CALCULUS

Definite integration and Riemann sum problems

OLD2

0590-1. Let $f(x) = 5 - 5x^2$.

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a. Compute $L_3S_{-1}^1 f$.
Sketch f over $[-1, 1]$ and add,
into your sketch, the three
rectangles represented by $L_3S_{-1}^1 f$.

b. Compute $M_3S_{-1}^1 f$.
Sketch f over $[-1, 1]$ and add,
into your sketch, the three
rectangles represented by $M_3S_{-1}^1 f$.

c. Compute $R_3S_{-1}^1 f$.
Sketch f over $[-1, 1]$ and add,
into your sketch, the three
rectangles represented by $R_3S_{-1}^1 f$.

0590-2. Let $f(x) = e^x + 4$.
OLD2

a. Compute $L_3 S_0^3 f$ to three decimal places.

b. Compute $M_3 S_0^3 f$ to three decimal places.

c. Compute $R_3 S_0^3 f$ to three decimal places.

0590-3. Let $f(x) = \sin^2 x$.
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a. Compute $L_4 S_0^\pi f$ to three decimal places.

b. Compute $M_4 S_0^\pi f$ to three decimal places.

c. Compute $R_4 S_0^\pi f$ to three decimal places.

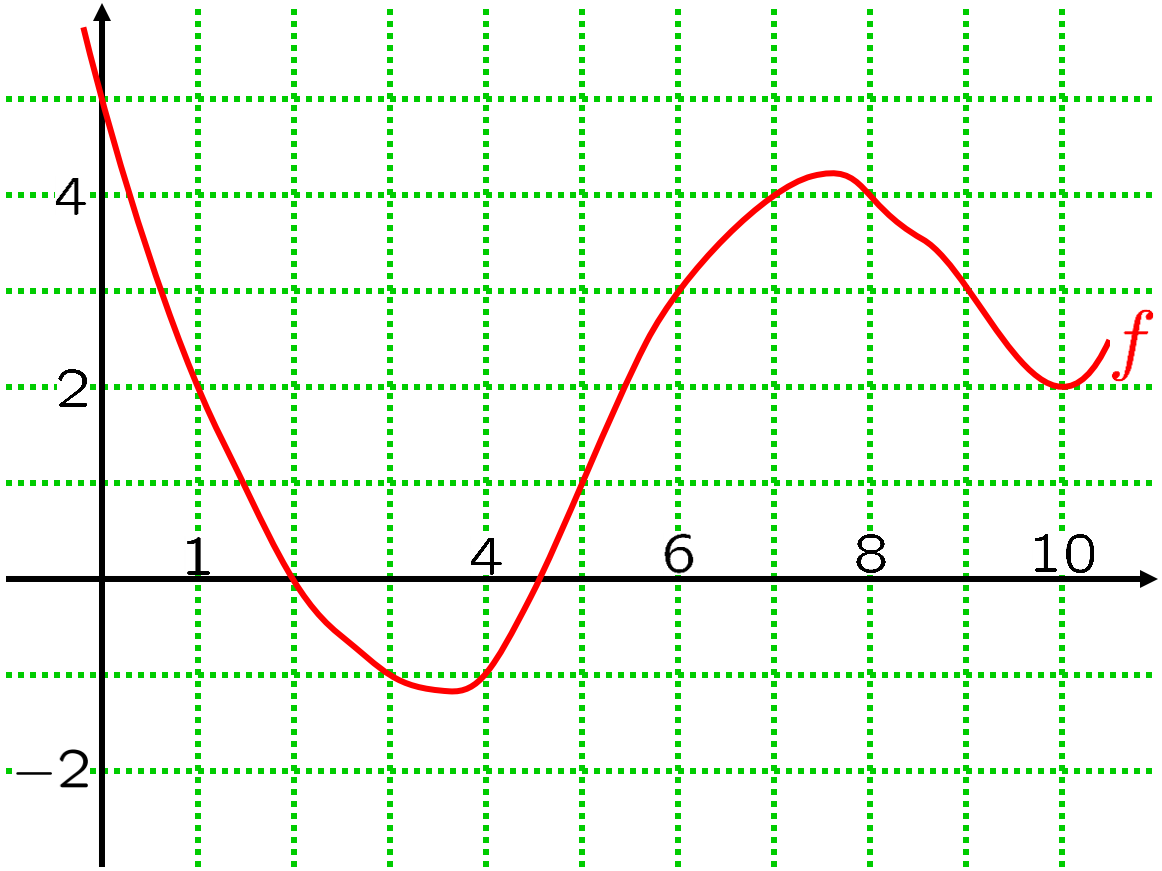
0590-4. OLD2 A car's acceleration is positive from time 0 to time 18 seconds, and its velocity at various times is given in the table below.

time (secs)	0	3	6	9	12	15	18
velocity (ft/sec)	0	30	50	65	75	80	82

Find upper and lower estimates for the distance traveled by the car over these 18 seconds.

0590-5. The gph of a function f appears below.

OLD2



Estimate $\int_0^{10} f(x)\,dx$ by computing

(a) $L_5 S_0^{10} f$, (b) $M_5 S_0^{10} f$

and (c) $R_5 S_0^{10} f$.

0590-6. Express the area under $y = e^{-x^2/2}$
OLD2
from $x = -1$ to $x = 3$ as a limit of midpoint Riemann sums. (Don't evaluate the limit.)

0590-7. Express the area under $y = \sqrt{x^3 + x + 9}$
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from $x = 2$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don't evaluate the limit.)

0590-8. Express the area under $y = \cos\left(x^3 + x\right)$
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from $x = -3$ to $x = 5$ as a limit of right endpt Riemann sums. (Don't evaluate the limit.)

0590-9. Express $\displaystyle\int_4^6 \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$ as a limit of midpoint Riemann sums. (Don't evaluate the limit.)

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0590-10. Let $f(x) = x^3 - x$.

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a. Write $R_n S_0^2 f$ as a rational expression in n (*i.e.*, as one polynomial in n divided by another).

b. Compute $\lim_{n\to\infty} R_n S_0^2 f$.

0590-11. The limit

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$$\lim_{n\to\infty}\left[\frac{9}{n}\sum_{j=0}^{n-1}\left(e^{-\cos(7+j(9/n))}\right)\right]$$

represents the area under $y = f(x)$ from $x = a$ to $x = b$, for some choice of $f(x)$, a and b.

a. Find $f(x)$, a and b.

b. Express the limit as a definite integral.

0590-12. The limit

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$$\lim_{n\to\infty}\left[\frac{5}{n}\sum_{j=1}^{n}\left(\sin\left(e^{-2+j(5/n)}\right)\right)\right]$$

represents the area under $y = f(x)$ from $x = a$ to $x = b$, for some choice of $f(x)$, a and b.

a. Find $f(x)$, a and b.

b. Express the limit as a definite integral.

0590-13. Let $f(x) = 2 + \sqrt{9 - x^2}$.

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a. Sketch the graph of $y = f(x)$.

b. Compute $\int_{-3}^{3} f(x)\,dx$, by interpreting this integral as an area.