CALCULUS
Derivatives and rates of change
NEW

WARNING: In this homework, derivatives must be computed from the defintion, *i.e.*, as the limit of the difference quotient. Do NOT use product, quotient or chain rules, or any other technique coming from a later topic.

Let C be the curve $y = -x^2 + 2x + 3$.

Let L be the tangent line to C

at the point $(0, 3)$.

a. Find the slope of L, by computing a limit of slopes of secant lines.

b. Find an equation of L.

c. Graph C and L in the rectangle
$$-2 \leq x \leq 4, \qquad -2 \leq y \leq 5.$$

d. Graph C and L in the rectangle
$$-1 \leq x \leq 1, \qquad 1 \leq y \leq 5.$$

e. Graph C and L in the rectangle
$$-0.1 \leq x \leq 0.1, \qquad 2.8 \leq y \leq 3.2.$$

In c, d and e, note that, as you "zoom in", the tangent line looks more and more like the curve.

0270-2. a. Compute $\lim\limits_{h \to 0} \dfrac{\sqrt{4+3h}-2}{h}$.

    b. Find the slope of the secant line to $y = \sqrt{3x+1}$ through the points $(1,2)$ and $(1+h, \sqrt{4+3h})$.

    c. Find an equation of the tangent line to $y = \sqrt{3x+1}$ at the point $(1,2)$.

0270-3. A particle moves on a number line. Its position at any time t is $\sqrt{3t+1}$.

    a. Find the average velocity between time $t = 1$ and time $t = 1+h$.

    b. Find the instantaneous velocity at time $t = 1$.

A heavy object is taken to the top of a building 200 feet high. At time $t = 0$, it is thrown upward at 10 feet/second. We engage the services of two Nobel prize-winning physicists who confer (*i.e.*, yell and scream at one another). After several hours of scholarly study, followed by minor medical treatment for blunt trauma, lacerations and contusions, they hold a joint press conference, and inform their public that, t seconds after release, the object will be located

$$200 + 10t - 16t^2 \quad \text{feet}$$

above the ground. Based on this, find the the velocity of the object 0.4 seconds after release. Give your answer in feet per second.

4
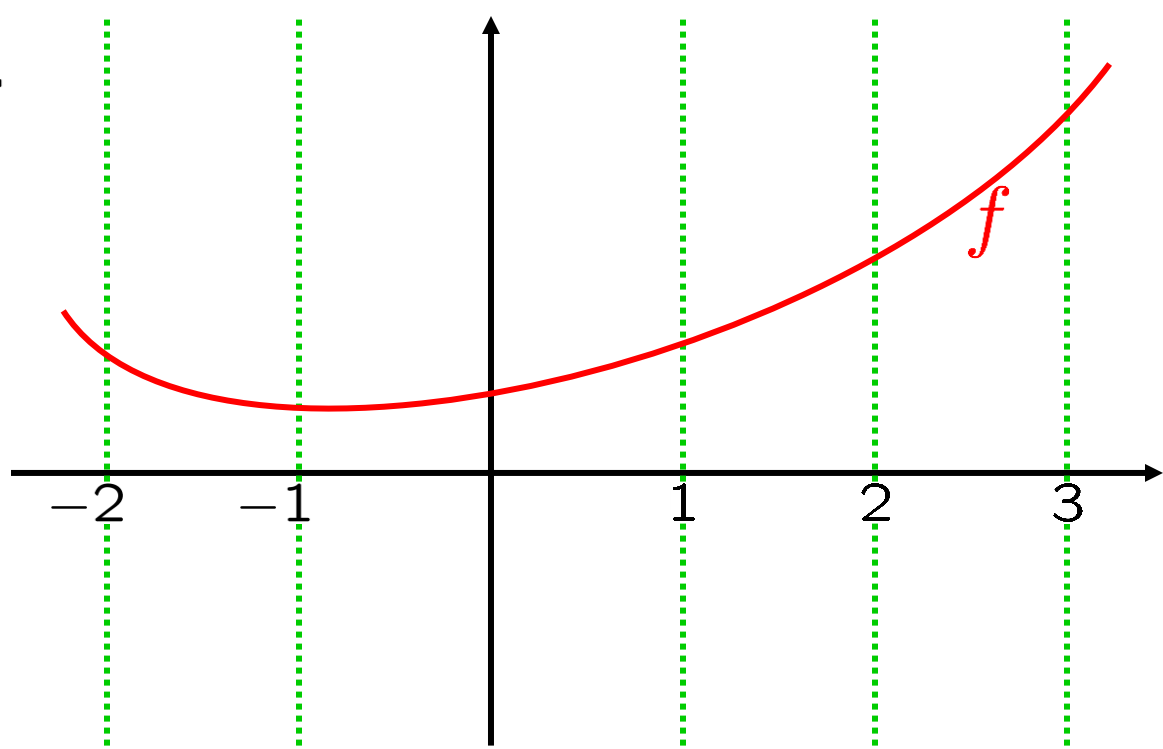
Order these numbers, from smallest to largest:
$$f'(-2), \; f'(-1), \; f'(0), \; f'(1), \; f'(2), \; f'(3)$$
Note that we are asking about f', not f.

NEW Let $f(x) = \dfrac{7x+5}{2x+1}$.

Do NOT use the quotient rule. Use only the definition of the derivative as the limit of the difference quotient.

a. Compute $f'(2)$.

b. Compute $f'(3)$.

c. Compute $f'(4)$.

d. Compute $f'(a)$, for an arbitrary number a.

6

Find a function f and a number a s.t.

$$f'(a) = \lim_{h \to 0} \frac{[\ln(3+h)] - [\ln(3)]}{h}.$$