$ ?

- (a) 2.995
- (b) 3
- (c) 3.005
- (d) -2.995
- (e) NONE OF THE ABOVE

**ANSWER:** (c)

**TOPIC:** 0050(7)

a. (F12 Midterm 01A, Topic 0050) For every  $x < 0, \sqrt{x^4} = -x^2$ .

**ANSWER:** False

**TOPIC:** 0050

d. (S11 Midterm 02, Topic 0050) If |x-2| < 0.0001, then x < 100.

ANSWER: True

**TOPIC:** 0050

# **END TOPIC: 0050**

### **BEGIN TOPIC: 0060**

8. (S11 Midterm 01, Topic 0060) Find an expression L(x) such that y = L(x) is the equation for the line through (2,5) and (3,2). (Note: For full credit, the left hand side should be y alone, and  $not \ y-5$  or y-2. The form of the equation needs to be y=L(x).)

**ANSWER:** 
$$L(x) = \left[\frac{2-5}{3-2}\right][x-2] + 5$$

**TOPIC:** 0060 (31-36)

TO-

## END TOPIC: 0060

#### **BEGIN TOPIC: 0070**

## END TOPIC: 0070

## **BEGIN TOPIC: 0080**

0. (never used, Topic 0080) Compute  $\sum_{j=1}^{3} \left[ \frac{1}{3} \cdot \left( 1 + \frac{j}{3} \right)^2 \right]$ . Write your answer as a fraction a/b, where a and b are integers.

**ANSWER:** 
$$(1/3)((4/3)^2+(5/3)^2+(6/3)^2)=(1/3)(16+25+36)/9=77/27$$

**PIC:** 0090 (27)

## END TOPIC: 0080

#### **BEGIN TOPIC: 0090**

d. (S11 Midterm 01, Topic 0090)  $1 + \tan^2(\alpha) = \sec^2(\alpha)$ .

ANSWER: True	<b>TOPIC:</b> 0090 (27)	
END TOPIC: 0090  BEGIN TOPIC: 0100		
	BEGIN TOPIC: 0110	
b. (S13 Midterm 01A, Top one-to-one function.	oic 0110(8)) Let $f$ be the restriction of sin to $[0, \pi]$ . Then $f$ is a	
ANSWER: False	<b>TOPIC:</b> 0110(8)	
b. (F12 Midterm 01A, Top	pic 0110) Let $f(x) = x^3$ . Then $f$ is a one-to-one function.	
ANSWER: True	<b>TOPIC:</b> 00110	
e. (S11 Midterm 02, Topic	0110) The domain of arcsin is $[-\pi/2, \pi/2]$ .	
ANSWER: False	<b>TOPIC:</b> 0110 (21)	
c. (S10 Midterm 01, Topic	0110) Let $f(x) = \sin x$ . Then $f$ is a one-to-one function.	
ANSWER: False	<b>TOPIC:</b> 0110	
c. (S09 Midterm 01, Topic one-to-one function.	c 0110) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$ . Then $f$ is a	
ANSWER: False	<b>TOPIC:</b> 0110	
	END TOPIC: 0110	
	BEGIN TOPIC: 0120	

# **BEGIN TOPIC: 0130**

**END TOPIC: 0120** 

D. (S13 Midterm 01A, Topic 0130(11)) (no partial credit) (no partial credit) A line passes through (1,40) and (3,80). Find its slope. Circle one of the following answers:

- (a) 5
- (b) 10
- (c) 15
- (d) 20
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0130(11)

4. (S12 Midterm 01A, Topic 0130) On the planet of Gallifrey, in an alternate universe, a dropped object travels  $t^3 + t^2$  feet during its first t seconds of free fall.

a. For  $h \neq 0$ , the average velocity between time t = 2 seconds and time t = 2 + h seconds is given by a quadratic polynomial in h of the form  $ah^2 + bh + c$ . Find the coefficients a, b and c.

**ANSWER:** a = 1, b = 7, c = 16 **TOPIC:** 0130 (11)

b. Find the instantaneous velocity at time t=2 seconds.

**ANSWER:** 16 ft/sec **TOPIC:** 0130 (11)

C. (S12 Midterm 01A, Topic 0130) (no partial credit) A line passes through (1,40) and (3,80). Find its slope. Circle one of the following answers:

- (a) 10
- (b) 20
- (c) 30
- (d) 40
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0130 (11)

B. (S12 Midterm 01A, Topic 0130) (no partial credit) A particle travels along a number line. Its position at time 1 is 40 and its position at time 3 is 80. Find its average velocity between time 1 and time 3. Circle one of the following answers:

- (a) 10
- (b) 20

- (c) 30
- (d) 40
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0130 (11)

4. (F11 Midterm 01, Topic 0130) On the planet of Gallifrey, in an alternate universe, a dropped object travels  $t^3$  feet during its first t seconds of free fall.

a. For  $h \neq 0$ , the average velocity between time t = 2 seconds and time t = 2 + h seconds is given by a quadratic polynomial in h of the form  $ah^2 + bh + c$ . Find the coefficients a, b and c.

**ANSWER:**  $a = 1, b = 3 \cdot 2 = 6, c = 3 \cdot 2^2 = 3 \cdot 4 = 12$ 

**TOPIC:** 0130

b. Find the instantaneous velocity at time t=2 seconds.

**ANSWER:**  $3 \cdot 2^2 = 3 \cdot 4 = 12$ 

**TOPIC:** 0130

C. (S11 Midterm 02, Topic 0130) (no partial credit) On the graph of  $y = -2x^2$ , find the slope of the secant line between (2, -8) and  $(2 + h, -2(2 + h)^2)$ . Your formula should be equal (for all  $h \neq 0$ ) to exactly one of the following answers. Circle that answer.

- (a) -8 2h
- (b)  $-2(2+h)^2 (-8)$
- (c) h
- (d)  $\frac{-2(4+4h+h^2)}{h}$
- (e) -8

ANSWER: (a)

**TOPIC:** 0130

END TOPIC: 0130

**BEGIN TOPIC: 0140** 

d. (S13 Midterm 01A, Topic 0140(3-4)) Let f be any function. If  $\lim_{x\to 3} f(x)$  exists, then 3 is in the domain of f.

ANSWER: False

**TOPIC:** 0140(3-4)

c. (S12 Midterm 01A, Topic 0140) A tangent line to the graph of a function cannot intersect the graph of the function more than once.

**ANSWER:** False **TOPIC:** 0140 (12)

e. (S10 Midterm 01, Topic 0140) A tangent line to the graph of a function can intersect the graph of the function no more than two times.

**ANSWER:** False **TOPIC:** 0140 (12)

### END TOPIC: 0140

## **BEGIN TOPIC: 0150**

E. (F14 Midterm 01A, Topic 0150) (no partial credit) Let  $g(x) = [8-3x] \left[\frac{x-5}{x-5}\right]$ . What is the largest  $\delta > 0$  such that  $0 < |x-5| < \delta \implies |(g(x)) + 7| < 0.6$ ? Circle one of the following answers:

- (a) 0.3
- (b) -0.3
- (c) 1.8
- (d) -0.2
- (e) NONE OF THE ABOVE

**ANSWER:** (e), 0.2

**TOPIC:** 0150

E. (F14 Midterm 01C, Topic 0150(23)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to\infty} (f(x)) = -\infty$ ? Circle one of the following answers:

- (a) If f(x) is very negative, then x is very positive.
- (b) If f(x) is very positive, then x is very negative.
- (c) If x is very positive, then f(x) is very negative.
- (d) If x is very negative, then f(x) is very positive.
- (e) NONE OF THE ABOVE

**ANSWER:** (c) **TOPIC:** 0150(23)

A. (F14 Midterm 01A, Topic 0150(21)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to-\infty} (f(x)) = \infty$ ? Circle one of the following answers:

- (a) If f(x) is very positive, then x is very negative.
- (b) If f(x) is very negative, then x is very positive.
- (c) If x is very positive, then f(x) is very negative.
- (d) If x is very negative, then f(x) is very positive.
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0150(21)

C. (F13 Midterm 01D, Topic 0150(13)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to 8^+} (H(x)) = \infty$ ? Circle one of the following answers:

- (a) If H(x) is very positive, then x is close to 8.
- (b) If x is close to 8, then H(x) is very positive.
- (c) If x is close to 8, but greater than 8, then H(x) is very positive.
- (e) NONE OF THE ABOVE

 $\mathbf{ANSWER}$ : (c)

**TOPIC:** 0150(13)

C. (F13 Midterm 01B, Topic 0150(10)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to 8} (H(x)) = 4$ ? Circle one of the following answers:

- (a) If x is close to 8, but not equal to 8, then H(x) is close to 4.
- (b) If x is close to 8, then H(x) is close to 4.
- (c) If x is close to 8, but not equal to 8, then H(x) is close to 4, but not equal to 4.
- (d) If x is close to 8, then H(x) is close to 4, but not equal to 4.
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0150(10)

A. (F13 Midterm 01A, Topic 0150(12)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to 8^+} (H(x)) = 4$ ? Circle one of the following answers:

- (a) If x is close to 8, but not equal to 8, then H(x) is close to 4, but not equal to 4.
- (b) If H(x) is close to 8, then x is close to 4.
- (c) If H(x) is close to 4, then x is close to 8, but greater than 8.
- (d) If x is close to 8, but greater than 8, then H(x) is close to 4.
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0150(12)

A. (S13 Midterm 01A, Topic 0150(11)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to 4^-} (h(x)) = 7$ ? Circle one of the following answers:

- (a) If x is close to 4, but not equal to 4, then h(x) is close to 7, but not equal to 7.
- (b) If x is close to 4, but less than 4, then h(x) is close to 7.
- (c) If h(x) is close to 7, but not equal to 7, then x is close to 4, but less than 4.
- (d) If h(x) is close to 4, then x is close to 7.
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0150(11)

C. (F12 Midterm 01A, Topic 0150(10)) (no partial credit) Which is the intuitive definition of  $\lim_{x\to 3} (g(x)) = 8$ ? Circle one of the following answers:

- (a) If g(x) is close to 3, then x is close to 8.
- (b) If x is close to 3, but not equal to 3, then g(x) is close to 8, but not equal to 8.
- (c) If g(x) is close to 8, but not equal to 8, then x is close to 3.
- (d) If x is close to 3, but not equal to 3, then g(x) is close to 8.
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0150(10)

F. (S12 Midterm 01A, Topic 0150) (no partial credit) Compute the largest  $\delta > 0$  such that:  $0 < |x - 1| < \delta$  implies |(2x + 7) - 9| < 0.05. Circle one of the following answers:

- (a) 0.2
- (b) 0.1
- (c) 0.025
- (d) 0.01
- (e) NONE OF THE ABOVE

ANSWER: (c)

**TOPIC:** 0150 (10)

a. (F11 Midterm 01, Topic 0150) If  $\lim_{x\to a} f(x) = \infty$ , then  $\lim_{x\to a^-} f(x) = \infty$ .

ANSWER: True

**TOPIC:** 0150

F. (F11 Midterm 01, Topic 0150) (no partial credit) Compute the largest  $\delta > 0$  such that:  $0 < |x-1| < \delta$  implies |(2x+4)-6| < 0.1. Circle one of the following answers:

- (a) 0.2
- (b) 0.1
- (c) 0.025
- (d) 0.01
- (e) NONE OF THE ABOVE

**ANSWER:** (e), 0.05

**TOPIC:** 0150

8. (S10 Midterm 01, Topic 0150) Give the intuitive definition of  $\lim_{x\to a^-} f(x) = \infty$ .

Do not use any verb except "is". If you do, you won't get more than half credit. In particular, if you say "approaches", or "gets close to" or "comes toward", you won't get more than half credit.

Do not use the words "left" or "right". Again, if you do, you won't get more than half credit. You may use "less than" or "greater than".

Do not use the words "large" or "small". Again, if you do, you won't get more than half credit. You may use "very positive", "very negative" and "very close to".

You may draw a graph to help you, but all graphing will be ignored in grading.

**ANSWER:** When [x is close to a, but less than a], [f(x) is very positive].

**TOPIC:** 0150

d. (S10 Midterm 01, Topic 0150) There is a function with three vertical asymptotes.

ANSWER: True TOPIC: 0150

b. (S10 Midterm 01, Topic 0150) If  $\lim_{x\to a} f(x) = L$ , then  $\lim_{x\to a^-} f(x) = L$ .

ANSWER: True TOPIC: 0150

a. (S10 Midterm 01, Topic 0150) If  $\lim_{x\to a^+} f(x) = L$ , then  $\lim_{x\to a^-} f(x) = L$ .

ANSWER: False TOPIC: 0150

8. (S10 Midterm 02, Topic 0150) Give the intuitive definition of  $\lim_{x\to-\infty} f(x) = L$ .

Do not use any verb except "is". If you do, you won't get more than half credit. In particular, if you say "approaches", or "gets close to" or "comes toward", you won't get more than half credit.

Do not use the words "left" or "right". Again, if you do, you won't get more than half credit. You may use "less than" or "greater than".

Do not use the words "large" or "small". Again, if you do, you won't get more than half credit. You may use "very positive", "very negative" and "very close to".

You may draw a graph to help you, but all graphing will be ignored in grading.

**ANSWER:** When [x is very negative], [f(x)] is close to L] **TOPIC:** 0150 (25)

END TOPIC: 0150

**BEGIN TOPIC: 0160** 

2. (F13 Midterm 01A, Topic 0160(23-26)) Compute  $\lim_{n\to\infty} \left(1 + \frac{0.045}{n}\right)^n$ .

**ANSWER:**  $e^{0.045}$  **TOPIC:** 0160(23-26)

2. (S13 Midterm 01A, Topic 0160(23-26)) Compute  $\lim_{n\to\infty} \left(1+\frac{45}{n}\right)^n$ .

**ANSWER:**  $e^{45}$  **TOPIC:** 0160(23-26)

**END TOPIC: 0160** 

**BEGIN TOPIC: 0170** 

**END TOPIC: 0170** 

**BEGIN TOPIC: 0180** 

END TOPIC: 0180

**BEGIN TOPIC: 0190** 

3. (F12 Midterm 01A, Topic 0190) Compute  $\lim_{x\to\infty} \left[ \frac{x^2 + \sin^2 x}{2x^2 + 1} \right]$ .

**ANSWER:** 1/2 **TOPIC:** 0190

A. (S12 Midterm 01A, Topic 0190) (no partial credit) Assume that  $\lim_{x\to 200}(f(x))=4$  and  $\lim_{x\to 200}(g(x))=5$ . At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

- (a)  $\lim_{x \to 200} [(f(x)) + (g(x))] = 9$
- (b)  $\lim_{x \to 400} [(f(x)) + (g(x))] = 9$
- (c)  $\lim_{x \to 1} \left[ \frac{f(x)}{g(x)} \right] = \frac{4}{5}$
- (d)  $\lim_{x\to 300} [(f(x)) + (g(x))]$  does not exist
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0190 (2)

e. (S10 Midterm 02, Topic 0190) If  $\lim_{x \to 5} f(x) = 1$  and  $\lim_{x \to 5} g(x) = \infty$ , then  $\lim_{x \to 5} \frac{f(x)}{g(x)} = 0$ .

**ANSWER:** True **TOPIC:** 0190 (20)

**END TOPIC: 0190** 

**BEGIN TOPIC: 0200** 

F. (F13 Midterm 01A, Topic 0200(22-23)) (no partial credit) Compute  $\lim_{x\to 0} \left[ \frac{x^5 + 2x^3 - 4x^2}{2x^4 - 7x^2} \right]$ . Circle one of the following answers:

- (a) 4/7
- (b) -4/7
- (c) 1/2
- (d) -1/2
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0200(22-23)

3. (S13 Midterm 01A, Topic 0200(30-34)) Compute  $\lim_{x\to\infty} \left[ \frac{2x^2 + \cos^2 x}{4x^2 + 2} \right]$ .

