CALCULUS

Problems involving horizontal asymptotes

OLD2

WARNING: In this homework, do NOT use l'Hôpital's rule. It has not been covered yet.

0250-1. Using the graph below of f,

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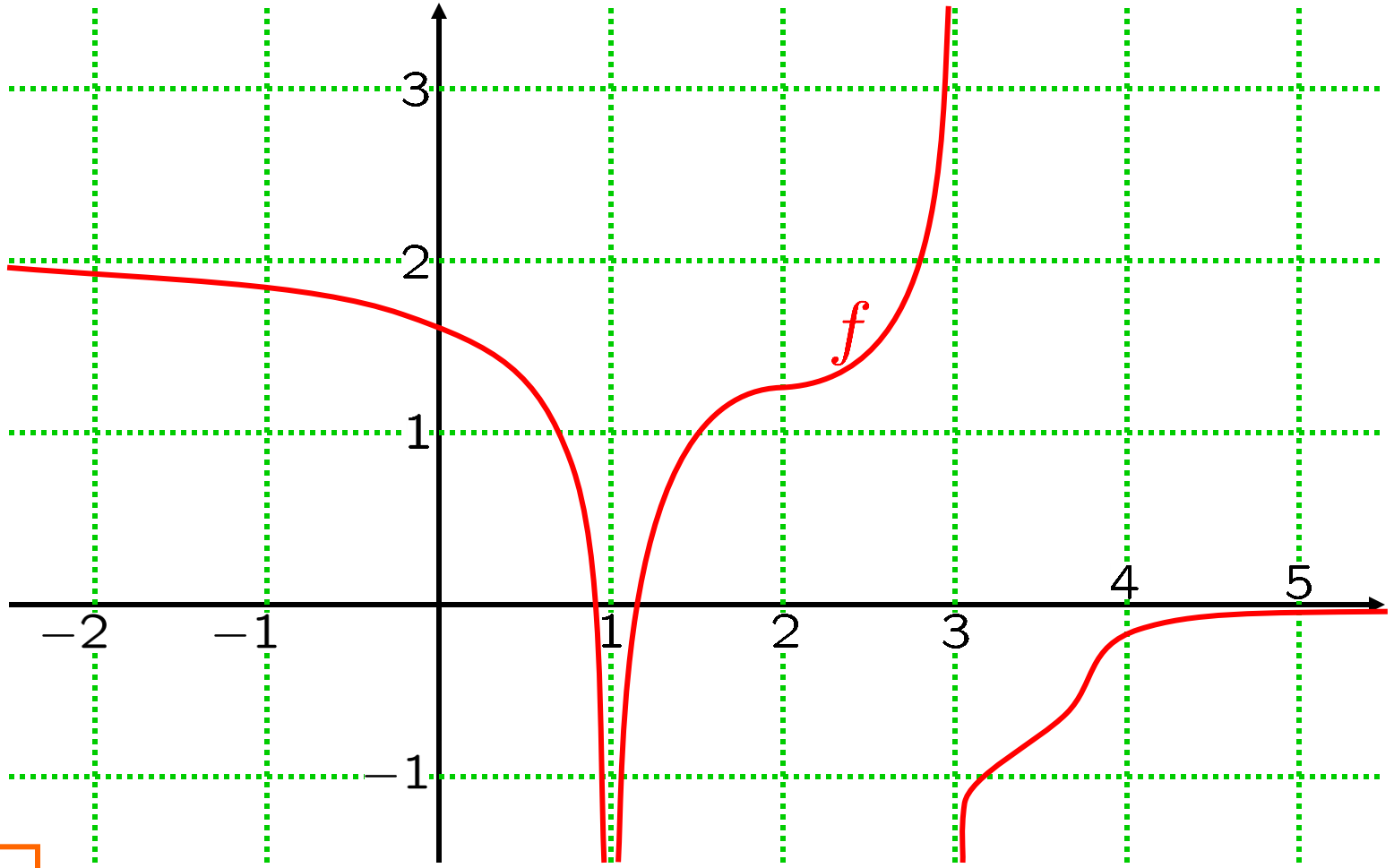
find these limits: a. $\lim_{x \to -\infty} f(x)$ b. $\lim_{x \to 1} f(x)$

c. $\lim_{x \to 3^-} f(x)$ d. $\lim_{x \to 3^+} f(x)$ e. $\lim_{x \to \infty} f(x)$



0250-2.

