MATH 1271 Spring 2014, Midterm #1
Handout date: Thursday 27 February 2014
Instructor: Scot Adams

PRINT YOUR NAME:

SOLUTIONS
Version D

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I. Multiple choice

A. (5 pts) (no partial credit) Compute $\underbrace{[d/dx][2e^3 + 5\sin x]}_{= 0 + 5\cos x}$. Circle one of the following answers:

(a) $5\cos x$

(b) $-5\cos x$

(c) $6e^2 + 5\cos x$

(d) $6e^3 + 5\cos x$

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $\left[\frac{d}{dx}\right]\left[\frac{e^x}{x^4 - 8x}\right]$. Circle one of the following answers:

(a) $\frac{(e^x)(4x^3 - 8) - (x^4 - 8x)(e^x)}{(x^4 - 8x)^2}$

(b) $\frac{(x^4 - 8x)(e^x) - (e^x)(4x^3 - 8)}{(x^4 - 8x)^2}$

(c) $\frac{xe^{x-1}}{4x^3 - 8}$

(d) $\frac{e^x}{4x^3 - 8}$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x\to\infty}(f(x)) = -\infty$? Circle one of the following answers:

(a) If x is very positive, then $f(x)$ is very negative.

(b) If x is very negative, then $f(x)$ is very positive.

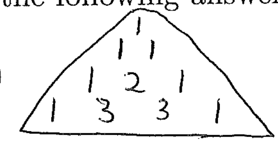
(c) If $f(x)$ is very negative, then x is very positive.

(d) If $f(x)$ is very positive, then x is very negative.

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Compute $\triangle(x^3 - x^2)$. Circle one of the following answers:

$$= (\Delta(x^3) - \Delta(x^2)) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2x(\Delta x) - (\Delta x)^2$$



(a) $3x^2 - 2x$

(b) $3x^2 + 3x(\triangle x) + (\triangle x)^2 - 2x - (\triangle x)$

(c) $3x^2(\triangle x) + 3x(\triangle x)^2 + (\triangle x)^3 - 2x(\triangle x)$

(d) $(3x^2 - 2x)(\triangle x)$

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Let $f(t) = \tan^2 t$. Compute $f'(\pi/4)$.
(Hint: $f(t) = (\tan t)(\tan t)$.) Circle one of the following answers:

$$f'(t) = (\sec^2 t)(\tan t) + (\tan t)(\sec^2 t)$$
$$= 2(\tan t)(\sec^2 t)$$
$$f'(\pi/4) = 2(1)\left(\frac{1}{1/2}\right) = 4$$

(a) $-\sqrt{2}/2$

(b) -1

(c) 1

(d) 4

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Let $g(x) = [8 - 3x]\left[\dfrac{x-5}{x-5}\right]$. What is the largest $\delta > 0$ such that $0 < |x - 5| < \delta \quad \Rightarrow \quad |(g(x)) + 7| < 0.6$? Circle one of the following answers:

$$\frac{\pm 0.6}{-3} = \mp 0.2$$

$$\delta = 0.2$$

(a) 0.3

(b) -0.3

(c) 1.8

(d) 0.2

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) $\dfrac{d}{dx}\left[\dfrac{\sin x}{x^2}\right] = \dfrac{\cos x}{2x}$.

False

b. (5 pts) If f and g are both differentiable at 3, then $2f^9g^8$ is also differentiable at 3.

True

c. (5 pts) If P is any polynomial of degree 5 and Q is any polynomial of degree 3, then $\displaystyle\lim_{x\to-\infty}\left[\frac{P(x)}{Q(x)}\right] = \infty$.

$$\frac{-x^5}{x^3} = -x^2 \xrightarrow{x\to-\infty} -\infty$$

False

d. (5 pts) $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{x} = 0$.

$$\frac{1-\cos x}{x} \underset{x\to 0}{\sim} \frac{x^2/2}{x} \overset{x\neq 0}{=} \frac{x}{2} \xrightarrow{x\to 0} 0$$

True

e. (5 pts) Let f and g be any two functions such that $f'(5) = 50$ and $g'(3) = 30$. Then $(f-g)'(2) = 20$.

False

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3ab

III. 4abc

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute $\underbrace{\frac{d}{dx}\left[\frac{(2x^3+x)(4+7e^x)}{\cot x}\right]}_{=}$.

$$\frac{[\cot x][(6x^2+1)(4+7e^x)+(2x^3+x)(7e^x)]-[(2x^3+x)(4+7e^x)][-\csc^2 x]}{\cot^2 x}$$

2. (10 pts) Compute $\lim_{x\to 0}\left[\underbrace{\frac{(\sin^2(4x))(\tan x)}{(\sin(2x))(\cos(3x))(3x^5-2x^4-4x^2)}}\right]$.

$$\Big\downarrow x\to 0$$

$$\frac{(4x)^2\left(\frac{x}{1}\right)}{(2x)(1)(-4x^2)}$$

$$\|$$

$$\frac{16x^3}{-8x^3}$$

$$\|\, x\neq 0$$

$$-2$$

$$\Big\downarrow x\longrightarrow 0$$

$$-2$$

3. Let $f(x) = -x^6 + 6x^4 + (\tan(e))$.

a. (5 pts) Find all $a \in \mathbb{R}$ such that the graph of f has a horizontal tangent line at $(a, f(a))$.

$$f'(x) = -6x^5 + 24x^3$$
$$= -6x^3(x^2 - 4)$$
$$= -6x^3(x+2)(x-2)$$

$(a = 0)$ OR $(a = -2)$ OR $(a = 2)$

b. (5 pts) Find all the maximal intervals on which f' is negative.

f' pos $\quad 0 \quad$ neg $\quad 0^3 \quad$ pos $\quad 0 \quad$ neg

$\qquad -2 \qquad\qquad 0 \qquad\qquad 2$

f' is negative on $(-2, 0)$

and on $(2, \infty)$.

4. Let $y = 3x^3 - 5x$. Then $\triangle y = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3 + k(\triangle x)$, for some real numbers a, b, c, k.

a. (5 pts) Compute a, b, c and k.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$\Delta(x^3) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$$

$$\Delta y = 9x^2(\Delta x) + 9x(\Delta x)^2 + 3(\Delta x)^3 - 5(\Delta x)$$

$$a = 9 \qquad b = 9 \qquad c = 3 \qquad k = -5$$

b. (5 pts) Assuming $\triangle x \neq 0$, compute $\underbrace{\dfrac{\triangle y}{\triangle x}}$.

$\| \; \Delta x \neq 0$

$$9x^2 + 9x(\Delta x) + 3(\Delta x)^2 - 5$$

c. (5 pts) Compute $\underbrace{\lim_{\triangle x \to 0} \dfrac{\triangle y}{\triangle x}}$.

$\|$

$$9x^2 + 0 + 0 - 5 = 9x^2 - 5$$