**ANSWER:** 1/2 **TOPIC:** 0200(30-34)

C. (S13 Midterm 01A, Topic 0200(10-18)) (no partial credit) Compute  $\lim_{t\to 3} \left[\frac{t^2+t-12}{t-3}\right]$ . Circle one of the following answers:

- (a) 5
- (b) 6
- (c) 7
- (d) 8
- (e) NONE OF THE ABOVE

**ANSWER:** (c)

**TOPIC:** 0200(10-18)

F. (F12 Midterm 01A, Topic 0200(26-29)) (no partial credit) Compute  $\lim_{h\to 0} \left[ \frac{\sqrt{9+h} - \sqrt{9+4h}}{3h} \right]$ . Circle one of the following answers:

- (a) 1/6
- (b) -1/6
- (c) 1/9

(d) This limit does not exist.

(e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0200(26-29)

D. (F12 Midterm 01A, Topic 0200(10-18)) (no partial credit) Compute  $\lim_{t\to 3} \left[\frac{t^2+t-12}{t-3}\right]$ . Circle one of the following answers:

(a) 3

(b) 4

(c) 5

(d) 6

(e) NONE OF THE ABOVE

**ANSWER:** (e), the limit is equal to 7

**TOPIC:** 0200(10-18)

1. (S12 Midterm 01A, Topic 0200) a. Compute  $\lim_{h\to 0} \frac{\sqrt{5+2h}-\sqrt{5-h}}{h}$ .

ANSWER:  $\frac{3}{2\sqrt{5}}$ 

**TOPIC:** 0200 (26-29)

b. Compute  $\lim_{h\to 0} \frac{\frac{1}{5+2h} - \frac{1}{5-h}}{h}.$ 

**ANSWER:** -3/25

**TOPIC:** 0200 (22-23)

2. (F11 Midterm 01, Topic 0200) a. Compute  $\lim_{h\to 0} \frac{\sqrt{7+h}-\sqrt{7-h}}{h}$ .

**ANSWER:**  $1/\sqrt{7}$ 

**TOPIC:** 0200

b. Compute  $\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x-h}}{h}$ .

**ANSWER:**  $1/\sqrt{x}$ 

**TOPIC:** 0200

A. (F11 Midterm 01, Topic 0200) (no partial credit) Assume that  $\lim_{x\to 100} (f(x)) = 4$  and  $\lim_{x\to 200} (g(x)) = 5$ . At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

(a) 
$$\lim_{x \to 300} [(f(x)) + (g(x))] = 9$$

(b) 
$$\lim_{x \to 4} (f(x)) = 100$$

(c) 
$$\lim_{x \to 2} \frac{f(x)}{g(x)} = 4/5$$

(d) 
$$\lim_{x\to 300} [(f(x)) + (g(x))]$$
 does not exist

(e) NONE OF THE ABOVE

**ANSWER:** (e), none of them follow

3. (S11 Midterm 02, Topic 0200) Let  $f(x) = \frac{\frac{1}{x-2} - 1}{x-3}$ .

a. Express f(x) as a quotient of two polynomials.

**ANSWER:**  $\frac{-x+3}{x^2-5x+6}$  **TOPIC:** 0030

b. Compute  $\lim_{x\to 3} f(x)$ .

**ANSWER:** -1 **TOPIC:** 0200

1. (S11 Midterm 02, Topic 0200) Compute  $\lim_{x\to 1} \frac{\sqrt{3+x}-2}{x-1}$ .

**ANSWER:** 1/4 **TOPIC:** 0200

F. (S11 Midterm 02, Topic 0200) (no partial credit) Compute  $\lim_{x\to 1^-} \frac{(x-1)^2(x-2)}{(x-1)^3}$ . Circle one of the following answers:

**TOPIC:** 0200

- (a) 0
- (b) 1
- (c) 2
- (d)  $\infty$
- (e) does not exist

**ANSWER:** (d) **TOPIC:** 0200

D. (S11 Midterm 02, Topic 0200) (no partial credit) Compute  $\lim_{h\to 0} \frac{-h(1+h)}{h}$ . Circle one of the following answers:

- (a) -1
- (b) -1/2
- (c) 1
- (d) 1/2
- (e) does not exist

ANSWER: (a)

**TOPIC:** 0200

B. (S11 Midterm 02, Topic 0200) (no partial credit) Assume that  $\lim_{x\to 100} (f(x)) = 4$  and  $\lim_{x\to 200} (g(x)) = 5$ . At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

- (a)  $\lim_{x \to 800} [(f(x)) + (g(x))] = 9$
- (b)  $\lim_{x \to 4} (f(x)) = 100$
- (c)  $\lim_{x \to 1/2} \frac{f(x)}{g(x)} = 4/5$
- (d)  $\lim_{x\to 300}[(f(x))+(g(x))]$  does not exist
- (e) NONE OF THE ABOVE

ANSWER: (e)

**TOPIC:** 0200

A. (S11 Midterm 02, Topic 0200) (no partial credit) Assume that  $\lim_{x\to 100}(f(x))=4$  and  $\lim_{x\to 100}(g(x))=5$ . At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

- (a)  $\lim_{x \to 9} [(f(x)) + (g(x))] = 400$
- (b)  $\lim_{x \to 200} [(f(x)) + (g(x))] = 9$
- (c)  $\lim_{x \to 200} [(f(x)) + (g(x))] = 9.$
- (d)  $\lim_{x \to 100} [(f(x))(g(x))] = 20$
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0200

a. (S11 Midterm 03, Topic 0200) If f is a rational function and a is any real number, then  $\lim_{x\to a^+} f(x)$  exists; it might be  $\infty$ , it might be  $-\infty$  or it might be a finite number, but it

will exist.

**ANSWER:** True **TOPIC:** 0200 (9)

### END TOPIC: 0200

## **BEGIN TOPIC: 0210**

c. (F13 Midterm 01A, Topic 0210(16))  $\lim_{x\to 4\pi} \frac{\sin x}{x} = 1$ .

**ANSWER:** False **TOPIC:** 0210(16)

e. (F13 Midterm 01D, Topic 0210(9)) If f and g are continuous at 3, then  $f^2g^3$  MUST be continuous at 3 as well.

**ANSWER:** True **TOPIC:** 0210(9)

a. (F13 Midterm 01A, Topic 0210(9)) If f and g are continuous at 4, then f - g MUST be continuous at 4 as well.

**ANSWER:** True **TOPIC:** 0210(9)

c. (no partial credit) Let f(x) = |x|. Then f is continuous at every real number.

ANSWER: True TOPIC: 0210

d. (F11 Midterm 01, Topic 0210) Every polynomial is continuous.

ANSWER: True TOPIC: 0210

c. (F11 Midterm 01, Topic 0210) If f and g are continuous at 3, then f+g MUST be continuous at 3 as well.

ANSWER: True TOPIC: 0210

c. (S11 Midterm 02, Topic 0210) Every jump discontinuity is a removable discontinuity.

**ANSWER:** False **TOPIC:** 0210 (8)

b. (S11 Midterm 02, Topic 0210) The function  $f(x) = \sqrt{x-2}$  is continuous from the right at x=2.

ANSWER: True TOPIC: 0210

a. (S11 Midterm 02, Topic 0210) If f and g are continuous at 3, then f + g MUST be continuous at 3 as well.

ANSWER: True

**TOPIC:** 0210

d. (S09 Midterm 01, Topic 0210) If x = 3 is a vertical asymptote of the curve y = f(x), then f is not continuous at 3.

ANSWER: True

**TOPIC:** 0210

b. (S09 Midterm 01, Topic 0210) If a function f is continuous at a number a, then  $\lim_{x\to a} f(x)$  exists.

ANSWER: True

**TOPIC:** 0210

END TOPIC: 0210

**BEGIN TOPIC: 0220** 

**END TOPIC: 0220** 

**BEGIN TOPIC: 0230** 

2. (F14 Midterm 01A, Topic 0230) Compute  $\lim_{x\to 0} \left[ \frac{(\sin(2x))(\cos(3x))(3x^5 - 2x^4 - 4x^2)}{(\sec(4x))(\tan^3 x)} \right]$ .

ANSWER: -8

**TOPIC:** 0230

c. (F14 Midterm 01A, Topic 0230(19-20))  $\lim_{x\to 0} \frac{1-\cos x}{x} = 1.$ 

ANSWER: False

**TOPIC:** 0230(19-20)

c. (F13 Midterm 01D, Topic 0230(19))  $\lim_{x\to 0} \frac{\sin^2 x}{x^2} = 1$ .

ANSWER: True

**TOPIC:** 0230(19)

c. (S13 Midterm 01A, Topic 0230(19-20))  $\lim_{x\to 0} \frac{1-\cos x}{x} = 1$ .

ANSWER: False

**TOPIC:** 0230(19-20)

B. (S13 Midterm 01A, Topic 0230) (no partial credit) Compute  $\lim_{x\to 0} \left[ \frac{(3x^5 - 8x^4)(\cos x)}{4x^3(\sin x)} \right]$ . Circle one of the following answers:

(a) 3/4

- (b) -2
- (c) 0
- (d) This limit does not exist.
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0230

E. (F12 Midterm 01A, Topic 0230) (no partial credit) Compute  $\lim_{x\to 0} \left[ \frac{x^3 + 2x^2 - 4x}{\sin(8x)} \right]$  Circle one of the following answers:

- (a) 2/3
- (b) -1/2
- (c) 1/2
- (d) -2/3
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0230

A. (F12 Midterm 01A, Topic 0230) (no partial credit) Compute  $\lim_{x\to 0} \left[ \frac{3x^4 + 2x^3}{7x(\sin^2 x)} \right]$ . Circle one of the following answers:

- (a) 0
- (b)  $\infty$
- (c) 5/7
- (d) 2/7
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0230

3. (S12 Midterm 01A, Topic 0230) Compute  $\lim_{x\to 0} \left(\frac{7x^3+4x^2}{8x\sin x}\right)$ .

ANSWER: 1/2

**TOPIC:** 0230 (22)

E. (S12 Midterm 01A, Topic 0230) (no partial credit) Compute  $\lim_{x\to 0} \frac{2x^3 - 5x^2}{7x(\sin x)}$ . Circle one of the following answers:

- (a) 0
- (b)  $\infty$
- (c) 5/7
- (d) 2/7
- (e) NONE OF THE ABOVE

**ANSWER:** (e), -5/7 **TOPIC:** 0230 (22)

E. (F11 Midterm 01, Topic 0230) (no partial credit) Compute  $\lim_{x\to 0} \frac{2x^3 + 5x^2}{7x(\sin x)}$ . Circle one of the following answers:

- (a) 2/7
- (b) 5/7
- (c)  $\infty$
- (d) 0
- (e) NONE OF THE ABOVE

**ANSWER:** (b) **TOPIC:** 0230

- 2. (S11 Midterm 02, Topic 0230) A particle is moving along a number line, and its position at time t is  $2(\sin t) + 3(\cos t)$ .
- a. Find its average velocity betweeen time 0 and time  $\pi/4$ . Your final answer should not have any trigonometric functions in it. That is, if, say,  $\sin(\pi/3)$  were part of your answer, you should evaluate that value. On the other hand, you need not do any arithmetic; an answer of the form  $(2/\pi)/(1+\sqrt{5})$  would be acceptable.

**ANSWER:** 
$$2\left[\frac{\sqrt{2}/2}{\pi/4}\right] + 3\left[\frac{(\sqrt{2}/2) - 1}{\pi/4}\right]$$
 **TOPIC:** 0130

b. Find its average velocity betweeen time 0 and time h. (Your answer should be an expression of h, and trigonometric functions are acceptable here.)

**ANSWER:** 
$$2\left[\frac{\sin h}{h}\right] + 3\left[\frac{(\cos h) - 1}{h}\right]$$

**TOPIC:** 0130

c. Find its instantaneous velocity at time 0. That is, find the limit, as  $h \to 0$ , of your answer to Part b.

**ANSWER:** 2

**TOPIC:** 0230

E. (S11 Midterm 02, Topic 0230) (no partial credit) Compute  $\lim_{x\to 0} \frac{(\sin x)(\tan x)(\sec x)}{x^2}$  Circle one of the following answers:

- (a) 0
- (b) 1
- (c) 2
- (d)  $\infty$
- (e) does not exist

ANSWER: (b)

**TOPIC:** 0230

8. (S09 Midterm 01, Topic 0230) Compute  $\lim_{x\to 0} \frac{[\sin^8(3x)][\cos^5(x)][(x+1)^4]}{(3x)^8}$ .

**ANSWER:** 1

**TOPIC:** 0230 (23)

**END TOPIC: 0230** 

### **BEGIN TOPIC: 0240**

e. (F14 Midterm 01A, Topic 0340) If f and g are both differentiable at 3, then  $f^2 - fg$  is also differentiable at 3.

ANSWER: True

**TOPIC:** 0340

a. (F14 Midterm 01B, Topic 0240(22-25)) If P is any polynomial of degree 3 and Q is any polynomial of degree 2, then  $\lim_{x\to -\infty}\left[\frac{P(x)}{Q(x)}\right]=-\infty$ .

ANSWER: False

**TOPIC:** 0240(22-25)

d. (F14 Midterm 01A, Topic 0240(20-21)) If P is any polynomial of degree 3 and Q is any polynomial of degree 4, then  $\lim_{x\to -\infty}\left[\frac{P(x)}{Q(x)}\right]=0$ .

ANSWER: True

**TOPIC:** 0240(20-21)

2. (never used, Topic 0240(19)) Compute 
$$\lim_{n\to\infty}\frac{1+2+\cdots+n}{n^2}$$
. (Hint:  $1+2+\cdots+n=n(n+1)/2$ .)

ANSWER: 1/2

**TOPIC:** 0240(19)

e. (S13 Midterm 01A, Topic 0240(19-25)) Let f be any rational function. If  $\lim_{x\to\infty} f(x)=2$ , then  $\lim_{x\to-\infty} f(x)=2$ .

ANSWER: True

**TOPIC:** 0240(19-25)

F. (S13 Midterm 01A, Topic 0240(20-21)) (no partial credit) Compute  $\lim_{x\to-\infty}\left[\frac{x^3+2x^2-4x}{2x^4-7x^2}\right]$  Circle one of the following answers:

- (a) 4/7
- (b) -4/7
- (c) 1/2
- (d) -1/2
- (e) NONE OF THE ABOVE

**ANSWER:** (e), the answer is 0

**TOPIC:** 0240(20-21)

b. (S12 Midterm 01A, Topic 0240) There is a function with three vertical asymptotes.

ANSWER: True

**TOPIC:** 0240 (7)

9. (S09 Midterm 01, Topic 0240) Find all horizontal asymptotes of; the curve  $y = \frac{(\sin^3 x)(\cos^5 x)}{r}$ .

**ANSWER:** y = 0

**TOPIC:** 0240

e. (S09 Midterm 01, Topic 0240) There is a function with three horizontal asymptotes.

ANSWER: False

**TOPIC:** 0240

**END TOPIC: 0240** 

**BEGIN TOPIC: 0250** 

4. (F13 Midterm 01A, Topic 0340) Suppose f(0) = 2 and f'(0) = 3. Suppose g(0) = 4 and g'(0) = 5. Let h = fg. Compute h(0) and h'(0).

**ANSWER:**  $h(0) = 2 \cdot 4 = 8$  and  $h'(0) = 3 \cdot 4 + 2 \cdot 5 = 22$ 

**TOPIC:** 0250(6)

3. (F13 Midterm 01C, Topic 0250(6)) Find all horizontal asymptotes to

$$y = \frac{\sqrt{x^4 + 4x - 7}}{3x^2 + 5}.$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines,  $\mathbf{NOT}$  numbers.)