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Sketch a graph of a function $h : \mathbb{R} \to \mathbb{R}$ s.t.

h is even, *i.e.*, $h(-x) = h(x)$,

$\lim_{x \to -\infty} h(x) = 1$, $\lim_{x \to -3} h(x) = \infty$,

$\lim_{x \to -2^-} h(x) = 3 = h(-2)$, $\lim_{x \to -2^+} h(x) = -\infty$,

$\lim_{x \to -1} h(x) = \infty$ and $h(0) = 2$.

0250-3.

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Sketch a graph of a function $f : \mathbb{R} \to \mathbb{R}$ s.t.

f is odd, *i.e.*, $f(-x) = -(f(x))$,

$\lim_{x \to -\infty} f(x) = 1$, $\lim_{x \to -3} f(x) = \infty$,

$\lim_{x \to -2^-} f(x) = 3 = f(-2)$, $\lim_{x \to -2^+} f(x) = -\infty$,

and $\lim_{x \to -1} f(x) = \infty$.

0250-4. Compute

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a. $\lim_{x\to\infty} \dfrac{x^3+2x-1}{-7x^2+5}$

Do NOT use l'Hôpital's rule.

b. $\lim_{x\to\infty} \dfrac{x^3+1000000}{x^5-x^4+1}$

Do NOT use l'Hôpital's rule.

c. $\lim_{x\to\infty} \dfrac{x^5-x^4+1}{x^3+1000000}$

Do NOT use l'Hôpital's rule.

d. $\lim_{x\to\infty} \dfrac{8x^3+1}{4x^3+1000000}$

Do NOT use l'Hôpital's rule.

0250-5. Compute

OLD2

a. $\lim_{x \to -\infty} \dfrac{x^3 + 2x - 1}{-7x^2 + 5}$ Do NOT use l'Hôpital's rule.

b. $\lim_{x \to -\infty} \dfrac{x^3 + 1000000}{x^5 - x^4 + 1}$ Do NOT use l'Hôpital's rule.

c. $\lim_{x \to -\infty} \dfrac{x^5 - x^4 + 1}{x^3 + 1000000}$ Do NOT use l'Hôpital's rule.

d. $\lim_{x \to -\infty} \dfrac{8x^3 + 1}{4x^3 + 1000000}$ Do NOT use l'Hôpital's rule.

NOTE: These are limits at $-\infty$; in the last problem, they were at ∞.

0250-6. Compute

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a. $\lim_{x \to -\infty} \sqrt[3]{\dfrac{16x^3 + 1}{2x^3 + 1000000}}$

Do NOT use l'Hôpital's rule.

b. $\lim_{x \to \infty} \sqrt[4]{\dfrac{81x^4 - x^3 + 1}{2 + 5x + x^4}}$

Do NOT use l'Hôpital's rule.

c. $\lim_{x \to -\infty} \dfrac{\sqrt[3]{27x^6 + 5x + 1}}{x^2 - 4x + 5000000}$

Do NOT use l'Hôpital's rule.

d. $\lim_{x \to -\infty} \left(\sqrt{x^2 + 8x + 4} - \sqrt{x^2 + 7x - 3} \right)$

Do NOT use l'Hôpital's rule.

0250-7. Find the (maximal) intervals where $x(x-1)^2(x-2)^3$ is positive and negative,

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then compute $\lim_{x \to -\infty} x(x-1)^2(x-2)^3$,

then compute $\lim_{x \to \infty} x(x-1)^2(x-2)^3$.

0250-8. Suppose, $\forall x > 100$, that

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$$\frac{4x^3 - 2x^2 - 8}{x^3 + 5x + 7} < f(x) < \frac{32x^4 - 16x^3 - 48x}{8x^4 + 10x + 30}.$$

Compute $\lim_{x \to \infty} f(x)$.