

# CALCULUS

## Derivatives of exponential functions

### OLD2

0330-1. Differentiate  $f(x) = e^{-4/7}$ .

OLD2

0330-2. Differentiate  $f(x) = x^{-4/7}$ .

OLD2

0330-3. Differentiate  $f(x) = \frac{2x^3 - 5x^2 + \pi}{\sqrt[5]{x}}$ .

OLD2

0330-4. Differentiate  $f(x) = 2\sqrt[3]{x^2} + 8e^x$ .

OLD2

0330-5. Let  $C$  be the graph of

OLD2

$$y = 6e^x + 2x^3 - x^{9/7} + 3,$$

and let  $p = (0, 9) \in C$ .

- Find the slope of the tangent line to  $C$  at  $p$ .
- Find an equation of the tangent line to  $C$  at  $p$ .
- Find the slope of the normal line to  $C$  at  $p$ .
- Find an equation of the normal line to  $C$  at  $p$ .

**NOTE:** **Normal** means  
perpendicular to the tangent.

0330-6. The position of a particle at time  $t$   
is given by  $p(t) = 3e^t - 6t^2 - t + 9$ .

a. Find a formula for its velocity

$$v(t) = p'(t) \text{ at time } t.$$

b. Find a formula for its acceleration

$$a(t) = v'(t) = p''(t) \text{ at time } t.$$

0330-7. Find all points on the graph of

OLD2

$$y = 2x^3 + 15x^2 + 36x - 5$$

where the tangent line is horizontal.

(To specify a point, you must give

*both* its  $x$ -coordinate *and* its  $y$ -coordinate.)

0330-8. Find a linear polynomial  $P$

OLD2

s.t.  $P(-2) = 3$  and s.t.  $P'(-2) = 5$ .

0330-9. Find a quadratic polynomial  $Q$

OLD2

s.t.  $Q(0) = 6$ , s.t.  $Q'(0) = 7$  and s.t.  $Q''(0) = 8$ .

0330-10. A population of microbes has growth rate always proportional to its size.  
OLD2

(This implies that the size of the population at time  $t$  is given by  $Ce^{kt}$ , for unknown constants  $C$  and  $k$ .)

Assume that this size, at time  $t = 0$ , is 150.

Assume that this size, at time  $t = 3$ , is 600.

- a. Find  $C$  and  $k$ .
- b. Find the size of the population at time  $t = 9$ .

0330-11. According to one set of estimates,  
OLD2 the world population

in the year 2000 was 6,070,000,000,  
and in the year 2008 was 6,707,000,000.

Use an exponential model, in which the  
population in the year  $2000 + t$  is  $Ce^{kt}$ .

- a. Find  $C$  and  $k$  to fit the data given above.
- b. Predict the world population in 2020.



0330-12. Many radioactive isotopes exhibit exponential decay, which means that the amount of the isotope is given by  $Ce^{-kt}$  for constants  $C$  and  $k$ , with  $k > 0$ . In particular, this is true of carbon-14. Carbon-14 has a half-life of 5730 years, which means that  $e^{-k \cdot 5730} = 1/2$ .

a. Find  $k$ .

Animals and plants replace their carbon (including carbon-14) all through their lives, but when they die, that replacement stops and their amount of carbon-14 decays exponentially.

b. An animal body is discovered with 65% of the carbon-14 found in a living animal of the same size and species.

Estimate how many years ago it died.

0330-13. The temperature of our Thanksgiving turkey is  $325^{\circ}\text{F}$ , as it's brought out of the oven. The temperature decays exponentially, toward the room temperature of  $72^{\circ}\text{F}$ .

That is, if  $f(t)$  denotes the temp (in  $^{\circ}\text{F}$ ) of the turkey  $t$  hours after it's removed from the oven, then  $\exists$  constants  $C \in \mathbb{R}$  and  $k > 0$  s.t.

$$f(t) = 72 + Ce^{-kt}.$$

Suppose the temperature of the turkey is  $150^{\circ}\text{F}$  45 minutes after it's removed from the oven.

- a. Find  $C$  and  $k$ .
- b. When will the temperature of the turkey reach  $100^{\circ}\text{F}$ ?

0330-14. We invest 240 dollars in a bank.  
OLD2 Say the account earns 5% nominally.

- a. How much will we have in the bank after one year, if the compounding occurs
- i. once per year?
  - ii. twice per year?
  - iii. four times per year?
  - iv. 365 times per year?
  - v. continuously?
- b. How much will we have in the bank after  $t$  years, assuming continuous compounding?