**ANSWER:** y = 1/3 **TOPIC:** 0250(6)

3. (F13 Midterm 01A, Topic 0250(6)) Find all horizontal asymptotes to

$$y = \frac{\sqrt{9x^2 + 2x + 5}}{2x - 3}.$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines,  $\mathbf{NOT}$  numbers.)

**ANSWER:** y = -3/2 and y = 3/2 **TOPIC:** 0250(6)

F. (never used, Topic 0250) Compute  $\lim_{x\to\infty} \frac{2x^2+4x-3}{5x^3-x^2-x-1}$ . Circle one of the following answers:

- (a) 3
- (b) 2/5
- (c)  $\infty$
- (d) 0
- (e) NONE OF THE ABOVE

1. (S13 Midterm 01A, Topic 0250) Find all horizontal asymptotes to

$$y = \frac{\sqrt[3]{8x^3 + 2x + 5}}{7x - 3}.$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines,  $\mathbf{NOT}$  numbers.)

**ANSWER:** y = 2/7

**TOPIC:** 0250

1. (F12 Midterm 01A, Topic 0250) Find all horizontal asymptotes to

$$y = \frac{\sqrt{9x^4 + 2x + 5}}{2x^2 - 3}.$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines,  $\mathbf{NOT}$  numbers.)

**ANSWER:** y=3/2

**TOPIC:** 0250

B. (F12 Midterm 01A, Topic 0250) (no partial credit) Compute  $\lim_{x\to-\infty}\left[\frac{\sqrt{16x^6-x}}{16x^3+x}\right]$ . Circle one of the following answers:

- (a) 1/4
- (b) -1/4
- (c) 1/2
- (d) -1/2
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0250

2. (S12 Midterm 01A, Topic 0250) Find all the horiztonal asymmtotes to  $y = \frac{\sqrt{9x^2 + 5}}{x + 1}$ .

**ANSWER:** y = -3 and y = 3

**TOPIC:** 0250 (6)

3. (F11 Midterm 01, Topic 0250) Compute  $\lim_{x \to -\infty} \left( \sqrt{x^2 + 4x} - \sqrt{x^2 - 5x} \right)$ .

**ANSWER:** -9/2

**TOPIC:** 0250

b. (F11 Midterm 01, Topic 0250) There is a function with three horizontal asymptotes.

ANSWER: False TOPIC: 0250

C. (F11 Midterm 01, Topic 0250) (no partial credit) Compute  $\lim_{x\to-\infty}\left[\frac{\sqrt{4x^4-x}}{8x^2+x}\right]$ . Circle one of the following answers:

(a) 1/4

- (b) -1/4
- (c) 1/2
- (d) -1/2
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0250

B. (F11 Midterm 01, Topic 0250) (no partial credit) Compute  $\lim_{x\to-\infty}\left[\frac{2x^2-x}{4x^2+x}\right]$ . Circle one of the following answers:

- (a)  $\infty$
- (b)  $-\infty$
- (c) 1/2
- (d) -1/2
- (e) NONE OF THE ABOVE

ANSWER: (c)

**TOPIC:** 0250

5. (S10 Midterm 01, Topic 0250) (no partial credit) Compute  $\lim_{x\to-\infty}\left[\frac{2x^3-x}{4x^2+x}\right]$ . Circle one of the following answers:

- (a)  $\infty$
- (b)  $-\infty$
- (c) 2/4
- (d) -2/4
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0250

**END TOPIC: 0250** 

**BEGIN TOPIC: 0260** 

ANSWER: True	<b>TOPIC:</b> 0260 (10)	
D. (F11 Midterm 01, Top following answers:	ic 0260) (no partial credit) Compute $\ln(e^{-\epsilon})$	(5 <sup>2</sup> ). Circle one of the
(a) 25		
(b) $-10$		
(c) $-25$		
(d) DOES NOT EXI	$\operatorname{ST}$	
(e) NONE OF THE	ABOVE	
<b>ANSWER:</b> (c)	<b>TOPIC:</b> 0260	
	ic 0260) (no partial credit) What is $\log_{10}$ (the number 1000? Circle one of the followi	
(a) $10^{1000}$		
(b) 3		
(c) <i>e</i>		
(d) it is undefined		
(e) NONE OF THE	ABOVE	
ANSWER: (b)	<b>TOPIC:</b> 0260	
d. (S11 Midterm 03, Top:	ic 0260) For any $b > 0$ , the function $f(x) =$	$= b^x$ is one-to-one.
ANSWER: False (but b	=1 is the only counterexample)	<b>TOPIC:</b> 0260
	END TOPIC: 0260	
	BEGIN TOPIC: 0270	
	END TOPIC: 0270	
	BEGIN TOPIC: 0280	

a. (S12 Midterm 01A, Topic 0260) For every real number x,  $\ln(e^x) = x$ .

4. (F14 Midterm 01A, Topic 0280(8-10)) Let  $y = 3x^3 - 5x$ . Then  $\triangle y = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3 - k(\triangle x)$ , for some real numbers a, b, c, k.

a. Compute a, b, c and k.

**ANSWER:** 
$$a = 9$$
,  $b = 9$ ,  $c = 3$ ,  $k = -5$ 

**TOPIC:** 0280(8)

b. Assuming  $\triangle x \neq 0$ , compute  $\frac{\triangle y}{\triangle x}$ .

**ANSWER:** 
$$9x^2 + 9x(\triangle x) + 3(\triangle x)^2 - 5$$

**TOPIC:** 0280(9)

c. Compute  $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ .

**ANSWER:** 
$$9x^2 - 5$$

**TOPIC:** 0280(10)

4. (F14 Midterm 01A, Topic 0280(8-10)) Let  $y = x^3 + 2x$ . Then  $\triangle y = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3 + k(\triangle x)$ , for some real numbers a, b, c, k.

a. Compute a, b, c and k.

**ANSWER:** 
$$a = 3, b = 3, c = 1, k = 2$$

**TOPIC:** 0280(8)

b. Assuming  $\triangle x \neq 0$ , compute  $\frac{\triangle y}{\triangle x}$ .

**ANSWER:** 
$$3x^2 + 3x(\triangle x) + (\triangle x)^2 + 2$$

**TOPIC:** 0280(9)

c. Compute  $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ .

**ANSWER:** 
$$3x^2 + 2$$

**TOPIC:** 0280(10)

C. (F14 Midterm 01A, Topic 0280(8)) (no partial credit) Compute  $\triangle(x^3-x^2)$ . Circle one of the following answers:

(a) 
$$3x^2 - 2x$$

(b) 
$$3x^2(\triangle x) + 3x(\triangle x)^2 + (\triangle x)^3 - 2x(\triangle x) - (\triangle x)^2$$

(c) 
$$3x^2 + 3x(\triangle x) + (\triangle x)^2 - 2x - (\triangle x)$$

(d) 
$$(3x^2 - 2x)(\triangle x)$$

(e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0280(8)

A. (F13 Midterm 02A, Topic 0280) (no partial credit) Let f be a function such that  $f'(x) = 4(\cos(4x))$ . Suppose, also, that f(0) = 1. Which of the following is an equation of the tangent line to the graph of f at (0,1). Circle one of the following answers:

(a) 
$$y = 4(x - 1)$$

(b) 
$$y - 1 = 4(\cos(4x))x$$

(c) 
$$y - 1 = 4x$$

(d) 
$$y = -4(\sin(x))(x-1)$$

(e) NONE OF THE ABOVE

ANSWER: c

**TOPIC:** 0280

5. (S13 Midterm 02D, Topic 0280(8-10)) Let  $y = x^4$ . Then  $\triangle y = px^3(\triangle x) + qx^2(\triangle x)^2 + rx(\triangle x)^3 + s$  for some real numbers p, q, r, s.

a. (5 pts) Compute p, q, r and s.

**ANSWER:** p = 4, q = 6, r = 4, s = 1

**TOPIC:** 0280(8)

b. (5 pts) Assuming  $\triangle x \neq 0$ , compute  $\frac{\triangle y}{\triangle x}$ .

**ANSWER:**  $4x^3 + 6x^2(\triangle x) + 4x(\triangle x)^2 + (\triangle x)^3$ 

**TOPIC:** 0280(9)

c. (5 pts) Compute  $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ .

**ANSWER:**  $4x^3$ 

**TOPIC:** 0280(10)

5. (S13 Midterm 02A, Topic 0280(8-10)) Let  $y = x^3$ . Then  $\triangle y = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3$ , for some real numbers a, b, c.

a. (5 pts) Compute a, b and c.

**ANSWER:** a = 3, b = 3, c = 1

**TOPIC:** 0280(8)

b. (5 pts) Assuming  $\triangle x \neq 0$ , compute  $\frac{\triangle y}{\triangle x}$ .

**ANSWER:**  $3x^2 + 3x(\triangle x) + (\triangle x)^2$ 

**TOPIC:** 0280(9)

c. (5 pts) Compute  $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ .

**ANSWER:**  $3x^2$ 

**TOPIC:** 0280(10)

A. (S13 Midterm 02A, Topic 0280) (no partial credit) Let f be a function such that  $f'(x) = 4e^{4x}$ . Suppose, also, that f(0) = 1. Which of the following is an equation of the

tangent line to the graph of f at (0,1). Circle one of the following answers:

(a) 
$$y = 4(x - 1)$$

(b) 
$$y = 1 + 4x$$

(c) 
$$y - 1 = 4e^{4x}x$$

(d) 
$$y = 4e^{4x}(x-1)$$

(e) NONE OF THE ABOVE

# ANSWER: b

**TOPIC:** 0280

F. (never used, Topic 0280(17-18)) (no partial credit) Find the slope of the tangent line to y = 1/x at (3, 1/3) Circle one of the following answers:

- (a) 1/3
- (b) ln 3
- (c) 1/9
- (d) -1/9
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0280(17-18)

a. (S13 Midterm 01A, Topic 0280(23)) Let f(x) = |x|. Then f(x) is differentiable at x = 1.

ANSWER: True

**TOPIC:** 0280(23)

- 2. (F12 Midterm 01A, Topic 0280(20-18)) Draw a single graph showing a function f:  $[3,5] \to \mathbb{R}$  with all of the following properties. Indicate, both on the x-axis and on the y-axis, where the number 1 appears.
  - ( $\bullet$ ) Its domain is the interval [3, 5].
  - ( $\bullet$ ) It is continuous on [3, 5].
  - $(\bullet)$  It is differentiable on (3,4) and on (4,5).
  - (•) For all  $x \in (3,4)$ , we have: f'(x) = -1.
  - (•) For all  $x \in (4,5)$ , we have: f'(x) = 1.
  - (•) It is not differentiable at 4.
  - $(\bullet) f(4) = 0.$

**ANSWER:** graph of y = |x - 4|, restricted to  $3 \le x \le 5$  **TOPIC:** 0280(20-18)

d. (F12 Midterm 01A, Topic 0280(28)) If a function f is continuous at a number a, then f is differentiable at a.

**ANSWER:** False **TOPIC:** 0280(28)

e. (F12 Midterm 01A, Topic 0280(28)) Let f(x) = |x|. Then the domains of f and of f' are equal.

**ANSWER:** False **TOPIC:** 0280(28)

0. (never used, Topic 0280) Let  $y = x^3$ . Then

$$[\triangle y]_{x:\to 3} = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3,$$

for some real numbers a, b and c.

a. Compute a, b and c.

**ANSWER:** a = 3, b = 3, c = 1 **TOPIC:** 0280 (8)

b. Compute  $\frac{\triangle y}{\triangle x}$ .

**ANSWER:**  $3x^2 + 3x(\triangle x) + (\triangle x)^2$  **TOPIC:** 0280 (8)

c. Compute  $\lim_{\triangle x \to 0} \frac{\triangle y}{\triangle x}$ .

**ANSWER:**  $3x^2$  **TOPIC:** 0280 (8)

e. (S12 Midterm 01A, Topic 0280) If a function is differentiable at 0, then it is continuous at 0.

**ANSWER:** True **TOPIC:** 0280 (29-33)

d. (S12 Midterm 01A, Topic 0280) The function f(x) = |x| is differentiable at every real number.

**ANSWER:** False **TOPIC:** 0280 (25-28)

D. (S12 Midterm 01A, Topic 0280) (no partial credit) Compute Let  $y = x^2 - x$ . Find  $\triangle y$ . Circle one of the following answers:

(a) 
$$(x + \triangle x)^2 - (x + \triangle x)$$

(b) 
$$[(x + \triangle x)^2 - (x + \triangle x)] + [x^2 - x]$$

(c) 
$$[(x + \Delta x)^2 - (x + \Delta x)] - [x^2 - x]$$

(d) 
$$[x^2 - x] - [(x + \triangle x)^2 - (x + \triangle x)]$$

(e) NONE OF THE ABOVE

**ANSWER:** (c)

**TOPIC:** 0280 (8)

- 1. (F11 Midterm 01, Topic 0280) Draw a single graph showing a function  $f:[3,5] \to \mathbb{R}$  with *all* of the following properties. Indicate, both on the x-axis and on the y-axis, where the number 1 appears.
  - ( $\bullet$ ) Its domain is the interval [3, 5].
  - $(\bullet)$  It is continuous on [3, 5].
  - $(\bullet)$  It is differentiable on (3,4) and on (4,5).
  - (•) It is not differentiable at 4.

**ANSWER:** many possible answers

**TOPIC:** 0280

e. (F11 Midterm 01, Topic 0280) The function f(x) = |x| is differentiable at 0.

**ANSWER:** False

**TOPIC:** 0280

a. (S09 Midterm 01, Topic 0280) If a function f is differentiable at a number a, then f is continuous at a.

ANSWER: True

**TOPIC:** 0280 (29-33)

**END TOPIC: 0280** 

**BEGIN TOPIC: 0290** 

**END TOPIC: 0290** 

**BEGIN TOPIC: 0300** 

END TOPIC: 0300

**BEGIN TOPIC: 0310** 

d. (F13 Midterm 01A, Topic 0310(3)) If two functions have the same derivative, then they must be equal.

ANSWER: False

**TOPIC:** 0310(3)

END TOPIC: 0310

# **BEGIN TOPIC: 0320**

- 3. (F14 Midterm 01C, Topic 0320) Let  $f(x) = x^6 6x^4 e^{-3}$ .
- a. Find all  $a \in \mathbb{R}$  such that the graph of f has a horizontal tangent line at (a, f(a)).

**ANSWER:** -2, 0, 2

**TOPIC:** 0320

b. Find all the maximal intervals on which f' is negative.

**ANSWER:**  $(-\infty, -2)$  and (0, 2)

**TOPIC:** 0320

- 3. (F14 Midterm 01D, Topic 0320) Let  $f(x) = -x^6 + 6x^4 + (\tan(e))$ .
- a. Find all  $a \in \mathbb{R}$  such that the graph of f has a horizontal tangent line at (a, f(a)).

**ANSWER:** -2, 0, 2

**TOPIC:** 0320

b. Find all the maximal intervals on which f' is negative.

**ANSWER:** (-2,0) and  $(2,\infty)$ 

**TOPIC:** 0320

- 3. (F14 Midterm 01A, Topic 0320) Let  $f(x) = 3x^5 5x^3 + e^7$ .
- a. Find all  $a \in \mathbb{R}$  such that the graph of f has a horizontal tangent line at (a, f(a)).

**ANSWER:** -1, 0, 1

**TOPIC:** 0320

b. Find all the maximal intervals on which f' is negative.

**ANSWER:** (-1,0) and (0,1)

**TOPIC:** 0320

b. (F14 Midterm 01C, Topic 0320(3)) Let f and g be any two functions such that f'(4) = 10 and g'(4) = 20. Then (f + g)'(4) = 30.

ANSWER: True

**TOPIC:** 0320(3)

a. (F14 Midterm 01A, Topic 0320(3)) Let f and g be any two functions such that f'(4) = 10 and g'(5) = 20. Then (f + g)'(9) = 30.

ANSWER: False

**TOPIC:** 0320(3)

e. (F13 Midterm 01A, Topic 0320(6-7)) If f is a polynomial of degree 7, then f'' is a polynomial of degree 5.

ANSWER: True

**TOPIC:** 0320(6-7)

C. (F13 Midterm 01A, Topic 0320(8-10)) (no partial credit) Compute  $[d/dx][3x^4 + 2x^{1/2} - \pi]$ . Circle one of the following answers:

(a) 
$$4x^3 + x^{-1/2} - \pi$$

(b) 
$$12x^3 + x^{-1/2} - \pi$$

(c) 
$$12x^3 + x^{1/2} + \pi$$

(d) 
$$3x^3 + x^{1/2} + \pi$$

(e) NONE OF THE ABOVE

**ANSWER:** (e),  $12x^3 + x^{-1/2}$ 

**TOPIC:** 0320(8-10)

2. (S11 Midterm 03, Topic 0320) Find an equation of the tangent line to the graph of  $y^3 + y - 8 = x^7 + x$  at the point (1, 2).

**ANSWER:**  $y - 2 = \frac{8}{13}(x - 1)$ 

**TOPIC:** 0320

D. (S11 Midterm 03, Topic 0320) (no partial credit) Compute  $\frac{d}{dx} \left[ (\ln 4)^{\cos 7} \right]$ .

(a) 
$$\left[ (\ln 4)^{\cos 7} \right] \left[ (-\sin 7)(\ln(\ln(4)) + (\cos 7)\frac{1/4}{\ln 4} \right]$$

(b) 
$$\left[ (\ln 4)^{\cos 7} \right] \left[ (-\sin 7)(\ln(\ln(4)) + (\cos 7)\frac{1/4}{\ln(\ln 4)} \right]$$

(c) 
$$(1/4)^{\sin 7}$$

(d) 
$$(1/4)^{-\sin 7}$$

(e) NONE OF THE ABOVE

**ANSWER:** (e),  $\frac{d}{dr} [(\ln 4)^{\cos 7}] = 0$ 

**TOPIC:** 0320

6. (S10 Midterm 01, Topic 0320) (no partial credit) Find the equation of the tangent line, at (1,4), to the graph of  $y=x^3-x^2+4x$ . Circle one of the following answers:

(a) 
$$y - 4 = 2(x - 1)$$

(b) 
$$y - 4 = 3(x - 1)$$

(c) 
$$y - 4 = 4(x - 1)$$

(d) 
$$y - 4 = 5(x - 1)$$

(e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0320

1. (S10 Midterm 01, Topic 0320) (no partial credit) Compute  $(d/dx)(2x^3-4x^2+7x-e^{\pi})$ . Circle one of the following answers:

(a) 
$$6x^2 - 8x + 7 - e^{\pi}$$

(b) 
$$6x^2 - 8x + 7$$

(c) 
$$6x^2 - 8x + 7 - \pi e^{\pi - 1}$$

(d) 
$$6x^2 - 8x + 7 - e^0$$

(e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0320

1. (S09 Midterm 01, Topic 0320) Compute  $(d/dx)(x^3 + 4x^2 + 3x - \ln \pi)$ .

**ANSWER:**  $3x^2 + 8x + 3$ 

**TOPIC:** 0320

## **END TOPIC: 0320**

### **BEGIN TOPIC: 0330**

D. (F13 Midterm 01A, Topic 0330) (no partial credit) Compute  $[d/dx][2e^x + 5e]$ . Circle one of the following answers:

(a) 
$$2e^x + 5$$

(b) 
$$2e^x$$

(c) 
$$2xe^{x-1} + 5$$

(d) 
$$2xe^{x-1}$$

(e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0330

END TOPIC: 0330

**BEGIN TOPIC: 0340** 

a. (F12 Midterm 02A, Topic 0340) If f and g are differentiable at a number a, then fg + f + g is differentiable at a.

ANSWER: True TOPIC: 0340

e. (F11 Midterm 02A, Topic 0340) If f and g are differentiable, then  $\frac{d}{dx}[(f(x))(g(x))] = [f'(x)][g'(x)]$ .

ANSWER: False TOPIC: 0340

C. (F11 Midterm 02A, Topic 0340) (no partial credit) Find the slope of the tangent line to  $y = (x^3 + 4)e^{2x}$  at the point (0, 4).

- (a) 2
- (b) 4
- (c) 6
- (d) 8
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0340

e. (S11 Midterm 03, Topic 0340) For any two functions f and g, we have (fg)' = (f')(g'). That is, the derivative of the product is the product of the derivatives.

ANSWER: False TOPIC: 0340

B. (S11 Midterm 03, Topic 0340) (no partial credit) Compute  $\frac{d}{dx} \left[ \frac{e^x}{x^2 + 5} \right]$ .

(a) 
$$\frac{(x^2+5)(e^x)-(e^x)(2x)}{(x^2+5)^2}$$

- (b)  $\frac{e^x}{2x}$
- (c)  $\frac{(e^x)(2x) (x^2 + 5)(e^x)}{(x^2 + 5)^2}$
- (d)  $\frac{2x}{e^x}$
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0340

2. (S10 Midterm 02, Topic 0340) (no partial credit) Let u be a function of t. Let  $\dot{u} := du/dt$ . Compute  $(d/dt)(ue^u)$ . Circle one of the following answers:

- (a)  $e^u + ue^u$
- (b)  $\dot{u}e^u + ue^u\dot{u}$
- (c)  $\dot{u}e^{\dot{u}}$
- (d)  $\dot{u}e^u + ue^u$
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0340

**END TOPIC: 0340** 

**BEGIN TOPIC: 0350** 

B. (F14 Midterm 01A, Topic 0350) (no partial credit) Compute  $\left[\frac{d}{dx}\right]\left[\frac{e^x}{x^4-8x}\right]$ . Circle one of the following answers:

(a) 
$$\frac{(x^4 - 8x)(e^x) - (e^x)(4x^3 - 8)}{(x^4 - 8x)^2}$$

(b) 
$$\frac{(e^x)(4x^3-8)-(x^4-8x)(e^x)}{(x^4-8x)^2}$$

(c) 
$$\frac{e^x}{4x^3 - 8}$$

(d) 
$$\frac{xe^{x-1}}{4x^3-8}$$

(e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0350

C. (S13 Midterm 02A, Topic 0350(7)) (no partial credit) The Quotient Rule says that (f/g)' is equal to what? Circle one of the following answers:

- (a) f'/g'
- (b) g'/f'
- (c)  $(gf' fg')/g^2$
- (d)  $(fg' gf')/g^2$

ANSWER: c

**TOPIC:** 0350(7)

4. (S10 Midterm 01, Topic 0350) (no partial credit) Compute  $\frac{d}{dx} \left[ \frac{e^x - \sin x}{x^4 + 2x + 5} \right]$ . Circle one of the following answers:

(a) 
$$\frac{(e^x - \sin x)(4x^3 + 2) - (x^4 + 2x + 5)(e^x - \cos x)}{(x^4 + 2x - 5)^2}$$

(b) 
$$\frac{xe^{x-1} - \cos x}{4x^3 + 2}$$

(c) 
$$\frac{e^x - \cos x}{4x^3 + 2}$$

(d) 
$$\frac{(e^x - \sin x)(4x^3 + 2) + (x^4 + 2x + 5)(e^x - \cos x)}{(x^4 + 2x - 5)^2}$$

(e) NONE OF THE ABOVE

**ANSWER:** (e), 
$$\frac{(x^4 + 2x + 5)(e^x - \cos x) - (e^x - \sin x)(4x^3 + 2)}{(x^4 + 2x + 5)^2}$$

**TOPIC:** 0350

## **END TOPIC: 0350**

## **BEGIN TOPIC: 0360**

1. (F14 Midterm 01D, Topic 0360) Compute  $\frac{d}{dx} \left[ \frac{(2x^3 + x)(4 + 7e^x)}{\cot x} \right].$ 

# **ANSWER:**

$$\frac{[\cot x][(6x^2+1)(4+7e^x)+(2x^3+x)(7e^x)]-[(2x^3+x)(4+7e^x)][-\csc^2 x]}{\cot^2 x}$$

**TOPIC:** 0360

1. (F14 Midterm 01A, Topic 0360) Compute  $\frac{d}{dx} \left[ \frac{(2x^2 - 8x)(\sec x)}{5 - e^x} \right]$ .

## **ANSWER:**

$$\frac{[5 - e^x][(4x - 8)(\sec x) + (2x^2 - 8x)((\sec x)(\tan x))] - [(2x^2 - 8x)(\sec x)][-e^x]}{[5 - e^x]^2}$$

b. (F14 Midterm 01A, Topic 0360)  $\frac{d}{dx} \left[ \frac{\sin x}{x^2} \right] = \frac{\cos x}{2x}$ 

ANSWER: False

**TOPIC:** 0360

F. (F14 Midterm 01A, Topic 0360) (no partial credit) Let  $f(t) = \cot^2 t$ . Compute  $f'(\pi/4)$ . (Hint:  $f(t) = (\cot t)(\cot t)$ .) Circle one of the following answers:

- (a)  $-\sqrt{2}/2$
- (b) -4
- (c) -1
- (d) 1
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0360

F. (F14 Midterm 01C, Topic 0360) (no partial credit) Let  $f(t) = \tan^2 t$ . Compute  $f'(\pi/4)$ . (Hint:  $f(t) = (\tan t)(\tan t)$ .) Circle one of the following answers:

- (a) 4
- (b) -1
- (c)  $-\sqrt{2}/2$
- (d) 1
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0360

D. (F14 Midterm 01A, Topic 0360) (no partial credit) Compute  $[d/dx][2e^3 + 5\sin x]$ . Circle one of the following answers:

- (a)  $6e^2 + 5\cos x$
- (b)  $6e^3 + 5\cos x$
- (c)  $5\cos x$
- (d)  $-5\cos x$

**ANSWER:** (c)

**TOPIC:** 0360

1. (F13 Midterm 01A, Topic 0360) Compute

$$\frac{d}{dx} \left[ \frac{(x^2 + 3x)(\sin x)}{1 + e^x} \right].$$

**ANSWER:**  $\frac{[1+e^x][(2x+3)(\sin x)+(x^2+3x)(\cos x)]-[(x^2+3x)(\sin x)][e^x]}{[1+e^x]^2}$ 

**TOPIC:** 0360

b. (F13 Midterm 01A, Topic 0360)  $\frac{d}{dx} \left[ \frac{\sin x}{x^2} \right] = \frac{\cos x}{2x}$ .

ANSWER: False

TOPIC: 0360

B. (F13 Midterm 01A, Topic 0360) (no partial credit) Compute  $[d/dx][(\sin x)(\cos x)]$ . Circle one of the following answers:

- (a)  $(\cos x)(\sin x)$
- (b)  $(\cos x)(-\sin x)$
- (c)  $(\cos^2 x) (\sin^2 x)$
- (d)  $(\sin^2 x) (\cos^2 x)$
- (e) NONE OF THE ABOVE

**ANSWER:** (c)

**TOPIC:** 0360

10. (S10 Midterm 01, Topic 0360) Compute  $\frac{d}{dx} \left[ \frac{e^x \tan x}{e^2 x^3 \csc x} \right]$ .

ANSWER:  $\frac{(e^2x^3\csc x)(e^x\tan x + e^x\sec^2 x) - (e^x\tan x)(3e^2x^2\csc x + e^2x^3(-\csc x\cot x))}{e^4x^6\csc^2 x}$ 

**TOPIC:** 0360

3. (S10 Midterm 01, Topic 0360) (no partial credit) Compute  $(d/dx)[e^x \csc x]$ . Circle one of the following answers:

41

- (a)  $-e^x[\csc x][\cot x]$
- (b)  $e^x[\csc x][\cot x]$

- (c)  $e^x[\csc x][1 + (\cot x)]$
- (d)  $e^x[\csc x][1 (\cot x)]$
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0360

2. (S10 Midterm 01, Topic 0360) (no partial credit) Compute  $(d/dx)[(\sin x)(\cos x)]$ . Circle one of the following answers:

- (a)  $(\cos^2 x) (\sin^2 x)$
- (b)  $-(\cos x)(\sin x)$
- (c) 1
- (d)  $(\cos x) + (\sin x)$
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0360

**END TOPIC: 0360** 

**BEGIN TOPIC: 0370** 

a. (S13 Midterm 02A, Topic 0370) Let u be any expression of x. Then  $(d/dx)(e^u) = e^u$ .

ANSWER: False

**TOPIC:** 0370

E. (S12 Midterm 02A, Topic 0370) (no partial credit) Compute  $[d/dx][\sin(\cos(e^x+3))]$ .

- (a)  $\cos(\cos(e^x + 3))$
- (b)  $[\cos(\cos(e^x+3))][-\sin(e^x+3)][e^x+3]$
- (c)  $[\cos(\cos(e^x + 3))][\cos(e^x + 3)][e^x + 3]$
- (d) 0
- (e) NONE OF THE ABOVE

**ANSWER:** (e),  $[\cos(\cos(e^x + 3))][-\sin(e^x + 3)][e^x]$ 

**TOPIC:** 0370

3. (S10 Midterm 02, Topic 0370) (no partial credit) Compute  $\frac{d}{dx} \left[ \frac{\sec x}{\sqrt{x^2 + 1}} \right]$ . Circle one of the following answers:

(a) 
$$\frac{\sec x \tan x}{\frac{1}{2}(x^2+1)^{-1/2}(2x)}$$

(b) 
$$\frac{(x^2+1)^{1/2}(\tan^2 x) - (\sec x)\frac{1}{2}(x^2+1)^{-1/2}}{x^2+1}$$

(c) 
$$\frac{\sqrt{x^2+1}(-\sec x \tan x) - (\sec x)\sqrt{2x}}{x^2+1}$$

(d) 
$$\frac{\tan^2 x}{\sqrt{2x}}$$

**ANSWER:** (e), 
$$\frac{\sqrt{x^2+1}(\sec x \tan x) - (\sec x)(1/2)(x^2+1)^{-1/2}(2x)}{x^2+1}$$
 **TOPIC:** 0370

1. (S10 Midterm 02, Topic 0370) (no partial credit) Compute  $(d/dx) \left(e^{[x^2]}\right)$ . Circle one of the following answers:

- (a)  $e^{2x}$
- (b)  $e^{[x^2]}$
- (c)  $2xe^{2x}$
- (d)  $2xe^{[x^2]}$
- (e) NONE OF THE ABOVE

ANSWER: (d)

**END TOPIC: 0370** 

**TOPIC:** 0370

#### **BEGIN TOPIC: 0380**

2. (F13 Midterm 02A, Topic 0380) Say  $f'(x) = e^{x^2}$ . (Here  $e^{x^2}$  means  $e^{(x^2)}$ .) Let g(x) = f(2x+1). Find g'(1).

**ANSWER:**  $g'(1) = 2e^9$  **TOPIC:** 0380

1. (S13 Midterm 02A, Topic 0380) Compute  $\frac{d}{dx} \left[ \frac{e^{-x^4}}{4 + \tan(x^2)} \right]$ . (Here  $e^{-x^4}$  means  $e^{\left(-x^4\right)}$ .)

**ANSWER:** 
$$\frac{[4 + \tan(x^2)][-4x^3e^{-x^4}] - [e^{-x^4}][2x(\sec^2(x^2))]}{(4 + \tan(x^2))^2}$$

**TOPIC:** 0380

1. (F12 Midterm 02A, Topic 0380) Compute 
$$\frac{d}{dx} \left[ \frac{e^{x^4} - 8}{5 + \sec(x^2)} \right]$$
. (Here  $e^{x^4}$  means  $e^{\left(x^4\right)}$ .)

**ANSWER:** 
$$\frac{[5+\sec(x^2)][4x^3e^{x^4}]-[e^{x^4}-8][\sec(x^2)][\tan(x^2)][2x]}{[5+\sec(x^2)]^2}$$

TOPIC:

0380

A. (S11 Midterm 03, Topic 0380) (no partial credit) Compute  $\frac{d}{dx} \left[ (1+x^2)^{100} (\sin x) \right]$ .

- (a)  $100(1+x^2)^{99}(\cos x)$
- (b)  $100(1+x^2)^{99}(2x)(\cos x)$
- (c)  $100(1+x^2)^{99}(\sin x) + (1+x^2)^{100}(\cos x)$
- (d)  $100(1+x^2)^{99}(2x)(\sin x) + (1+x^2)^{100}(\cos x)$
- (e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0380

3. (S09 Midterm 01, Topic 0380) Compute  $(d/dx)(\sec(x^4+2x-1))$ .

**ANSWER:**  $[\sec(x^4 + 2x - 1)][\tan(x^4 + 2x - 1)][4x^3 + 2]$ 

**TOPIC:** 0380

b. (S09 Midterm 02, Topic 0380) If f(x) > 0, then  $\frac{d}{dx}[\ln(f(x))] = \frac{f(x)}{f'(x)}$ .

ANSWER: False

**TOPIC:** 0380

END TOPIC: 0380

BEGIN TOPIC: 0390

**END TOPIC: 0390** 

### **BEGIN TOPIC: 0400**

C. (S14 Midterm 02A, Topic 0400) (no partial credit) Find the derivative of  $(2 + x^4)^{\cos x}$  w.r.t. x. Circle one of the following answers:

(a) 
$$[(2+x^4)^{\cos x}][(\cos x)(\ln(2+x^4)) + (-\sin x)(4x^3/(2+x^4))]$$

(b) 
$$[(2+x^4)^{\cos x}][(-\sin x)(4x^3/(2+x^4))]$$

(c) 
$$[(2+x^4)^{\cos x}][(-\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))]$$

(d) 
$$[(2+x^4)^{\cos x}][(\cos x)(\ln(2+x^4))]$$

ANSWER: c

**TOPIC:** 0400

B. (S14 Midterm 02C, Topic 0400) (no partial credit) Find the derivative of  $(2 + x^4)^{\sin x}$  w.r.t. x. Circle one of the following answers:

(a) 
$$[(2+x^4)^{\sin x}][(\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))]$$

(b) 
$$[(2+x^4)^{\sin x}][(\cos x)(\ln(2+x^4)) + (\sin x)(4x^3/(2+x^4))]$$

(c) 
$$[(2+x^4)^{\sin x}][(\sin x)(\ln(2+x^4))]$$

(d) 
$$[(2+x^4)^{\sin x}][(\cos x)(4x^3/(2+x^4))]$$

(e) NONE OF THE ABOVE

ANSWER: b

**TOPIC:** 0400

2. (S13 Midterm 02A, Topic 0400) Compute  $\frac{d}{dx} \left[ (2 - \cos x)^{4+x} \right]$ .

**ANSWER:**  $[[2-\cos x]^{4+x}][[\ln(2-\cos x)]+[4+x][(\sin x)/(2-\cos x)]]$ 

**PIC:** 0400

D. (S13 Midterm 02A, Topic 0400(13)) (no partial credit) Compute  $\frac{d}{dx} \left[ \ln |(2x+1)(3x-4)| \right]$ . Circle one of the following answers:

(a) 
$$\frac{2}{2x+1} + \frac{3}{3x-4}$$

(b) 
$$\left| \frac{2}{2x+1} + \frac{3}{3x-4} \right|$$

(c) 
$$\frac{6}{(2x+1)(3x+4)}$$

(d) 
$$\left| \frac{6}{(2x+1)(3x+4)} \right|$$

(e) NONE OF THE ABOVE

ANSWER: a

**TOPIC:** 0400(13)

B. (F12 Midterm 02A, Topic 0400) (no partial credit) Find the derivative of  $(2+\sin(2x))^{\cos x}$  w.r.t. x. Circle one of the following answers:

(a) 
$$[(2 + \sin(2x))^{\cos x}] \left[ (-\sin x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \right]$$

(b) 
$$[(2+\sin(2x))^{\cos x}] \left[ (\cos x)(\ln(2+\sin(2x))) + (-\sin x) \left( \frac{2\cos(2x)}{2+\sin(2x)} \right) \right]$$

(c) 
$$[(2 + \sin(2x))^{\cos x}][(\cos x)(\ln(2 + \sin(2x)))]$$

(d) 
$$[(2+\sin(2x))^{\cos x}] \left[ (-\sin x)(\ln(2+\sin(2x))) + (\cos x) \left( \frac{2\cos(2x)}{2+\sin(2x)} \right) \right]$$

(e) NONE OF THE ABOVE

ANSWER: d

**TOPIC:** 0400

A. (F12 Midterm 02A, Topic 0400) (no partial credit) Find the logarithmic derivative of  $(2 + \sin(2x))^{\cos x}$  w.r.t. x. Circle one of the following answers:

(a) 
$$(-\sin x)$$
  $\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)$ 

(b) 
$$(\cos x)(\ln(2+\sin(2x))) + (-\sin x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)$$

(c) 
$$(\cos x)(\ln(2 + \sin(2x)))$$

(d) 
$$(-\sin x)(\ln(2+\sin(2x))) + (\cos x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)$$

(e) NONE OF THE ABOVE

ANSWER: d

**TOPIC:** 0400

F. (S12 Midterm 02A, Topic 0400) (no partial credit) Find the logarithmic derivative of  $x^2 + 7x - 8$  w.r.t. x.

(a) 
$$\frac{x^2 + 7x - 8}{2x + 7}$$

(b) 
$$ln(2x+7)$$

(c) 
$$\frac{2x+7}{x^2+7x-8}$$

(d) 
$$(\ln(x^2)) + 7(\ln x) - (\ln 8)$$

(e) NONE OF THE ABOVE

**ANSWER:** (c)

**TOPIC:** 0400

B. (S12 Midterm 02A, Topic 0400) (no partial credit) Find the derivative of  $(2 + x^4)^{\cos x}$  w.r.t. x.

(a) 
$$[(2+x^4)^{\cos x}][(-\sin x)(4x^3/(2+x^4))]$$

(b) 
$$[(2+x^4)^{\cos x}][(\cos x)(\ln(2+x^4)) + (-\sin x)(4x^3/(2+x^4))]$$

(c) 
$$[(2+x^4)^{\cos x}][(\cos x)(\ln(2+x^4))]$$

(d) 
$$[(2+x^4)^{\cos x}][(-\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))]$$

(e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0400

A. (S12 Midterm 02A, Topic 0400) (no partial credit) Find the logarithmic derivative of  $(2 + x^4)^{\cos x}$  w.r.t. x.

(a) 
$$(-\sin x)(4x^3/(2+x^4))$$

(b) 
$$(\cos x)(\ln(2+x^4)) + (-\sin x)(4x^3/(2+x^4))$$

(c) 
$$(\cos x)(\ln(2+x^4))$$

(d) 
$$(-\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))$$

(e) NONE OF THE ABOVE

ANSWER: (d)

**TOPIC:** 0400

E. (F11 Midterm 02A, Topic 0400) (no partial credit) Find the derivative of  $(2 + \sin x)^x$  w.r.t. x.

(a) 
$$[(2 + \sin x)^x] \left[ (\ln(2 + \sin x)) + \left( \frac{x \cos x}{2 + \sin x} \right) \right]$$

(b) 
$$(\ln(2+\sin x)) + \left(\frac{x\cos x}{2+\sin x}\right)$$

- (c)  $\ln(\cos x)$
- (d)  $\cos x$
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0400

D. (F11 Midterm 02A, Topic 0400) (no partial credit) Find the logarithmic derivative of  $(2 + \sin x)^x$  w.r.t. x.

(a) 
$$[(2 + \sin x)^x] \left[ (\ln(2 + \sin x)) + \left( \frac{x \cos x}{2 + \sin x} \right) \right]$$

(b) 
$$(\ln(2+\sin x)) + \left(\frac{x\cos x}{2+\sin x}\right)$$

- (c)  $\ln(\cos x)$
- (d)  $\cos x$
- (e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0400

B. (F11 Midterm 02A, Topic 0400) (no partial credit) Find the logarithmic derivative of  $x^2 + 3x - 8$  w.r.t. x.

(a) 
$$\frac{2x+3}{x^2+3x-8}$$

(b) 
$$\frac{x^2 + 3x - 8}{2x + 3}$$

(c) 
$$(\ln(x^2)) + 3(\ln x) - (\ln 8)$$

(d) 
$$ln(2x+3)$$

(e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0400

E. (S11 Midterm 03, Topic 0400) (no partial credit) Compute  $\frac{d}{dx} \left[ \ln(x^2 + 2x + 5) \right]$ .

(a) 
$$\frac{2x+2}{x^2+2x+5}$$

(b) 
$$\frac{2x+2}{1+(x^2+2x+5)^2}$$

(c) 
$$\frac{2x+2}{\sqrt{1-(x^2+2x+5)^2}}$$

(d) 
$$\frac{1}{x^2 + 2x + 5}$$

(e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0400

C. (S11 Midterm 03, Topic 0400) (no partial credit) Compute  $\frac{d}{dx} \left[ (1+x^2)^{2x+5} \right]$ .

(a) 
$$(2x)^2$$

(b) 
$$(2x+5)^2 + (2x)^{2x+5}$$

(c) 
$$[(1+x^2)^{2x+5}]$$
  $\left[2\left(\ln\left(1+x^2\right)\right) + (2x+5)\left(\frac{2x}{1+x^2}\right)\right]$ 

(d) 
$$[(1+x^2)^{2x+5}][2x][2(\ln(1+x^2))+(2x+5)(\frac{2x}{1+x^2})]$$

ANSWER: (c)

**TOPIC:** 0400

4. (S09 Midterm 01, Topic 0400) (no partial credit) Compute  $\frac{d}{dx} \frac{\ln |e^x - \sin x|}{x^4}$ . Circle one of the following answers:

(a) 
$$\frac{\left[x^4(e^x - \cos x)/(e^x - \sin x)\right] - \left[\ln|e^x - \sin x|\right][4x^3]}{x^8}$$

(b) 
$$\frac{1/(e^x - \cos x)}{4x^3}$$

(c) 
$$\frac{x^4(\ln|e^x - \sin x|) - (4x^3/(e^x - \cos x))}{x^8}$$

(d) 
$$\frac{\left[x^4/(e^x - \cos x)\right] - \left[\ln|e^x - \sin x|\right][4x^3]}{x^8}$$

(e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0400

END TOPIC: 0400

## **BEGIN TOPIC: 0410**

c. (S14 Midterm 02C, Topic 0410) Assume that  $\lim_{x\to a} [f(x)] = 1 = \lim_{x\to a} [g(x)]$ . Assume also that  $\lim_{x\to a} \frac{f'(x)}{g'(x)} = 3$ . Then  $\lim_{x\to a} \frac{f(x)}{g(x)} = 3$ .

ANSWER: False

**TOPIC:** 0410

e. (S14 Midterm 02A, Topic 0410) Assume that  $\lim_{x\to a} [f(x)] = 0 = \lim_{x\to a} [g(x)]$ . Assume also that  $\lim_{x\to a} \frac{f'(x)}{g'(x)} = -\infty$ . Then  $\lim_{x\to a} \frac{f(x)}{g(x)} = -\infty$ .

ANSWER: True

**TOPIC:** 0410

e. (F13 Midterm 02A, Topic 0410(7)) Let f and g be any two functions such that  $\lim_{x\to 7} f(x) = \infty$  and  $\lim_{x\to 7} g(x) = \infty$ . Then  $\lim_{x\to 7} [1/(f(x))]^{g(x)} = 0$ .

ANSWER: True

**TOPIC:** 0410(7)

d. (S13 Midterm 02B, Topic 0410(27)) Let f and g be any two functions such that  $\lim_{x\to 5} f(x) = 0$  and  $\lim_{x\to 5} g(x) = \infty$ . Then  $\lim_{x\to 5} \frac{f(x)}{g(x)} = 0$ .

ANSWER: True

**TOPIC:** 0410(27)

e. (S13 Midterm 02A, Topic Topic 0410(7)) Let f and g be any two functions such that  $\lim_{x\to 5} f(x) = 1$  and  $\lim_{x\to 5} g(x) = 0$ . Then  $\lim_{x\to 5} \frac{f(x)}{g(x)} = \infty$ .

ANSWER: False

**TOPIC:** Topic 0410(7)

e. (F12 Midterm 02A, Topic 0410) Assume that  $\lim_{x\to 0} [f(x)] = 0 = \lim_{x\to 0} [g(x)]$ . Assume also that  $\lim_{x\to 0} \left[\frac{f'(x)}{g'(x)}\right]$  does not exist. Then  $\lim_{x\to 0} \left[\frac{f(x)}{g(x)}\right]$  does not exist.

ANSWER: False

**TOPIC:** 0410

d. (F12 Midterm 02A, Topic 0410) Assume that  $\lim_{x\to 3} [f(x)] = 0 = \lim_{x\to 3} [g(x)]$ . Assume also that  $\lim_{x\to 3} \frac{f'(x)}{g'(x)} = 7$ . Then  $\lim_{x\to 3} \frac{f(x)}{g(x)} = 7$ .

ANSWER: True

**TOPIC:** 0410

5. (S12 Midterm 02A, Topic 0410) Compute  $\lim_{x\to 1} \left[ \frac{\ln x}{\cos(\pi x/2)} \right]$ .

ANSWER:  $-2/\pi$ 

**TOPIC:** 0410

e. (S12 Midterm 02A, Topic 0410) Assume that  $\lim_{x\to a} [f(x)] = 0 = \lim_{x\to a} [g(x)]$ . Assume also that  $\lim_{x\to a} \frac{f'(x)}{g'(x)} = 7$ . Then  $\lim_{x\to a} \frac{f(x)}{g(x)} = 7$ .

ANSWER: True

**TOPIC:** 0410

9. (S10 Midterm 02, Topic 0410) Compute  $\lim_{x\to 1} \left[ \frac{\ln x}{\cos(\pi x/2)} \right]$ .

**ANSWER:**  $-2/\pi$ 

**TOPIC:** 0410

d. (S10 Midterm 02, Topic 0410) If  $\lim_{x \to 5} f(x) = 1$  and  $\lim_{x \to 5} g(x) = 0$ , then  $\lim_{x \to 5} \frac{f(x)}{g(x)} = \infty$ .

**ANSWER:** False **TOPIC:** 0410(5)

END TOPIC: 0410

# **BEGIN TOPIC: 0420**

4. (S13 Midterm 02A, Topic 0420(10-16)) Compute  $\lim_{x\to 0} (e^x + \sin x)^{5/x}$ .

**ANSWER:**  $e^{10}$ 

**TOPIC:** 0420(10-16)

d. (S13 Midterm 02A, Topic Topic 0420(2)) Let g be any function such that  $\lim_{x\to\infty}[g(x)]=\infty$ . Then  $\lim_{x\to\infty}[(1/x)^{g(x)}]=0$ .

**TOPIC:** Topic 0420(2)

ANSWER: True

c. (S13 Midterm 02A, Topic 0420(2)) Let f and g be any two functions such that  $\lim_{x\to a} [f(x)] = \infty$  and  $\lim_{x\to a} [g(x)] = \infty$ . Then  $\lim_{x\to a} [(f(x)) - (g(x))] = 0$ .

**ANSWER:** False **TOPIC:** 0420(2)

F. (S13 Midterm 02A, Topic 0420(24)) (no partial credit) Compute  $\lim_{x\to\infty} (2x^2 + 4x - 3)e^{-x}$ . Circle one of the following answers:

- (a) 2
- (b) -3
- (c)  $\infty$
- (d) 0
- (e) NONE OF THE ABOVE

ANSWER: d

**TOPIC:** 0420(24)

A. (S13 Midterm 02D, Topic 0420(2)) (no partial credit) Compute  $\lim_{x\to-\infty} (2x^3+4x-3)e^{-x}$ . Circle one of the following answers:

- (a) -3
- (b) 0
- (c) 2

- (d)  $\infty$
- (e) NONE OF THE ABOVE

**ANSWER:**  $e, -\infty$ 

**TOPIC:** 0420(2)

4. (F12 Midterm 02A, Topic 0420) Compute  $\lim_{x\to 0} ((\cos x) + (\sin x))^{5/x}$ .

**ANSWER:**  $e^5$ 

**TOPIC:** 0420

- F. (F11 Midterm 02A, Topic 0420) (no partial credit) Compute  $\lim_{x\to 0} \left[ \frac{\sin^2 x}{4x^3 + 2x^2} \right]$ .
  - (a) 2
  - (b) 1
  - (c) 1/2
  - (d) 1/4
  - (e) NONE OF THE ABOVE

ANSWER: (c)

**TOPIC:** 0420

1. (S11 Midterm 03, Topic 0420) Compute  $\lim_{x\to 0} [2x + \cos x]^{1/x}$ .

**ANSWER:**  $e^2$ 

**TOPIC:** 0420

c. (S10 Midterm 02, Topic 0420) If  $\lim_{x \to 1} f(x) = 1$  and  $\lim_{x \to 1} g(x) = \infty$ , then  $\lim_{x \to 1} [f(x)]^{g(x)} = 1$ .

ANSWER: False

**TOPIC:** 0420

**END TOPIC: 0420** 

# **BEGIN TOPIC: 0430**

3. (S13 Midterm 02A, Topic 0430(26-29)) Find an equation for the tangent line to  $7x^3 - 5xy + y^2 = 4x - y$  at (1,3).

**ANSWER:** y - 3 = -(x - 1)

**TOPIC:** 0430(26-29)

- E. (S13 Midterm 02A, Topic 0430) (no partial credit) Compute  $[d/dx][\sin^2(xy)]$ . Circle one of the following answers:
  - (a)  $2[\sin(xy)][\cos(xy)]$

- (b)  $[\cos^2(xy)][y + xy']$
- (c)  $2[\sin(xy)][y + xy']$
- (d)  $2[\sin(xy)][\cos(y+xy')]$
- (e) NONE OF THE ABOVE

**ANSWER:** e,  $2[\sin(xy)][\cos(xy)][y + xy']$ 

so tengent line to  $x^3 \perp xy \perp y^3 = 1$ 

**TOPIC:** 0430

3. (F12 Midterm 02A, Topic 0430) Find an equation for the tangent line to  $x^3 + xy + y^3 = 11$  at (2, 1).

**ANSWER:** y - 1 = -(13/5)(x - 2)

**TOPIC:** 0430

E. (F12 Midterm 02A, Topic 0430) (no partial credit) Compute  $[d/dx][\sin^2(xy)]$ . Circle one of the following answers:

- (a)  $2[\sin(xy)][\cos(xy)]$
- (b)  $[\cos^2(xy)][y + xy']$
- (c)  $2[\sin(xy)][y + xy']$
- (d)  $2[\sin(xy)][\cos(y+xy')]$
- (e) NONE OF THE ABOVE

**ANSWER:** e,  $2[\sin(xy)][\cos(xy)][y + xy']$ 

**TOPIC:** 0430

2. (S12 Midterm 02A, Topic 0430) Using implicit differentiation, find y' = dy/dx, assuming that  $(x - y^2)^5 = x$ .

**ANSWER:**  $\frac{1-5(x-y^2)^4}{-10y(x-y^2)^4}$ 

**TOPIC:** 0430

D. (S12 Midterm 02A, Topic 0430) (no partial credit) Find an equation of the tangent line to  $4x^2y - 2y^3 = 2$  at the point (1, 1).

- (a) y 1 = 4(x 1)
- (b) y 1 = 3(x 1)
- (c) y 1 = 2(x 1)
- (d) y 1 = x 1
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0430

2. (F11 Midterm 02A, Topic 0430) Using implicit differentiation (and logarithmic differentiation), find y' = dy/dx, assuming that  $(2 + y^2)^{xy} = 9$ .

**ANSWER:** 
$$y' = \frac{-y(\ln(2+y^2))}{x(\ln(2+y^2) + \frac{2xy^2}{2+y^2})}$$
 **TOPIC:** 0430

10. (S10 Midterm 02, Topic 0430) Find an equation for the tangent line to  $3x^3y - 2xy^7 + 2\cos y = x$  at the point (2,0).

**ANSWER:** 
$$y = (x - 2)/24$$
 **TOPIC:** 0430

6. (S09 Midterm 01, Topic 0430) Find the equation of the tangent line, at (0,0), to

$$y + y^2 + e^y = x^2 + (\cos x) + 4(\sin x).$$

**ANSWER:** 
$$y = 2x$$
 **TOPIC:** 0430

8. (S09 Midterm 02, Topic 0430) Find the equation of the tangent line to  $3x^2 + 4y^4 = 7$  at the point (1,1).

**ANSWER:** 
$$y - 1 = -\frac{3}{8}(x - 1)$$
 **TOPIC:** 0430

7. (S09 Midterm 02, Topic 0430) Differentiate  $3x^2 + 4y^4 = 7$  implicity, writing y' = dy/dx as an expression in x and y.

**ANSWER:** 
$$y' = -\frac{3x}{8y^3}$$
 **TOPIC:** 0430

## **BEGIN TOPIC: 0440**

2. (F12 Midterm 02A, Topic 0440) Compute  $\frac{d}{dx} \left[ (5 - \sin x)^{7 \arctan x} \right]$ .

**ANSWER:** 
$$[(5 - \sin x)^{7 \arctan x}][[\frac{7}{1 + x^2}][\ln(5 - \sin x)] + [7 \arctan x][\frac{-\cos x}{5 - \sin x}]]$$
  
**PIC:** 0440

F. (F12 Midterm 02A, Topic 0440) (no partial credit) Compute the derivative of  $\ln (x^{\arctan x})$ , with respect to x, on the interval x > 0. Circle one of the following answers:

TO

(a) 
$$\frac{1}{x^{\arctan x}}$$

(b) 
$$\frac{\ln x}{1+x^2} + \frac{\arctan x}{x}$$

(c) 
$$\frac{1}{x^{\sec^2 x}}$$

(d) 
$$x^{\sec^2 x}$$

ANSWER: b

**TOPIC:** 0440

3. (S12 Midterm 02A, Topic 0440) Let  $f(x) = 2x + 6x^5$ . Then f is a one-to-one function. Let  $g := f^{-1}$ . Then f(1) = 8, so g(8) = 1. Compute g'(8).

**ANSWER:** 1/32

**TOPIC:** 0440

1. (S12 Midterm 02A, Topic 0440) a. Compute  $\frac{d}{dx} \left[ \frac{2x^3 - 8}{3 + (\arctan(2x))} \right]$ .

ANSWER:  $\frac{[3 + (\arctan(2x))][6x^2] - [2x^3 - 8] \left[\frac{1}{1 + (2x)^2}\right][2]}{[3 + (\arctan(2x))]^2}$ 

**TOPIC:** 0440

b. Compute  $\frac{d}{dx} [(4 - \sin x)^x]$ .

**ANSWER:**  $[(4-\sin x)^x] \left[ (\ln(4-\sin x)) + x \left( \frac{-\cos x}{4-\sin x} \right) \right]$ 

**TOPIC:** 0400

3. (F11 Midterm 02A, Topic 0440) Suppose f is 1-1 and  $g = f^{-1}$  is the inverse of f. Suppose f(3) = 4 and f'(3) = 27. Compute g(4) and g'(4).

**ANSWER:** g(4) = 3 and g'(4) = 1/27

**TOPIC:** 0440

1. (F11 Midterm 02A, Topic 0440) Compute  $\frac{d}{dx} \left[ \frac{2x^3 - 8}{\arctan x} + xe^{\sin x} \right]$ 

**ANSWER:**  $\left[ \frac{(\arctan x)(6x^2) - (2x^3 - 8)\left(\frac{1}{1+x^2}\right)}{(\arctan x)^2} \right] + \left[ (e^{\sin x}) + x(e^{\sin x})(\cos x) \right]$ 

**TOPIC:** 0440

- 3. (S11 Midterm 03, Topic 0440) Let  $f(x) = x^3 + x + 1$ . Then  $f : \mathbb{R} \to \mathbb{R}$  turns out to be one-to-one and onto. Also, f(-2) = -9. Let  $g : \mathbb{R} \to \mathbb{R}$  be the inverse of f.
- a. Find f'(-2).

ANSWER: 13

**TOPIC:** 0320

b. Find g(-9).

ANSWER: -2

**TOPIC:** 0110

c. Find g'(-9).

**ANSWER:** 1/13

**TOPIC:** 0440

F. (S11 Midterm 03, Topic 0440) (no partial credit) Compute  $\frac{d}{dx} \left[ \arctan(x^2 + 2x + 5) \right]$ .

(a) 
$$\frac{2x+2}{x^2+2x+5}$$

(b) 
$$\frac{2x+2}{1+(x^2+2x+5)^2}$$

(c) 
$$\frac{2x+2}{\sqrt{1-(x^2+2x+5)^2}}$$

(d) 
$$\frac{1}{x^2 + 2x + 5}$$

(e) NONE OF THE ABOVE

ANSWER: (b)

**TOPIC:** 0440

11. (S10 Midterm 01, Topic 0440) Let  $f(x) = e^{2x}$ . It is a fact that f is a one-to-one function. Let g be the inverse function to f; that is, let  $g := f^{-1}$ . Note that f(0) = 1.

a. Compute g(1).

ANSWER: 0

**TOPIC:** 0110

b. Compute f'(0).

ANSWER: 2

**TOPIC:** 0370

c. It is a fact that, for all x, [g'(f(x))][f'(x)] = 1. Using this, compute g'(1).

ANSWER: 1/2

**TOPIC:** 0440

5. (S10 Midterm 02, Topic 0440) (no partial credit) Compute the derivative of  $x^{\arctan x}$ , with respect to x. Circle one of the following answers:

(a) 
$$[x^{\arctan x}] \left[ \frac{\arctan x}{x} + \frac{\ln x}{1+x^2} \right]$$

(b) 
$$[x^{\arctan x}] \left[ \frac{1}{x^{\arctan x}} \right]$$

(c) 
$$[x^{\arctan x}] \left[ \frac{1}{x^{\sec^2 x}} \right]$$

(d) 
$$[x^{\arctan x}][x^{\sec^2 x}]$$

ANSWER: (a)

**TOPIC:** 0440

4. (S10 Midterm 02, Topic 0440) (no partial credit) Compute the derivative of  $\ln \left[ x^{\arctan x} \right]$ , with respect to x. Circle one of the following answers:

(a) 
$$\frac{\arctan x}{x} + \frac{\ln x}{1+x^2}$$

(b) 
$$\frac{1}{x^{\arctan x}}$$

(c) 
$$\frac{1}{x^{\sec^2 x}}$$

(d) 
$$x^{\sec^2 x}$$

(e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0440

10. (S09 Midterm 01, Topic 0440) Let  $f(x) = e^{x^3+5x}$ . It is a fact that f is a one-to-one function. Let g be the inverse function to f; that is, let  $g := f^{-1}$ . Note that f(0) = 1.

a. Compute g(1).

ANSWER: 0

**TOPIC:** 0110

b. Compute g'(1).

ANSWER: 1/5 hskip1in TOPIC: 0440

2. (S09 Midterm 01, Topic 0440) Compute  $(d/dx)(\arctan(5x+7))$ .

**ANSWER:**  $\frac{5}{1 + (5x + 7)^2}$ 

**TOPIC:** 0440

END TOPIC: 0440

# TO-

#### **BEGIN TOPIC: 0450**

c. (F13 Midterm 02B, Topic 0450(13)) If f has a local minimum at c, then c is a critical number for f.

**ANSWER:** True **TOPIC:** 0450(13)

b. (F13 Midterm 02A, Topic 0450(13)) If f has a global maximum at c, then c is a critical number for f.

**ANSWER:** True **TOPIC:** 0450(13)

5. (F12 Midterm 02A, Topic 0450) Find the global maximum and minimum value of  $f(x) = x^3 - 3x^2 + 3x + 4$  on the interval  $0 \le x \le 2$ .

**ANSWER:** The minimum value is f(0) = 4; the maximum value is f(2) = 6.

**PIC:** 0450

- 4. (S11 Midterm 03, Topic 0450) Let  $h(x) = 3x^4 4x^3 + 36x^2$ .
- a. Find all critical numbers for h.

**ANSWER:** 0 is the only critical number for h. **TOPIC:** 0450

b. Find all critical numbers for h[-1, 1].

**ANSWER:** -1, 0 and 1 are the critical numbers for h[-1,1]. **TOPIC:** 0450

c. Find the global maximum and minimum values for h(x) on  $-1 \le x \le 1$ .

**ANSWER:** Global max value is 43 and global min value is 0. **TOPIC:** 0450

c. (S11 Midterm 03, Topic 0450) Every global extremum is a local extremum.

**ANSWER:** False **TOPIC:** 0450 (12)

b. (S11 Midterm 03, Topic 0450) Every local extremum occurs at a critical number.

**ANSWER:** True **TOPIC:** 0450 (13)

b. (S10 Midterm 02, Topic 0450) If f is a continuous function on [2,5], then f attains a global maximum and a global minimum on [2,5].

ANSWER: True

**TOPIC:** 0450 (15)

a. (S10 Midterm 02, Topic 0450) Any local maximum occurs at a critcal number.

ANSWER: True

**TOPIC:** 0450 (13)

6. (S10 Midterm 02, Topic 0450) (no partial credit) What are all the critical numbers of  $f(x) = x^3 - 3x + 8$ ?

- (a) x = 0 only
- (b) x = 1 and x = 2
- (c) x = -1 and x = 1
- (d) x = -2, x = 0 and x = 2
- (e) NONE OF THE ABOVE

ANSWER: (c)

**TOPIC:** 0450

10. (S09 Midterm 02, Topic 0450) Find the minimum value of  $f(x) = x^4 - 4x - 1$  with x ranging over [2,3].

**ANSWER:** min value = f(2) = 16 - 8 - 1 = 7

**TOPIC:** 0450

9. (S09 Midterm 02, Topic 0450) Find the minimum value of  $f(x) = x^4 - 4x - 1$ , with x ranging over all reals.

**ANSWER:** min value = f(1) = 1 - 4 - 1 = -4

**TOPIC:** 0450

e. (S09 Midterm 02, Topic 0450) If f has a local max at c, then c is a critical number for f.

ANSWER: True

**TOPIC:** 0450 (13)

d. (S09 Midterm 02, Topic 0450) If c is a critical number for f, then f either has a local max or a local min at c.

**ANSWER:** False, e.g.,  $f(x) = x^3$  at x = 0

**TOPIC:** 0450

**END TOPIC: 0450** 

# **BEGIN TOPIC: 0460**

d. (S14 Midterm 02A, Topic 0460(22)) Let  $f, g : \mathbb{R} \to \mathbb{R}$  be any two differentiable functions such that, for all  $x \in \mathbb{R}$ , f'(x) = g'(x). Then f - g is a constant.

**ANSWER:** True

**TOPIC:** 0460(22)

a. (S14 Midterm 02C, Topic 0460(22)) Let  $f, g : \mathbb{R} \to \mathbb{R}$  be any two differentiable functions such that, for all  $x \in \mathbb{R}$ , f'(x) = g'(x). Then f = g.

**ANSWER:** False

**TOPIC:** 0460(22)

d. (F13 Midterm 02A, Topic 0460(22)) Let  $f, g : \mathbb{R} \to \mathbb{R}$  be any two differentiable functions such that, for all  $x \in \mathbb{R}$ , f'(x) = g'(x). Then f - g is a constant.

ANSWER: True

**TOPIC:** 0460(22)

c. (F13 Midterm 02A, Topic 0460(29)) Let f be any function such that f(x) is increasing on 1 < x < 3. Then f'(2) > 0.

ANSWER: False

**TOPIC:** 0460(29)

a. (S12 Midterm 02A, Topic 0460) If f' = g' on an interval I, then f - g is constant on I.

ANSWER: True

**TOPIC:** 0460

c. (S09 Midterm 02, Topic 0460) Let f be differentiable at all real numbers. Suppose f(0) = 0 and f(2) = 2. Then, for some  $c \in (0, 2)$ , we have: f'(c) = 1.

ANSWER: True

**TOPIC:** 0460 (13)

a. (S09 Midterm 02, Topic 0460) If two functions have the same derivative, then they are equal.

ANSWER: False

**TOPIC:** 0460 (22)

**END TOPIC: 0460** 

## **BEGIN TOPIC: 0470**

b. (S14 Midterm 02A, Topic 0470(31)) Let  $f : \mathbb{R} \to \mathbb{R}$  be any function with a global maximum at 7. Assume that f'' is defined on  $\mathbb{R}$ . Then f'(7) = 0 and f''(7) < 0.

ANSWER: False

**TOPIC:** 0470(31)

b. (S14 Midterm 02C, Topic 0470(31)) Let  $f : \mathbb{R} \to \mathbb{R}$  be any function such that f'(2) = 0 and f''(2) > 0. Assume that f'' is defined on  $\mathbb{R}$ . Then f has a local minimum at 2.

ANSWER: True

**TOPIC:** 0470(31)

a. (S14 Midterm 02B, Topic 0470(31)) Let  $f : \mathbb{R} \to \mathbb{R}$  be any function such that f'(4) = 0 and f''(4) < 0. Assume that f'' is defined on  $\mathbb{R}$ . Then f has a global maximum at 4.

**ANSWER:** False **TOPIC:** 0470(31)

a. (S14 Midterm 02B, Topic 0470(31)) Let  $f : \mathbb{R} \to \mathbb{R}$  be any function such that f'(8) = 0 and f''(8) > 0. Assume that f'' is defined on  $\mathbb{R}$ . Then f has a local maximum at 8.

**ANSWER:** False **TOPIC:** 0470(31)

D. (S14 Midterm 02A, Topic 0470(21-22)) (no partial credit) Suppose  $f''(x) = -(x-7)^3(x-8)^4$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave down on  $(-\infty, 7]$  and up on  $[7, \infty)$ ].
- (b) f is concave up on  $(-\infty, 7]$  and down on  $[7, \infty)$ ].
- (c) f is concave up on  $(-\infty, 7]$ , down on [7, 8] and up on  $[8, \infty)$ .
- (d) f is concave down on  $(-\infty, 7]$ , up on [7, 8] and down on  $[8, \infty)$ .
- (e) NONE OF THE ABOVE

**ANSWER:** b **TOPIC:** 0470(21-22)

E. (S14 Midterm 02D, Topic 0470(21-22)) (no partial credit) Suppose  $f''(x) = (x-7)^2(x-8)^4$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave down on  $(-\infty, 7]$  and up on  $[7, \infty)$ ].
- (b) f is concave up on  $(-\infty, 7]$  and down on  $[7, \infty)$ ].
- (c) f is concave up on  $(-\infty, 7]$ , down on [7, 8] and up on  $[8, \infty)$ .
- (d) f is concave down on  $(-\infty, 7]$ , up on [7, 8] and down on  $[8, \infty)$ .
- (e) NONE OF THE ABOVE

**ANSWER:** e, f is concave up on  $\mathbb{R} = (-\infty, \infty)$ . **TOPIC:** 0470(21-22)

C. (S14 Midterm 02B, Topic 0470(21-22)) (no partial credit) Suppose  $f''(x) = -(x - 7)^3(x - 8)^3$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave down on  $(-\infty, 7]$  and up on  $[7, \infty)$ ].
- (b) f is concave up on  $(-\infty, 7]$  and down on  $[7, \infty)$ ].
- (c) f is concave up on  $(-\infty, 7]$ , down on [7, 8] and up on  $[8, \infty)$ .

- (d) f is concave down on  $(-\infty, 7]$ , up on [7, 8] and down on  $[8, \infty)$ .
- (e) NONE OF THE ABOVE

ANSWER: d

**TOPIC:** 0470(21-22)

C. (S14 Midterm 02C, Topic 0470(21-22)) (no partial credit) Suppose  $f''(x) = (x-7)^3(x-8)^4$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave down on  $(-\infty, 7]$  and up on  $[7, \infty)$ ].
- (b) f is concave up on  $(-\infty, 7]$  and down on  $[7, \infty)$ ].
- (c) f is concave up on  $(-\infty, 7]$ , down on [7, 8] and up on  $[8, \infty)$ .
- (d) f is concave down on  $(-\infty, 7]$ , up on [7, 8] and down on  $[8, \infty)$ .
- (e) NONE OF THE ABOVE

ANSWER: a

**TOPIC:** 0470(21-22)

a. (F13 Midterm 02A, Topic 0470(22)) Let f be any function such that f(3) = 5 and such that f''(3) = 0. Then (3,5) is a point of inflection for f.

ANSWER: False

**TOPIC:** 0470(22)

D. (F13 Midterm 02C, Topic 0470(21)) (no partial credit) Suppose  $f''(x) = -x^2 - 4x - 3$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave up on  $(-\infty, 1]$ , down on [1, 3] and up on  $[3, \infty)$ .
- (b) f is concave down on  $(-\infty, 1]$ , up on [1, 3] and down on  $[3, \infty)$ .
- (c) f is concave up on  $(-\infty, -3]$ , down on [-3, -1] and up on  $[-1, \infty)$ .
- (d) f is concave down on  $(-\infty, -3]$ , up on [-3, -1] and down on  $[-1, \infty)$ .
- (e) NONE OF THE ABOVE

ANSWER: d

**TOPIC:** 0470(21)

E. (F13 Midterm 02C, Topic 0470(31)) (no partial credit) Suppose

- f'(1) = 1, f''(1) = 3,
- f'(2) = 0, f''(2) = 2,
- f'(3) = 0, f''(3) = -6.

At which of the numbers 1, 2 and 3 does f have a local minimum? Circle one of the following answers:

- (a) 1, but not 2, and not 3
- (b) 2, but not 1, and not 3
- (c) 3, but not 1, and not 2
- (d) both 2 and 3, but not 1
- (e) NONE OF THE ABOVE

ANSWER: b

**TOPIC:** 0470(31)

E. (F13 Midterm 02B, Topic 0470(31)) (no partial credit) Suppose

- f'(1) = 1, f''(1) = 3,
- f'(2) = 0, f''(2) = -2,
- f'(3) = 0, f''(3) = 6.

At which of the numbers 1, 2 and 3 does f have a local maximum? Circle one of the following answers:

- (a) 1, but not 2, and not 3
- (b) 2, but not 1, and not 3
- (c) 3, but not 1, and not 2
- (d) both 2 and 3, but not 1
- (e) NONE OF THE ABOVE

ANSWER: b

**TOPIC:** 0470(31)

C. (F13 Midterm 02A, Topic 0470(31)) (no partial credit) Suppose

- f'(1) = 1, f''(1) = 3,
- f'(2) = 0, f''(2) = -2,
- f'(3) = 0, f''(3) = 6.

At which of the numbers 1, 2 and 3 does f have a local minimum? Circle one of the following answers:

- (a) 1, but not 2, and not 3
- (b) 2, but not 1, and not 3
- (c) 3, but not 1, and not 2
- (d) both 2 and 3, but not 1
- (e) NONE OF THE ABOVE

ANSWER: c

**TOPIC:** 0470(31)

c. (S13 Midterm 02D, Topic 0470(4)) If f' > 0 on an interval I, then f is increasing on I.

ANSWER: True

**TOPIC:** 0470(4)

b. (S13 Midterm 02A, Topic 0470(11)) If f is increasing on an interval I, then f' > 0 on I.

ANSWER: False TOPIC: 0470(11)

F. (S13 Midterm 02D, Topic Topic 0470(5-12)) (no partial credit) Suppose  $f'(x) = (x-1)^5(x-2)^6(x-3)^8$ . Which of the following is a maximal interval of decrease for f? Circle one of the following answers:

- (a)  $(-\infty, 1]$
- (b)  $[1, \infty)$
- (c)  $(-2, \infty)$
- (d)  $[2,\infty)$
- (e) NONE OF THE ABOVE

ANSWER: a

**TOPIC:** Topic 0470(5-12)

B. (S13 Midterm 02A, Topic 0470(5-12)) (no partial credit) Suppose  $f'(x) = (x-1)^2(x-2)(x-3)^2$ . Which of the following is a maximal interval of increase for f? Circle one of the following answers:

- (a)  $[2, \infty)$
- (b)  $(-2, \infty)$
- (c)  $[1, \infty)$
- (d)  $(-\infty, 1]$
- (e) NONE OF THE ABOVE

ANSWER: a

**TOPIC:** 0470(5-12)

4. (never used, Topic 0470) Find the maximal intervals of increase and decrease for  $f(x) = x^4 + 4x^3 + 8$ . For each interval, state clearly whether f is increasing or decreasing on that interval.

**ANSWER:** decreasing on  $(-\infty, -3]$ , increasing on  $[-3, \infty)$  **TOPIC:** 0470

4. (S12 Midterm 02A, Topic 0470) Find the maximal intervals of concavity for  $f(x) = -3x^5 + 20x^4 + 7x + 3$ . For each interval, state clearly whether f is concave up or concave down on that interval.

**ANSWER:** concave up on  $(-\infty, 4]$  and concave down on  $[4, \infty)$  **TOPIC:** 0470

d. (S12 Midterm 02A, Topic 0470) If f'' > 0 on an interval I, then f is concave up on I.

ANSWER: True TOI

**TOPIC:** 0470

c. (S12 Midterm 02A, Topic 0470) If f'(7) = 0 and f''(7) < 0, then f has a local maximum at 7.

ANSWER: True

**TOPIC:** 0470

b. (S12 Midterm 02A, Topic 0470) Every critical number occurs at local extremum.

ANSWER: False

**TOPIC:** 0470

C. (S12 Midterm 02A, Topic 0470) (no partial credit) Suppose  $f''(x) = -x^2 + 4x - 3$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave down on  $(-\infty, 1]$ , up on [1, 3] and down on  $[3, \infty)$ .
- (b) f is concave up on  $(-\infty, 1]$ , down on [1, 3] and up on  $[3, \infty)$ .
- (c) f is concave down on  $(-\infty, -3]$ , up on [-3, -1] and down on  $[-1, \infty)$ .
- (d) f is concave up on  $(-\infty, -3]$ , down on [-3, -1] and up on  $[-1, \infty)$ .
- (e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0470

3. (S09 Midterm 02, Topic 0470) Find the maximal intervals of concavity for  $x^3 - 9x^2 + 24x + 4$ . For each interval, state clearly whether f is concave up or concave down on that interval.

**ANSWER:** cc dn on  $(-\infty, 3]$ , cc up on  $[3, \infty)$ 

**TOPIC:** 0470 (21-22)

2. (S09 Midterm 02, Topic 0470) Find the maximal intervals of increase and decrease for  $x^3 - 9x^2 + 24x + 4$ . For each interval, state clearly whether f is increasing or decreasing on that interval.

**ANSWER:** incr. on  $(-\infty, 2]$ , decr. on [2, 4], incr. on  $[4, \infty)$ 

**TOPIC:** 0470 (5-8)

#### **END TOPIC: 0470**

#### **BEGIN TOPIC: 0480**

c. (F12 Midterm 02A, Topic 0480) If f' > 0 on an interval I, then f is increasing on I.

ANSWER: True

**TOPIC:** 0480

b. (F12 Midterm 02A, Topic 0480) If f is increasing on an interval I, then f' > 0 on I.

ANSWER: False

**TOPIC:** 0480

D. (F12 Midterm 02A, Topic 0480) (no partial credit) Suppose  $f'(x) = (x-1)^2(x-2)(x-3)^2$ . Which of the following is a maximal interval of increase for f? Circle one of the following answers:

- (a)  $(-\infty, 1]$
- (b)  $[1,\infty)$
- (c)  $(2, \infty)$
- (d)  $[2,\infty)$
- (e) NONE OF THE ABOVE

ANSWER: d

**TOPIC:** 0480

C. (F12 Midterm 02A, Topic 0480) (no partial credit) Suppose  $f''(x) = -x^2 + 4x - 3$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is concave down on  $(-\infty, 1]$ , up on [1, 3] and down on  $[3, \infty)$ .
- (b) f is concave up on  $(-\infty, 1]$ , down on [1, 3] and up on  $[3, \infty)$ .
- (c) f is concave down on  $(-\infty, -3]$ , up on [-3, -1] and down on  $[-1, \infty)$ .
- (d) f is concave up on  $(-\infty, -3]$ , down on [-3, -1] and up on  $[-1, \infty)$ .
- (e) NONE OF THE ABOVE

ANSWER: a

**TOPIC:** 0480

12. (S10 Midterm 02, Topic 0480) Determine the maximal intervals of concavity for  $f(x) = x^5 - 5x^4 + 4x - 3$ .

**ANSWER:** concave down on  $(-\infty, 3]$ , concave up on  $[3, \infty)$ 

**TOPIC:** 0480

**END TOPIC: 0480** 

**BEGIN TOPIC: 0490** 

END TOPIC: 0490

## **BEGIN TOPIC: 0500**

4. (F11 Midterm 02A, Topic 0500) Find the maximal intervals of increase and decrease for  $f(x) = x^3 - 6x^2 + 5$ .

**ANSWER:** f is increasing on  $(\infty, 0]$ , decreasing on [0, 4] and increasing on  $[4, \infty)$ .

**TOPIC:** 0500

d. (F11 Midterm 02A, Topic 0500) If f is increasing on an interval I, then f' > 0 on I.

ANSWER: False TOPIC: 0500

c. (F11 Midterm 02A, Topic 0500) Every global extremum occurs at a critical number.

ANSWER: True TOPIC: 0500

b. (F11 Midterm 02A, Topic 0500) Every local extremum occurs at a critical number.

ANSWER: True TOPIC: 0500

a. (F11 Midterm 02A, Topic 0500) If f'(3) = 0 and f''(3) < 0, then f has a local maximum at 3.

ANSWER: True TOPIC: 0500

A. (F11 Midterm 02A, Topic 0500) (no partial credit) Suppose  $f'(x) = -x^2 + 3x - 2$ . At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

- (a) f is increasing on  $(-\infty, 1]$ , decreasing on [1, 2] and increasing on  $[2, \infty)$ .
- (b) f is decreasing on  $(-\infty, 1]$ , increasing on [1, 2] and decreasing on  $[2, \infty)$ .
- (c) f is increasing on  $(-\infty, -2]$ , decreasing on [-2, -1] and increasing on  $[-1, \infty)$ .
- (d) f is decreasing on  $(-\infty, -2]$ , increasing on [-2, -1] and decreasing on  $[-1, \infty)$ .
- (e) NONE OF THE ABOVE

**ANSWER:** (b) **TOPIC:** 0500

## END TOPIC: 0500

## **BEGIN TOPIC: 0510**

3. (S14 Midterm 02D, Topic 0510) We are asked to design a large cup in the shape of a cylinder. The cup is to have an open top, and must contain  $2\pi$  cubic feet of volume inside. Let r be the radius of the top of the cup. On the interval r > 0, find the choice of r (in

feet) that minimizes the surface area, A, of the cup. (HINT: Our local precalculus expert shows us the formula that relates A to r. It is  $A = \pi r^2 + (4\pi/r)$ .)

**ANSWER:** On r > 0, A attains a global minimum only at  $r = 2^{1/3}$  ft.

**TOPIC:** 0510

3. (S14 Midterm 02A, Topic 0510) We are asked to design a large cup in the shape of an inverted (i.e., upside down) cone. The cup is to have an open top, and must contain  $\pi/3$  cubic feet of volume inside. Let r be the radius of the top of the cup. On the interval r > 0, find the choice of r (in feet) that minimizes the surface area, A, of the cup. (HINT: Our local precalculus expert shows us the formula that relates A to r. It is  $A = (\pi r)\sqrt{r^2 + r^{-4}}$ .)

**ANSWER:** On r > 0, A attains a global minimum only at  $r = (1/2)^{1/6}$  ft.

**TOPIC:** 0510

3. (F13 Midterm 02A, Topic 0510) We must design a cylindrical can that contains  $128\pi$  cubic feet of volume inside, and which minimizes surface area. Find the height, h, and the radius, r, of such a can. (Remember: The surface area includes the side of the can, the bottom of the can, and the top of the can).

**ANSWER:** r = 4 feet, h = 8 feet **TOPIC:** 0510

3. (F13 Midterm 02A, Topic 0510) We must design a cylindrical can with an open top that contains  $8\pi$  cubic feet of volume inside, and which minimizes surface area. Find the height, h, and the radius, r, of such a can. (Remember: The surface area includes the side of the can, and the bottom of the can, but the can has no top).

**ANSWER:** r = 2 feet, h = 2 feet **TOPIC:** 0510

5. (F11 Midterm 02A, Topic 0510) Among all pairs of positive numbers x and y such that xy = 100, find the global maximum value of x + 4y, provided it exists. Then find the global minimum value, provided it exists. (NOTE: If the global maximum value does not exist, you need to state that clearly to receive full credit. If it does exist, for full credit, you'll need to compute x + 4y; computing x and/or y alone is insufficient. These same comments apply to the global minimum value.)

**ANSWER:** f has no global maximum. f has a global minimum at x = 20, y = 5, with global minimum value = 40. **TOPIC:** 0510

11. (S10 Midterm 02, Topic 0510) A cylindrical can is to be constructed with a total surface area (including the side, the top and the bottom of the can) of 60 square inches. What should its radius and height be if we want to maximize the volume enclosed?

**ANSWER:**  $r = \sqrt{\frac{10}{\pi}}, \quad h = \frac{40}{2\sqrt{10\pi}}$  **TOPIC:** 0510

## END TOPIC: 0510

#### **BEGIN TOPIC: 0520**

4. (S14 Midterm 02A, Topic 0520) A square-based pyramid is growing. Its height is always equal to the length, s, of the sides of its base. Assume that its volume is always growing at a rate of 10 cubic feet per minute. Find the rate of growth in s (in feet per minute) at the moment when the volume is 9 cubic feet. (HINT: According to our local precalculus expert, its volume, V, is given by  $V = s^3/3$ .)

**ANSWER:** 10/9 ft/min **TOPIC:** 0520

4. (S14 Midterm 02D, Topic 0520) A conical pile of sand is growing. Its height is always equal to the radius, r, of its base. Assume that its volume is always growing at a rate of  $5\pi$  cubic feet per minute. Find the rate of growth in r (in feet per minute) at the moment when the volume is  $9\pi$  cubic feet. (HINT: According to our local precalculus expert, its volume, V, is given by  $V = \pi r^3/3$ .)

**ANSWER:** 5/9 ft/min **TOPIC:** 0520

4. (F13 Midterm 02A, Topic 0520) Sand is accumulating in a conical pile whose base radius is, at all times, twice its height. Sand is being added at 200 cubic feet per hour. At what rate is the height of the pile increasing, at the moment when the height is 5 feet?

**ANSWER:**  $2/\pi$  feet per hour **TOPIC:** 0520

6. (S09 Midterm 02, Topic 0520) A square is growing, with its interior area gaining 10 cm<sup>2</sup> per second. At a certain moment in time, its side length is 5 cm. At that moment, how fast is its side length growing (in cm per second)?

ANSWER: 1 cm per second TOPIC: 0520

#### **END TOPIC: 0520**

#### **BEGIN TOPIC: 0530**

E. (S14 Midterm 02A, Topic 0530) (no partial credit) Let  $f(x) = e^{2x} + 3x$ . What is the iterative formula of Newton's method used to solve f(x) = 0? Circle one of the following answers:

(a) 
$$x_{n+1} = x_n + \frac{e^{2x_n} + 3}{e^{2x_n} + 3x_n}$$

(b) 
$$x_{n+1} = x_n + \frac{2e^{2x_n} + 3}{e^{2x_n} + 3x_n}$$

(c) 
$$x_{n+1} = x_n + \frac{e^{2x_n} + 3x_n}{e^{2x_n} + 3}$$

(d) 
$$x_{n+1} = x_n + \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3}$$

**ANSWER:** e, 
$$x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3}$$

**TOPIC:** 0530

4. (S09 Midterm 02, Topic 0530) Apply Newton's method to  $f(x) = 3 + \tan x$ , with an initial guess  $x_1 = \pi/4$ . Find the second guess  $x_2$ . (Hint:  $\tan(\pi/4) = 1$  and  $\sec(\pi/4) = \sqrt{2}$ .) Circle one of the following answers:

(a) 
$$\frac{\pi}{4} - 2$$

(b) 
$$\frac{\pi}{4} - \frac{1}{2}$$

(c) 
$$\frac{\pi}{4} + 2$$

(d) 
$$\frac{\pi}{4} + \frac{1}{2}$$

(e) NONE OF THE ABOVE

ANSWER: (a)

**TOPIC:** 0530

**END TOPIC: 0530** 

## **BEGIN TOPIC: 0540**

F. (S14 Midterm 02A, Topic 0540(2)) (no partial credit) Let  $y = x^2 + x$ . Compute dy, evaluated at x = 10, dx = 0.1. Circle one of the following answers:

- (a) 1.2
- (b) 2.1
- (c) 1.22
- (d) 2.11
- (e) NONE OF THE ABOVE

ANSWER: b

**TOPIC:** 0540(2)

1. (F13 Midterm 02A, Topic 0540) Compute  $d\left[\frac{e^{-x^4}}{4 + (\tan(x^2))}\right]$ . (Here  $e^{-x^4}$  means  $e^{\left(-x^4\right)}$ .)

WARNING: You are asked to find the differential, d, and NOT the derivative, d/dx.

ANSWER: 
$$\frac{[4 + (\tan(x^2))][-4x^3e^{-x^4}] - [e^{-x^4}][2x(sec^2(x^2))]}{[4 + (\tan(x^2))]^2}$$
 TOPIC: 0540

- F. (F13 Midterm 02A, Topic 0540) (no partial credit) Let  $y=x^2+x$ . Compute  $\triangle y$ , evaluated at x=10,  $\triangle x=0.1$ . (Hint:  $\triangle(x^2)=2x(\triangle x)+(\triangle x)^2$ .)
  - (a) 1.22
  - (b) 2.11
  - (c) 2.1
  - (d) 1.2
  - (e) NONE OF THE ABOVE

ANSWER: b

**TOPIC:** 0540

c. Compute L(3.08, Topic0540). (Hint: Just substitute 3.08 for x in L(x).)

**ANSWER:** 2 + 5(3.08 - 3) = 2 + 5(0.08) = 2.4

**TOPIC:** 0540 (3)

- 9. (S10 Midterm 01, Topic 0540) Let y = L(x) be the equation of the tangent line to  $y = 5e^{-3}e^x$  at the point (3,5). Remember that  $5e^{-3}$  is a constant; it's derivative is zero.
- a. Find the slope of L, i.e., the slope of the tangent line to  $y = 5e^{-3}e^x$  at (3,5).

ANSWER: 5

**TOPIC:** 0330

b. Find L(x). (Hint: You know the slope of L and you know that it passes through (3,5).)

**ANSWER:** L(x) = 2 + 5(x - 3)

**TOPIC:** 0540 (3)

- 7. (S09 Midterm 01, Topic 0540) Let y = L(x) be the equation of the tangent line to  $y = \sqrt{x}$  at the point (4,2).
- a. Find L(x).

**ANSWER:**  $L(x) = 2 + \frac{1}{4}(x-4)$ 

**TOPIC:** 0540

b. Evaluate L at 4.01, *i.e.*, compute L(4.01).

**ANSWER:**  $L(4.01) = 2 + \frac{1}{4}(0.01) = 2.0025$ 

**TOPIC:** 0540

11. (S09 Midterm 02, Topic 0540) The side length of an isosceles right triangle is measured to be 4 inches with an error of 0.1 inch. Using differentials, estimate the error in the area (in square inches). ("Isosceles" means two of its sides have the same length; the hypotenuse is longer.)

**ANSWER:**  $A = s^2/2$ , so dA = s ds, so error  $\approx 4(0.1) = 0.4$ 

**TOPIC:** 0540

## END TOPIC: 0540

#### **BEGIN TOPIC: 0550**

1. (S14 Midterm 02C, Topic 0550(38)) Find an antiderivative w.r.t. x of  $\cos^2(x-3)$ . (Hint:  $\cos(2\theta) = -1 + 2(\cos^2\theta)$ .)

**ANSWER:**  $\frac{x}{2} + \frac{\sin(2x-6)}{4}$ 

**TOPIC:** 0550(38)

1. (S14 Midterm 02A, Topic 0550(38)) Find an antiderivative w.r.t. x of  $\sin^2(2x-1)$ . (Hint:  $\cos(2\theta) = 1 - 2(\sin^2\theta)$ .)

**ANSWER:**  $\frac{x}{2} - \frac{\sin(4x-2)}{8}$ 

**TOPIC:** 0550(38)

#### **END TOPIC: 0550**

#### **BEGIN TOPIC: 0560**

1. (S09 Midterm 02, Topic 0560) Find an antiderivative of  $x^2 + 4x - 5$ .

**ONE ANSWER:**  $(x^3/3) + 2x^2 - 5x$ 

**TOPIC:** 0560 (2)

#### **END TOPIC: 0560**

#### **BEGIN TOPIC: 0570**

#### END TOPIC: 0570

#### **BEGIN TOPIC: 0580**

c. (S14 Midterm 02A, Topic 0580(28)) If f is continuous on [a, b], then

$$\int_{a}^{b} (f(x)) dx = \lim_{n \to \infty} [M_n S_a^b f].$$

d. (S14 Midterm 02C, Topic 0580(28)) If f is continuous on [a, b], then

$$\int_{a}^{b} (f(x)) dx = \lim_{n \to \infty} [L_n S_a^b f].$$

ANSWER: True

**TOPIC:** 0580(28)

e. (S14 Midterm 02D, Topic 0580(28)) If f is continuous on [a, b], then

$$\int_{a}^{b} (f(x)) dx = \lim_{n \to \infty} [R_n S_a^b f].$$

ANSWER: True

**TOPIC:** 0580(28)

A. (S14 Midterm 02A, Topic 0580(30)) (no partial credit) Let  $f(x) = \sin^2(5x^4 + 1)$ . Compute  $\int_2^2 f(x) dx$ . Circle one of the following answers:

- (a) 0
- (b) 2
- (c) 6
- (d) 20
- (e) NONE OF THE ABOVE

ANSWER: a

**TOPIC:** 0580(30)

END TOPIC: 0580

## **BEGIN TOPIC: 0590**

B. (S14 Midterm 02A, Topic 0590(12)) (no partial credit) Let  $f(x) = e^{3x-4}$ . Recall that  $M_2S_1^5f$  denotes the midpoint Riemann sum, from 1 to 5, of f, with two subintervals. Which of these is equal to  $M_2S_1^5f$ ? Circle one of the following answers:

- (a)  $e^5 + e^{11}$
- (b)  $e^2 + e^8$
- (c)  $2(e^5 + e^{11})$
- (d)  $2(e^2 + e^8)$

ANSWER: d

**TOPIC:** 0590(12)

B. (S14 Midterm 02B, Topic 0590(12)) (no partial credit) Let  $f(x) = e^{3x-4}$ . Recall that  $L_2S_1^5f$  denotes the left endpoint Riemann sum, from 1 to 5, of f, with two subintervals. Which of these is equal to  $L_2S_1^5f$ ? Circle one of the following answers:

- (a)  $2(e^5 + e^{11})$
- (b)  $2(e^{-1} + e^5)$
- (c)  $e^5 + e^{11}$
- (d)  $2(e^2 + e^8)$
- (e) NONE OF THE ABOVE

ANSWER: b

**TOPIC:** 0590(12)

END TOPIC: 0590

**BEGIN TOPIC: 0600** 

END TOPIC: 0600

## **BEGIN TOPIC: 0610**

d. (S14 Midterm 02B, Topic 0610(20))  $\frac{d}{dx} \left[ \int_{1}^{x} \sin(e^{t}) dt \right] = \sin(e^{x}).$ 

ANSWER: True

**TOPIC:** 0610(20)

a. (S14 Midterm 02A, Topic 0610(20)) 
$$\frac{d}{dx} \left[ \int_1^x \sin(e^t) dt \right] = \cos(e^x).$$

ANSWER: False

**TOPIC:** 0610(20)

END TOPIC: 0610

# **BEGIN TOPIC: 0620**

2. (S14 Midterm 02D, Topic 0620(21-29)) Let  $f(x) = \int_{2+5x}^{1+e^x} \sqrt{t^3+1} \ dt$ . Compute f'(0).

2. (S14 Midterm 02B, Topic 0620(21-29)) Let  $f(x) = \int_{2x-1}^{e^{x-1}} \sqrt{2t^6 - 2t^2 + 4} dt$ . Compute f'(1).

**ANSWER:** -2 **TOPIC:** 0620(21-29)

2. (S14 Midterm 02A, Topic 0620(21-29)) Let  $f(x) = \int_x^{x^4} \sqrt{t^6 + 4t^2 + 4} dt$ . Compute f'(1).

**ANSWER:** 9 **TOPIC:** 0620(21-29)

**END TOPIC: 0620** 

**BEGIN TOPIC: 0630** 

**END TOPIC: 0630** 

**BEGIN TOPIC: 0640** 

END TOPIC: 0640

**BEGIN TOPIC: 0650** 

**END TOPIC: 0650** 

**BEGIN TOPIC: 0660** 

**END TOPIC: 0660** 

**BEGIN TOPIC: 0670** 

**END TOPIC: 0670** 

**BEGIN TOPIC: 0680** 

**END TOPIC: 0680** 

BEGIN TOPIC: 0690
END TOPIC: 0690
BEGIN TOPIC: 0700
END TOPIC: 0700
BEGIN TOPIC: 0710
END TOPIC: 0710
BEGIN TOPIC: 0720
END TOPIC: 0720
BEGIN TOPIC: 0730
END TOPIC: 0730
BEGIN TOPIC: 0740
END TOPIC: 0740
BEGIN TOPIC: 0750
END TOPIC: 0750